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Spare parts management under double demand uncertainty

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Abstract

Introducing capital goods brings a lot of uncertainties a company has to deal with. One of these uncertainties is managing the spare parts inventory required for the maintenance. When a capital good has just been introduced there is a large amount of uncertainty related to the demand rates. In this paper, we consider a multi-item inventory problem with a backorder constraint where the distribution of demand is known but the demand rate is uncertain. We show that when demand follows a Poisson distribution, a larger variance of the demand rate always results in higher costs. Next, we show that one may require over four times the initial holding costs investment for certain scenarios in order to deal with the additional demand uncertainty. Moreover, we show that if one would ignore the additional uncertainty, the solutions are in general far from feasible and thus leading to poor solutions.

Keywords: Inventory control, SKUs, system-oriented service constraints, heuristics

1. Introduction

Whenever an OEM introduces new capital goods there are many struggles to overcome when starting to use these capital goods in the field. Especially regarding the maintenance of capital goods there is a lot of uncertainty. Due to the high costs of the capital goods in general it is common to have very high service level targets in place, or strict agreements with customers using the capital goods regarding the time the capital goods should be running. Even when capital goods are being used for multiple years, spare parts fail randomly, but the rate at which the spare parts fail are better predictable. This allows for good decision making on the amount of stock to put on hand, despite the uncertainty that already exists in the distribution of the demand itself. For a new product introduction this is much more difficult since the demand rates are highly uncertain. These estimates of the demand rates are commonly based upon expert knowledge and/or limited qualitative research. As a result of this uncertainty it is not wise to use the expectation of the demand rate to make decisions as this may lead to results at which the downtime or number of backorders are too high, which would lead to additional costs and unsatisfied customers. Especially

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during the introduction of capital goods, it is bad advertisement if the downtime for customers are too long. Therefore, it is wise to take this uncertainty into consideration from the start. An example of this problem is when a company introduces a new lithography machine. Based on past information regarding previous similar machines, they know that demand would behave randomly and most likely according to follow a Poisson process. However, the rate at which these demands will take place are highly uncertain during the time of the introduction. If the uncertainty of the demand rate itself could also be modeled explicitly, better stocking decisions can be made. In this paper we do exactly that, we model the uncertainty about the actual demand rate besides having the uncertainty from the demand distribution only. We consider a single-location multi-item inventory problem with a constraint on the expected number of backorders, which is similar to having a service agreement. Demand follows a Poisson but we do not know the demand rate, we model the demand rate to follow a known and given distribution instead. Because the capital good has not been used in the field before, there usually is no statistical evidence about the demand rate. This makes it difficult to apply statistical methods such as a Bayesian approach to predict the demand rate (see e.g. Kamath and Pakkala, 1999a; Hayes, 1969). Another option in the case of limited data is presented by Akcay et al. (2011). These methods require at least some historical data, however, we do not have any historical data at all and rely on the expert knowledge who provide us with distributions of the demand rates. However, not for every component the amount of uncertainty is similar thus this distribution may also vary as for some parts the variance of this distribution is larger than for other parts as for some parts the failure rates are better predictable than others. Simply replacing the demand rate with the estimate of the parameter is not the best option here as we show that this leads to poor solutions. Hence, we model the uncertainty of the demand rate instead of taking a point estimate. One of the questions we would like to answer is whether a better predictability of the demand rate, hence a less variable demand rate, will always lead to a better performance in terms of costs when the expected demand rate is the same.

The impact of the variability of the demand rate has similarities with other papers in the process improvement sector (see e.g Hall, 1983; Graham, 1982; Shingo, 1989; Sarkar and Zangwill, 1991). These papers among others explore the effect of setup reductions and/or variability eliminations, which are related to the effect of reducing the variability of the demand rate. However, none of these papers consider the uncertainty of the demand rate itself.

