Optimal Commonality and Reliability – A Life Cycle Costs Perspective

J.P.C. Driessen, J.J. Arts, G.J. van Houtum

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Optimal Commonality and Reliability – A Life Cycle Costs Perspective

J.P.C. Driessen, J.J. Arts, G.J. van Houtum
Eindhoven University of Technology, School of Industrial Engineering, the Netherlands, j.p.c.driessen@tue.nl,j.j.arts@tue.nl g.j.v.houtum@tue.nl

Competitive Original equipment manufacturers (OEMs) do not only sell equipment, but also service contracts that ensure proper functioning and uptime of equipment after the sale. It then becomes the concern of OEMs to minimize equipment downtime by providing after-sales services such as repairs and spare parts over the lifetime of equipment. OEMs will therefore aim to minimize the total Life Cycle Costs (LCC) of their equipment by deciding for each component (1) whether to use a common component (one-for-all-systems) or a dedicated component (one-for-each-system), (2) the reliability and (3) the spare parts stock levels. We present life cycle cost functions in case of both dedicated and common components. These original life cycle cost functions can only be analyzed numerically. Therefore, we prove that there is a simpler expression that is asymptotically equivalent to the LCC in the regime where downtime of equipment is expensive. These asymptotics allow us to show that commonality can reduce the LCC even when the production costs under commonality are substantially higher than under dedicated components and characterize situations where this happens. Furthermore, we show that making commonality, reliability and spare parts stocking decisions sequentially can lead to arbitrarily large gaps compared to the optimal integrated decision. Finally we derive conditions for when a common component should be more (less) reliable than dedicated components for two practical cases.

Key words: After-sales Services; Asymptotics; Commonality; Life Cycle Costs; Reliability; Spare Parts

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1. Introduction

Production and service companies both use capital intensive systems to manufacture their products or render their services. For example, lithography systems are critical for operations in the semi-conductor industry, aviation engines and avionics are vital in the aviation industry, MRI scanners are critical in healthcare, and material handling systems are essential for distribution operations. The users of such capital intensive systems require high system availability, as unavailability results in production or service losses of millions of dollars (Kranenburg and Van Houtum 2009). However, realizing such high system availability at low costs is a major challenge without the help of the Original Equipment Manufacturers (OEMs). Therefore, the users of capital goods close full service contracts (Cohen et al. 2006) with the OEMs of these systems (e.g. ASML, Philips, GE, Pratt & Whitney, Vanderlande, and Bombardier). Under such a contract, the system users pay the OEM a periodic fee, and the OEM becomes responsible for the system availability during the usage period. As a consequence, modern OEMs are responsible for the entire life cycle of systems, and are therefore primarily interested in minimizing the total costs accrued throughout the systems' life cycle.

The Life Cycle Costs (LCC) have increased over the last years due to fact that OEMs offer a higher variety of systems. In an attempt to alleviate this burden, OEMs typically use common components in multiple different systems of their product portfolio. For example, identical rotor blades are used in multiple different aerospace engines or the same positioning sensors are used in various lithography systems. The main motivation for commonality usually comes from a marketing and production perspective: offering more different systems with a relatively small increase in the production costs. However, we show that even when the production costs increase substantially, component commonality still reduces the total LCC. Thus, the benefits of component commonality reach further than the ones considered from a marketing and production perspective.

Now, we explain how we model commonality in our setting. Any system consists of components, and we say that components – from different systems – belong to the same *component family* when they fulfill the same functionality, but are not necessarily identical (Meyer and Lehnerd 1997). Therefore, the OEM can decide per component family whether to use a single common component for all systems, or to use a dedicated component per system, as illustrated in Figure 1. Note that a component family may correspond to rotor blades in the case of aerospace engines, to positioning sensors in lithography systems, or to electric motors in MRI scanners.

Next to the decision to make a component common or not, there are two important decisions that impact the LCC. These decisions are the reliability of the component (how often will a component fail) and the amount of spare parts stock that is kept for after-sales services.

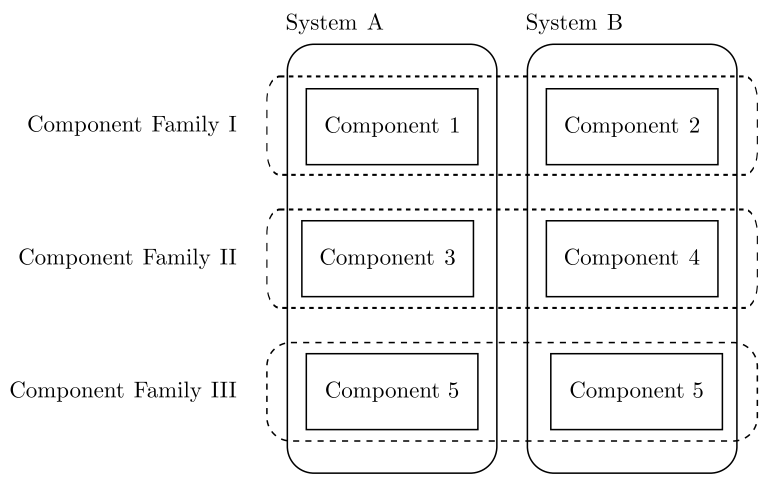


Figure 1 **A schematic representation of our concepts.**

The OEM's objective is to optimize this decision triad such that the LCC is minimized. In particular, this means that the OEM has to determine for each component family: (1) whether to use a common component or dedicated components, (2) the reliability for the components of each alternative, and (3) the spare parts stock levels for the components of each alternative.