There are many papers that study demand uncertainty, and in particular unknown demand and/or parameters, in the case of holding and backorering costs. Gerchak and Mossman (1991) analyze the effect of demand randomness on inventories and costs in relation to pooling. They consider a single period, single item newsvendor problem to decide upon the optimal order quantity and show that only after a certain threshold value the optimal order quantity increases but it may also decrease in some cases. For the same problem, Ridder et al. (1998) further show that larger demand variability may even lead to lower costs for particular cases of the newsvendor problem under some special conditions. These papers do not consider possible additional uncertainty of the demand rate during an introduction but only the uncertainty of the demand distribution itself. Other papers that study the variability of the demand distribution itself and the impact on inventory are Jemai and Karaesmen (2005); Xu et al. (2010).

As we are interested in the impact of uncertainty of the demand rate on the demand during the lead time, which we need for setting the optimal base stock levels, our problem is related to the problem where the impact of uncertainty in the lead time on the demand during the lead time is analyzed. Feeney and Sherbrooke (1966) consider a Poisson demand process but assume i.i.d leadtimes (as they assume an exogenous parallel supply process). This means that orders that are placed later may be fulfilled earlier. They show that the systems performance of any base stock policy only depends on the mean value in this case. For the inventory model with holding and backorder costs, Bagchi et al. (1986) also investigate the impact of leadtime variability on stockouts and stockout risk in the case that lead times are i.i.d random variables. However, we do not assume i.i.d. lead times thus results may be different in the case of sequential deliveries. See Svoronos and Zipkin (1991), and Zipkin (1991) for an extensive discussion about the difference between sequential and parallel supply processes.

Song (1994) investigate the effect of leadtime uncertainty when having sequential deliveries instead. They assume demand follows a (compound) Poisson distribution and first analyze the impact of a more variable lead time in the case of different average lead times. Next to this, they consider the case where the average is the same but one lead time is more variable than the other. They show that when the expected lead time is the same, a more variable lead time will lead to a more variable lead time demand and thus higher costs for any base stock level as well as for the optimal base stock levels. In this paper we show that in the case of Poisson demand, uncertainty in lead time has an identical effect as uncertainty of the demand rate in the case of sequential deliveries. However, we consider a multi-item problem with a backorder constraint instead of penalty costs. There are also many papers that consider inventory models with uncertain leadtimes under the assumption of sequential deliveries (see e.g Song et al., 2010; Zipkin, 1986; Kamath and Pakkala, 1999b; Nahmias, 1979; Ehrhardt, 1984; Chen and Yu, 2005).

Our main contribution of this paper is that we show for a multi item problem that more variability of the demand rate parameter always leads to higher costs when minimizing inventory holding costs subject to an expected mean number of backorders constraint. Secondly, we show the equivalence between lead time uncertainty and uncertainty of the demand rate parameter. When demand follows a Poisson distribution, both are equal in terms of impact on the lead time demand distribution. Moreover, we show numerically that if the uncertainty of the demand rate follows a Gamma distribution and each SKU has the same squared coefficient of variation of the demand rate uncertainty, the relative holding costs increase linear. Moreover, when demand rates are larger, the impact of the additional demand uncertainty is much bigger, whereas if the lead time is longer, the relative impact of the additional uncertainty is smaller. Finally, we find that if a company would decide to ignore the additional uncertainty of the demand rate, and simply take the expectation, this leads to poor solutions.

In Section 2 we give the description of our model and the corresponding problem that we want to solve. Then, in Section 3 we show the equivalence between demand rate uncertainty and lead time uncertainty, and show that higher variability always result in higher costs. In Section 4 we present our numerical results, where we first show how costs increase in terms of the additional uncertainty for different scenarios. We also show the impact of ignoring the demand rate uncertainty on the solution obtained. Finally, we give our conclusions in Section 5.

2. Model description

In this section, we introduce three models we use throughout the remainder of this paper, the *basic model*, *uncertain demand rate model*, and the *uncertain lead time model*. For the *basic model* we consider a single warehouse where several stock keeping units (SKUs) are kept on stock. The set of SKUs is denoted by I, and the number of SKUs is denoted by $|I|$. For convenience, the SKU's are numbered $i = 1, 2, \dots |I|$. For each SKU $i \in I$, demand occurs according to a Poisson process with rate m_i. The mean procurement or repair leadtime for SKU i is denoted by $t_i(> 0)$ are known and given. Demand is fulfilled immediately if possible and otherwise backordered and fulfilled as soon as possible. The inventory is managed by a continuous review base stock policy with base stock level S_i. The price of a part is denoted by $c_i(> 0)$.

Let us denote the demand during the leadtime by X_i which is Poisson distributed with mean

$m_i t_i$. Notice that X_i is integer valued.

The distribution of stock on hand OH and the number of backordered demands BO is then given by:

$$P\left\{OH_i = x\right\} = \begin{cases} \sum_{y=S}^{\infty} P\left\{X_i = y\right\} & \text{if } x = 0; \\ P\left\{X_i = S - x\right\} & \text{if } x \in \mathbb{N}, x \leq S; \end{cases}$$

$$P\left\{BO_i = x\right\} = \begin{cases} \sum_{y=0}^{S} P\left\{X_i = y\right\} & \text{if } x = 0 \\ P\left\{X_i = x + S\right\} & \text{if } x \in \mathbb{N}. \end{cases}$$

From these we can easily obtain the expected number of backorder $EBO(S)$:

$$EBO_i(S_i) = \sum_{x=S_i+1}^{\infty} (x - S) P\left\{X_i = x\right\} = m_i t_i - S_i + \sum_{x=0}^{S_i} (S_i - x) P\left\{X_i = x\right\}, S_i \in \mathbb{N}_0 \qquad (1)$$

Then the average inventory holding costs for item i under basestock level S_i are

$$C_i(S_i) = E[c_i(S - X_i)^+] = c_i \sum_{x=0}^{S_i} (S_i - x) P\{X_i = x\}. \qquad (2)$$

Let $C(\mathbf{S})$ be the total costs over all SKUs, which is the sum of all individual costs where $\mathbf{S} = (S_1, \ldots, S_{|I|})$ denotes a vector consisting of all basestock levels.

We are interested in the aggregate mean number of backorders which is denoted by $EBO(\mathbf{S})$. The aggregate mean number of backorders in steady state is:

$$EBO(\mathbf{S}) = \sum_{i \in I} EBO_i(S_i) \qquad (3)$$

Hence in mathematical terms our optimization problem is as follows:

$$\begin{array}{lll} \text{(A)} & \min & C(\mathbf{S}) \\ & \text{subject to} & EBO(\mathbf{S}) \leq EBO^{obj} \\ & & S_i \in \mathbb{N}_0, \forall\ i \in I \end{array}$$

where EBO^{obj} is our maximum level for the mean expected number of backorders.

Our second model, the *uncertain demand rate model* is similar to the *basic model* except we do not know our demand rate for certain. We still have the same expectation of the demand rate, m_i, but its exact value can be lower or higher with a certain probability. In other words, in our second model the demand rate m_i is multiplied by y, which is a realization of the random variable $Y \geq 0$ with distribution function G and mean value 1. In the case of the uncertain demand rate model, the leadtime demand distribution for SKU i, X_i^{ud}, follows a Poisson distribution with unknown mean ym_it_i and is calculated as follows:

$$P\left\{X_i^{ud}=x\right\}=\int_0^\infty \frac{(um_it_i)^x}{x!}e^{-um_it_i}f_g(u)du, x\in\mathbb{N}_0 \tag{4}$$

where $f_g(x)$ represent the probability density function of the random variable Y. Let $EBO_i^{ud}(S_i)$, and $C_i^{ud}(S_i)$ respectively denote the expected number of backorders for the uncertain demand rate model and costs, which are similar to equation 1 and 2 except that we replace X_i by X_i^{ud}.