The problem is typically tackled in a sequential fashion, because each decision is taken at a different time epoch, possibly by a different department. First, the OEM decides whether to use a common component for all systems, or a dedicated component for each. Secondly, the design department determines the reliability for the component(s) of the chosen alternative, and subsequently fixes the design. Finally, the spare parts stock levels are determined by the after-sales services department. Clearly, such a sequential approach is sub-optimal for minimizing the total LCC. After-sale services are typically not taken into account explicitly during the early design phase, at which time the OEM has the ability to determine 70-85% of the LCC (Asiedu and Gu 1998). Yet, after-sales services may constitute up to 70-80% of the LCC (Öner et al. 2007), and thus are pivotal in an effort to lower the LCC.

Therefore, we propose an alternative approach that takes after-sales services, component reliability, and the commonality decision into account simultaneously. We will refer to this approach as the *integrated LCC approach*. Our objective is to answer the following three main research questions for a single component family: (RQ1) Under the integrated LCC approach, when does commonality yield lower LCC than dedicated components, and by what factors is this affected? (RQ2) In which settings is it particularly important to consider an integrated LCC approach over the sequential approach? (RQ3) Under, the integrated LCC approach, is a common component more or less reliable than all dedicated components, at optimality? And what factors determine this?

We develop two LCC models to answer our questions; one for the common component, and one for the dedicated components. Our goal is to select the alternative – common or dedicated – with corresponding optimal reliabilities and spare parts stock levels, such that we minimize the total LCC of the component family. Based on these two models, we make four main contributions. First (1), we show how our original problem formulations, which appear to be intractable, can still be studied when considering asymptotically equivalent problem formulations as the cost of spare part unavailability approaches infinity. For these tractable problems (common and dedicated), we show that (2) the benefits of component commonality reach beyond the ones derived from a marketing and production perspective. That is, commonality reduces the LCC in many cases, even when the its production costs are higher than those of the dedicated components. Thirdly, (3) we illustrate that using the sequential approach underestimates the attractiveness of commonality, and it can yield arbitrary large LCC increases. Hence, we strongly recommend the use of the integrated LCC approach in the decision making process. Finally, (4) we prove – for two practical cases – that a common component is more (less) reliable, only if the initial production costs of all sold components can (cannot) be earned back throughout the time horizon that the components are used.

1.1. Outline

The remainder of this paper is organized as follows. We discuss related literature in Section 2. In Section 3, we present two optimization models: the commonality model and the dedicated components model.

In Section 4, we study the cost expressions of the common component model and the dedicated components model in more detail. First, we derive an expression for the optimal spare parts stock level of each component (common or dedicated). However, we conclude that the cost expressions are not amenable for further analysis upon inserting the optimal stock level expressions. At this point, we make the key observation that the costs of spare part unavailability is typically very high in capital good environments. Hence, we propose to study the asymptotic behavior of both models, and present tractable and asymptotically equivalent cost expressions for the common component and dedicated components model, as the cost of spare part unavailability approaches infinity. These tractable cost expressions allow for straightforward optimization of the problems.

In the following Section 5, we use the asymptotically equivalent models to answer our research questions. First, we define the attractiveness of commonality in terms of the maximum cost of a common component such that commonality yields lower LCC than the alternative of dedicated components. We use this measure to study which factors affect the attractiveness of commonality in Section 5.1. Secondly, we investigate how the sequential approach compares to the integrated LCC approach in Section 5.2. We study the differences in the commonality decision made, the cost differences resulting from this, and also settings in which the sequential approach may perform reasonable. Our final research question is addressed in Section 5.3, where we study two practical cases and explore conditions under which a common component is more reliable than all dedicated components, and when it is not. We conclude our paper in Section 6.

2. Literature

Our work focuses on the interaction between three literature streams: reliability optimization, after-sales services, and commonality. The literature on reliability optimization is typically studied from an engineering perspective. Design variables are chosen such that a specified cost function is minimized and reliability constraints are satisfied, or the reliability is maximized under specified cost constraints (Royset et al. 2001, Zou and Mahadevan 2006). Reliability optimization is also studied in the broader context of the warranty literature, e.g. Huang et al. (2007) and references therein. Work in this stream typically aim to optimize the reliability of a component such that the all costs that are accrued throughout the warranty period are minimized. The second related literature stream focuses on after-sales services, particularly on spare parts planning problems in complex (but also simple) supply chain structures; see e.g Sherbrooke (2004), Muckstadt (2005) and van Houtum and Kranenburg (2015). The third related literature area considers component commonality. This topic is studied from numerous different perspectives, e.g. marketing (Desai et al. 2001), new product development (Muffatto and Roveda 2000) and engineering (Fellini et al. 2004). We restrict our commonality review to an operations management perspective that focuses on cost minimization, and categorize this research stream into two classes. For a more elaborate review on commonality from an operations management perspective, see Labro (2004). The first class considers stylized Assemble-To-Order (ATO) models and focuses on the inventory implications of commonality, see e.g. Baker et al. (1986), Hillier (2000), and Song and Zhao (2009). The second class takes a combinatorial approach by studying large mathematical programming models, see e.g. Gupta and Krishnan (1999) and Thonemann and Brandeau (2000), and focuses on deriving efficient solution procedures.