Our third model, the *uncertain lead time model* is similar to the *basic model* except that we have uncertainty in the lead time. The lead time t_i is multiplied by z which is a realization of the random variable $Z\geq 0$ with distribution function W and mean value 1. In the case of the uncertain lead time model, the leadtime demand distribution for SKU i, X_i^{ul}, follows a Poisson distribution with unknown mean zm_it_i and is calculated as follows:

$$P\left\{X_i^{ul}=x\right\}=\int_0^\infty \frac{(m_ivt_i)^x}{x!}e^{-m_ivt_i}f_w(v)dv, x\in\mathbb{N}_0 \tag{5}$$

where $f_w(x)$ represent the probability density function of the random variable Z. Let $C_i^{ul}(S_i)$, $EBO_i^{ul}(S_i)$ respectively denote the expected number of backorders and the costs for the uncertain lead time model which are calculated as in equation 1 and 2 except that X_i is now replaced by X_i^{ul}.

3. Impact of a larger variance of the demand rate parameters

In this section we proof that both the costs as well as the expected number of backorders are increasing when the variance of the demand rate is increasing. A first key observation we need to make is that the *uncertain demand rate model* has an identical leadtime demand distribution as the *uncertain lead time model.*

Proposition 1. *Suppose* $G\overset{d}{=}W$, *then* $X_i^{ud}\overset{d}{=}X_i^{ul}$ *for all* $i\in I$

Proof.

$$P\left\{X_i^{ud}=x\right\}=\int_0^\infty \frac{(um_it_i)^x}{x!}e^{-um_it_i}f_g(u)du=\int_0^\infty \frac{(m_iut_i)^x}{x!}e^{-m_iut_i}f_w(u)du=P\left\{X_i^{ul}=x\right\}, x\in\mathbb{N}_0$$

□

Let us now introduce our definition of variability. Let $u(t)$ be a real function defined on an ordered set U of the real line. Let $H(u)$ be the number of sign changes of $u(t)$. Graphically, this is the number of time $u(t)$ crosses the t-axis when t ranges over the entire set U. More rigorously we have

$$H(u) = sup\ H[u(t_1), u(t_2), \ldots, u(t_k)],$$

where the supremum is extended over all sets $t_1 < t_2 < \cdots < t_k$ ($t_i \in U$), k is arbitrary but finite, and $\mathrm{H}(x_1, x_2, \ldots, x_k)$ is the number of sign changes of the sequence $x_1, \ldots, x_k$, zero terms being discarded.

Definition 3.1. Consider two random variables V_1 and V_2 with the same mean $E[V_1] = E[V_2]$, having distributions R_1 and R_2 and densities r_1 and r_2. Suppose V_1 and V_2 are either both continuous or both discrete. We say V_1 is more variable than V_2, denoted $V_1 \geq_{\mathrm{var}} V_2$, if

$$H(r_1 - r_2) = 2 \ \text{ with sign sequence } +, -, +.$$

That is, r_1 crosses r_2 exactly twice, first from above and then from below.

Examples under which Definition 3.1 holds are the Uniform, Gamma, and Normal distribution. See Song (1994) for more examples of distributions for which Definition 3.1 holds. Let us now introduce the definition of increasing convexity.

Definition 3.2. We say that a random variable V_1 is stochastically larger than a random variable V_2, denoted by $V_1 \geq_{\mathrm{ic}} V_2$, if

$$Pr\{V_1 \geq x\} \geq Pr\{V_2 \geq x\} \ \text{ for all } x$$

This is equivalent to

$$E[f(V_1)] \geq E[f(V_2)]$$

for all nondecreasing functions f

For the following propositions, we introduce two systems $j = 1, 2$ where each system is equal in number and characteristics of the SKUs except from the uncertainty of the demand rate. Let Y_j, Z_j denote the random variable for system j for the uncertain demand model and the uncertain lead time model respectively. Let $X_{i,j}^{ud}$ and $X_{i,j}^{ul}$ be the leadtime demand distribution of the uncertain demand model and the uncertain lead time model for SKU i under system j.

Proposition 2. *Suppose we have $Y_1 \geq_{var} Y_2$ and $E[Y_1] = E[Y_2]$ then $X_{i,1}^{ud} \geq_{ic} X_{i,2}^{ud}$ and $E[X_{i,1}^{ud} = E[X_{i,2}^{ud}]$.*

Proof. Let $Z_1 \overset{d}{=} Y_1$ and $Z_2 \overset{d}{=} Y_2$. For the uncertain lead time model we know that, by making use of Proposition 4.10 of Song (1994), $Z_1 \geq_{\mathrm{var}} Z_2$ implies

$$X_{i,1}^{ul} \geq_{\mathrm{ic}} X_{i,2}^{ul} \text{ and } E[X_{i,1}^{ul}] = E[X_{i,2}^{ul}] \ \forall\ i \in I.$$

Making use of Proposition 1 we are then able to show that

$$X_{i,1}^{ud} \stackrel{d}{=} X_{i,1}^{ul} \geq_{\text{ic}} X_{i,2}^{ul} \stackrel{d}{=} X_{i,2}^{ud} \ \forall \ i \in I$$

$$E[X_{i,1}^{ud}] = E[X_{i,1}^{ul}] = E[X_{i,2}^{ul}] = E[X_{i,2}^{ud}] \ \forall \ i \in I$$

□

Using this result we proof that both the costs as well as the expected number of backorders are increasing when the demand rate is more variable. First let us show that the costs are increasing if the leadtime demand distribution is more increasingly convex.

Proposition 3. *Let* $X_{i,1}^{ud} \geq_{ic} X_{i,2}^{ud}$ *and* $E[X_{i,1}^{ud}] = E[X_{i,2}^{ud}]$*, then*

$$C_{i,1}^{ud}(\boldsymbol{S}) \geq C_{i,2}^{ud}(\boldsymbol{S}) \text{ for all } i \in I$$

Proof. Observe that for each $i \in I$

$$R(S_i, u) = c_i[S_i - u]^+$$

is convex in u. Then, by definition 3.2, for each fixed $\mathbf{S}$,

$$C_{i,1}^{ud}(\mathbf{S}) = E[R(S_i, X_{i,1}^{ud})] \geq E[R(S_i, X_{i,2}^{ud})] = C_{i,2}^{ud}(\mathbf{S}), \ i \in \ I$$

□

Using this result we further extend our proof by showing that the mean expected number of backorders is also increasing in terms of increasing variance.

Proposition 4. *Let* $X_{i,1}^{ud} \geq_{ic} X_{i,2}^{ud}$ *and* $E[X_{i,1}^{ud}] = E[X_{i,2}^{ud}]$*, then*

$$EBO_1^{ud}(\boldsymbol{S}) \geq EBO_2^{ud}(\boldsymbol{S})$$

Proof.

$$EBO_1^{ud}(\mathbf{S}) - EBO_2^{ud}(\mathbf{S}) = \sum_{i \in I} \left\{ m_i t_i - S_i + \sum_{x=0}^{S_i - 1} (S_i - x) P\left\{X_{i,1}^{ud} = x\right\} - m_i t_i + S_i - \sum_{x=0}^{S_i - 1} (S_i - x) P\left\{X_{i,2}^{ud} = x\right\} \right\}$$

$$= \sum_{i \in I} \left\{ \sum_{x=0}^{S_i - 1} (S_i - x) P\{X_{i,1}^{ud} = x\} - \sum_{x=0}^{S_i - 1} (S_i - x) P\{X_{i,2}^{ud} = x\} \right\} = \sum_{i \in I} \frac{C_{i,1}^{ud}(\mathbf{S}) - C_{i,2}^{ud}(\mathbf{S})}{c_i} \geq 0$$

□

Let $\mathbf{S}_1^*$ and $\mathbf{S}_2^*$ be the optimal base stock levels that together satisfies the EBO constraint for the uncertain demand rate model for systems 1 and 2 respectively. By combining Proposition 3 and 4, it is clear that any feasible solution $\mathbf{S}_1^*$ is also feasible for the less variable system since $EBO_1(S_1^*) \geq EBO_2(S_1^*)$. Using Proposition 3 this implies $C_1(\mathbf{S}_1^*) \geq C_2(\mathbf{S}_2^*)$.

To summarize, we can combine Propositions 2, 3, 4 to show that:

Proposition 5. *Let $Y_{i,1}^{ud} \geq_{var} Y_{i,2}^{ud}$ and $E[Y_{i,1}^{ud}] = E[Y_{i,2}^{ud}]$ for all $i \in I$, then*

1. $X_{i,1}^{ud} \geq_{ic} X_{i,2}^{ud}$ *and* $E[X_{i,1}^{ud}] = E[X_{i,2}^{ud}]$
2. $C_1^{ud}(\boldsymbol{S}) \geq C_2^{ud}(\boldsymbol{S})$ *for all* $\boldsymbol{S}$
3. $EBO_1^{ud}(\boldsymbol{S}) \geq EBO_2^{ud}(\boldsymbol{S})$
4. $C_1^{ud}(\boldsymbol{S}_1^*) \geq C_2^{ud}(\boldsymbol{S}_2^*)$

4. Numerical experiments

In this section we present our numerical results to show the impact of the uncertainty of the demand rate. We first give two examples of distributions to model the demand rate and the resulting lead time demand distribution as a result of the double demand uncertainty. We then assume the demand rate is Gamma distributed and look at different scenarios to get an indication of the impact of the additional uncertainty. Finally, using the same scenarios we show the impact of ignoring the additional uncertainty.

4.1 Modeling uncertain demand rates

In this section we consider two distributions to model the uncertainty of the demand rate parameter and get expressions of the leadtime demand distribution. We first look at Uniformly distributed demand rates and then at Gamma distributed demand rates. Although we only consider these two distributions, any distribution that satisfies Definition 3.1 is possible.

Uniformly distributed demand rates

Suppose the demand rate is uniformly distributed on the interval $[1-\alpha_j, 1+\alpha_j)$ with $0 < \alpha_j \leq 1$, where $\alpha_{i,j}$ represent the amount of uncertainty for system j. Then one can show that (after some algebra)

$$P\{X_{i,j}^{ud} = x\} \quad = \quad \frac{1}{2\alpha_{i,j} m_i t} \left[E_{x+1,t}((1+\alpha_{i,j})m_i) - E_{x+1,t}((1-\alpha_{i,j})m_i)\right],$$

where $E_{k,\lambda}$ stands for the distribution function of an Erlang distribution with k phases and scale parameter λ, i.e.,

$$E_{k,\lambda}(x) = 1 - \sum_{j=0}^{k-1} \frac{(\lambda x)^j}{j!} e^{-\lambda x}, \quad x \geq 0.$$

The corresponding probability density function is given by

$$e_{k,\lambda}(x) = \frac{(\lambda^k x^{k-1})}{(k-1)!} e^{-\lambda x}, \quad x \geq 0.$$

Note that it holds that if $\alpha_{i,1} \geq \alpha_{i,2}$, then $Y_{i,1} \geq_{var} Y_{i,2}$.

Gamma distributed demand rates

Suppose the demand rate is gamma distributed with known mean m_i and variance $\sigma^2_{i,j}$. After some algebra it is possible to show that the demand during the lead time then has a Negative Binomial distribution with shape parameters r, and p, where $r = \frac{(tm_i)^2}{\sigma^2_{i,j}}$, and $p = \frac{1}{1+\frac{\sigma^2_{i,j}}{tm_i}}$. The probability density function of the Negative Binomial distribution is as follows:

$$P\{X = x\} = \frac{\Gamma(r+x)}{\Gamma(x+1)\Gamma(r)} p^r (1-p)^x I_{0,1,\ldots}(x)$$

Note that it holds that if $\sigma^2_{i,1} \geq \sigma^2_{i,2}$, then $Y_1 \geq_{var} Y_2$. After some rewriting it is also possible to obtain the variation of the lead time demand distribution, $\xi^2_{i,j}$:

$$\xi^2_{i,j} = tm_i + \sigma^2_{i,j} \tag{6}$$

From this equation it becomes clear that the variation of the demand rate is added to the initial uncertainty of the demand distribution (Poisson distribution). Moreover, in the next section, we look at the impact of having Gamma distributed demand rates to get insights on the most important aspects that determine the costs increase. We show that this equation is able to explain the relation between holding costs increase quite well.

4.2 Impact of uncertainty in a multi-item setting

To show how the additional uncertainty of the demand rate impacts the relative growth of the holding costs, we consider sixteen different scenarios for different amounts of uncertainty. For each SKU the uncertainty is modeled using the Gamma distribution (thus obtaining a Negative Binomial distribution for the leadtime demand distribution). Moreover, for each SKU the variation of the Gamma distributed demand rate is always equal to a fixed squared coefficient of variation of the demand. Thus SKUs for a given system j that have a higher value of m_i also have a higher value of $\sigma_{i,j}$. For each scenario we randomly draw combinations of expected demand rates and costs for 250 different parts, where for each part these values are drawn from different Uniform distributions. Moreover, we consider two different values for both the lead time as well as the value of EBO^{obj}. The base stock levels are set to near-optimal solutions making use of a greedy algorithm. We then repeat this process ten times and the results are averaged over these ten repetitions. The detailed scenario description and the final results are presented in Table 1.

Based on these results we can make a few interesting observations. First of all, when we increase the uncertainty using the squared coefficient of variation, the average relative holding costs of the solution seem to increase linear with respect to the scenario without the additional uncertainty. When we consider Equation 6, the standard deviation also increases linear for each part when we fill in the uncertainty of the demand rate. The relative increase in ξ_i^2 behaves similar as the relative holding costs increase for the multi-item case, as can be seen in Table 1, even though this equation is for every SKU separately, whereas the solution also contains the system approach. Another interesting observation is that the difference in the cost structure of the SKUs do not make a big difference in the relative cost increase. Finally, we find that for some scenarios, in particular the scenarios where demand rates can be larger on average, costs can increase significantly in order to compensate for the additional uncertainty.

4.3 Impact of ignoring uncertainty

In this Section we investigate the impact of ignoring the uncertainty in the demand rate for the same scenarios as presented in Table 1. We first calculate the base stock levels, S^*, using the greedy algorithm to obtain near-optimal solutions of our problem, assuming there is no uncertainty. We then calculate $EBO(\mathbf{S}*)$ based on the true model, in which we do have additional uncertainty, varying from a squared coefficient of variation of 0.25 to a squared coefficient of variation of 2 for each SKU. As the difference in holding costs to obtain a feasible solution are already presented

Table 1: Relative holding costs for different squared coefficients of variations of demand rate uncertainty

Scenario	m_i	c_i	t	EBO^{obj}	Holding costs without uncertainty	Squared coefficient of variation			
						0.25	0.5	1	2
1	$\mathcal{U}(0,1)$	$\mathcal{U}(5000,15000)$	1	1	$6.61 * 10^6$	+16.6%	+32.63%	+64.27%	+127.75
2	$\mathcal{U}(0,1)$	$\mathcal{U}(1000,19000)$	1	1	$6.26 * 10^6$	+16.33%	+32.49%	+63.65%	+127.6
3	$\mathcal{U}(0,10)$	$\mathcal{U}(5000,15000)$	1	1	$28.5 * 10^6$	+64.84%	+124.03%	+241.92%	+474.7
4	$\mathcal{U}(0,10)$	$\mathcal{U}(1000,19000)$	1	1	$27.7 * 10^6$	+64.16%	+123.14%	+240.56%	+470.06
5	$\mathcal{U}(0,1)$	$\mathcal{U}(5000,15000)$	3	1	$12.4 * 10^6$	+3.68%	+7.38%	+14.5%	+29.25
6	$\mathcal{U}(0,1)$	$\mathcal{U}(1000,19000)$	3	1	$12.5 * 10^6$	+3.63%	+7.28%	+14.67%	+28.84
7	$\mathcal{U}(0,10)$	$\mathcal{U}(5000,15000)$	3	1	$64.8 * 10^6$	+14.48%	+27.56%	+52.15%	+98.68
8	$\mathcal{U}(0,10)$	$\mathcal{U}(1000,19000)$	3	1	$64.5 * 10^6$	+14.25%	+27.47%	+52.34%	+96.81
9	$\mathcal{U}(0,1)$	$\mathcal{U}(5000,15000)$	1	0.1	$8.87 * 10^6$	+20.39%	+39.5%	+77.09%	+152.11
10	$\mathcal{U}(0,1)$	$\mathcal{U}(1000,19000)$	1	0.1	$8.71 * 10^6$	+19.69%	+38.85%	+75.91%	+151.25
11	$\mathcal{U}(0,10)$	$\mathcal{U}(5000,15000)$	1	0.1	$33.0 * 10^6$	+75.36%	+147.12%	+286.22%	+556.73
12	$\mathcal{U}(0,10)$	$\mathcal{U}(1000,19000)$	1	0.1	$32.8 * 10^6$	+75.35%	+146.71%	+282.9%	+550.44
13	$\mathcal{U}(0,1)$	$\mathcal{U}(5000,15000)$	3	0.1	$15.8 * 10^6$	+4.44%	+8.74%	+17.43%	+34.08
14	$\mathcal{U}(0,1)$	$\mathcal{U}(1000,19000)$	3	0.1	$15,7 * 10^6$	+4.37%	+8.78%	+17.16%	+34.19
15	$\mathcal{U}(0,10)$	$\mathcal{U}(5000,15000)$	3	0.1	$73,6 * 10^6$	+16.64%	+32.32%	+61.27%	+115.62
16	$\mathcal{U}(0,10)$	$\mathcal{U}(1000,19000)$	3	0.1	$72,7 * 10^6$	+16.64%	+32.13%	+61.32%	+114.76

in Table 1, we only present the actual values of the $EBO(\mathbf{S}^*)$ that we get for different levels of uncertainty. These results are presented in Table 2. Based on this table we can easily see that ignoring the uncertainty leads to a significantly bigger values of the expected number of backorders. Even for the scenario where we only have to increase the holding costs by 3.63%, we get an EBO value that is 30% bigger than the objective. As the EBO values obtained while ignoring the additional uncertainty are much larger than the objective, one can expect solutions that will result in much longer down times of the machines, for which the costs are most likely to exceed the additional costs of the base stock levels by far.

5. Conclusions

Modeling the uncertain demand rate using a distribution instead of a point estimate allows companies to take the uncertainty they are facing when introducing capital goods into consideration. For a multi item stocking problem with backordering and a constraint on the expected number of backorders we show that an increase of the uncertainty of the demand rate always leads to higher costs. Moreover, we show using numerical experiments that if the variation of the additional demand rate uncertainty is set using the squared coefficient of variation for each SKU, the relative costs of the near-optimal solution multi-item problem increase linear. However, the slope of these relative costs increase depend mainly on the expected demand rates, as well as the lead time relative to these demand rates. Finally, we consider what the impact is if a company would ignore the additional

Table 2: EBO values when ignoring additional demand rate uncertainty

Scenario	EBO^{obj}	Squared coefficient of variation			
		0.25	0.5	1	2
1	1	2.14	3.64	6.79	15.28
2	1	2.13	3.49	6.79	13.93
3	1	30.03	71.05	158.91	292.67
4	1	29.64	72.3	158.58	301.66
5	1	1.34	1.74	2.61	5.09
6	1	1.3	1.71	2.5	4.73
7	1	6.97	20.19	54.38	147.55
8	1	6.84	19.17	57.6	143.5
9	0.1	0.41	0.93	2.55	7.38
10	0.1	0.38	0.86	2.35	6.99
11	0.1	14.39	49.18	112.77	241.21
12	0.1	13.25	41.29	126.06	230.72
13	0.1	0.16	0.24	0.49	1.24
14	0.1	0.16	0.24	0.48	1.19
15	0.1	1.71	7.19	29.1	84.48
16	0.1	1.8	7.16	28.46	92.52

uncertainty, and show that this leads to poor solutions that are far from the objective.

For future research it may be interesting to further investigate the relation between the variation of individual SKUs and the holding costs for multi-item problems. Moreover, it would be interesting to find out how much a company can benefit if one has the possibility to reduce the additional uncertainty.

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