Our paper is closely related to research that focuses on the interaction between two of these three literature streams (commonality and after-sales services). Stylized inventory models with component commonality have mainly been studied for ATO systems. In an ATO setting, a product demand is satisfied if *all* components are on stock (coupled demand). This differs from an after-sales logistics setting, in which demand typically occurs for each of the individual components. Kranenburg and van Houtum (2007) take such an after-sales logistics perspective and focus on the spare parts inventory implications of component commonality. They only model the costs incurred after production of the component, i.e., reliability and commonality decisions are neglected, but downtimes and repairs are included. Thonemann and Brandeau (2000) explicitly model the commonality decision and consider the spare parts element of after-sales logistics, but no downtimes and repairs. Their problem is combinatorial and the authors determine which features one or more common components should have based on component requirements. Both papers, Thonemann and Brandeau (2000) and Kranenburg and van Houtum (2007), do not consider any reliability design decision, and focus on developing efficient solution techniques without presenting analytical insights regarding their solutions.

Another closely related literature stream studies reliability explicitly in combination with after-sales logistics. Research in this stream has not yet considered commonality. Huang et al. (2007) propose a profit maximization model that optimizes the reliability and considers sales revenues, production costs, and repair costs. They take a life cycle approach, but do not consider costs related to spare part logistics. Kim et al. (2017)[1] present an LCC minimization model that jointly optimizes reliability and spare part basestock levels. The LCC are comprised of design, production, spare parts storage, and backorder costs. Downtime and repair costs are not included. Kim et al. (2017) prove convexity of their LCC function, so that straightforward optimization techniques can be used to find the optimal solution to their problem. Subsequently, the authors derive analytical insights for different service contract types through the use of a game theoretical analysis. By contrast, Öner et al. (2010) do not take a game theoretic perspective, but focus on the after-sales logistics aspect of the LCC minimization problem. The authors extend the cost function from Kim et al. (2017) by also incorporating repairs and system downtimes, and assume that demand during a stockout is satisfied via an emergency procedure. Öner et al. (2010) develop an efficient algorithm to find an optimal solution, but do not provide further analytical insights regarding their solutions.

Our work uses similar modeling as Kim et al. (2017) and Öner et al. (2010), but we also include the commonality dimension combined with the explicit inclusion of three after-sales logistics aspects: spare parts, repairs and downtimes. To our best knowledge, we are the first to combine after-sales logistics, with commonality and reliability optimization into a single model. Secondly, we provide managerial insights via analytical analysis of our model instead of only developing an efficient solution procedure. We provide a comparison between our work and the most related research in the literature in Table 1.

[1] The work by Kim et al. (2017) has been first published as a working paper, see Kim et al. (2007).

Table 1 Comparison of most related papers.

Paper	Commonality	Reliability	Downtime	Repairs	Spare parts	Life Cycle Costs	Analytical Insights
Kranenburg and van Houtum (2007)	X		X	X	X		
Thonemann and Brandeau (2000)	X				X		
Huang et al. (2007)		X		X		X	X
Kim et al. (2017)		X			X	X	X
Öner et al. (2010)		X	X	X	X	X	
This paper	X	X	X	X	X	X	X

3. Model

We consider an OEM who offers her customers various systems $i \in J$, where J is to the set of different systems. The OEM expects to sell $N_i > 0$ units of each system $i \in J$ at time $t = 0$ with a supplementary Full Service Contract (FSC). The FSC states that the OEM will service the N_i units of each system $i \in J$ for a finite lifetime T. We assume that this time T is equal for all systems, and it is typically 10-30 years. Given the component structure for her systems (see Figure 1 and Section 1), the OEM has to determine for each component family whether to opt for a common component or dedicated components. Hence, we restrict ourselves to a single component family, which is critical for system functioning and repairable upon a failure. The latter implies that each failed item of a component is repaired and subsequently replenishes the spare parts stock after its repair, i.e., we have a closed loop supply chain with a constant turnaround stock level. Furthermore, we assume that exactly one component of the family occurs in system $i \in J$, e.g. one rotor blade assembly occurs in an aerospace engine or only one positioning sensor occurs in a lithography system. As a consequence, the sales quantity N_i of system i is equal to the number of units of a dedicated component – for system i – that is installed in the field at time $t = 0$. Furthermore, a system's identifier $i \in J$ is equivalent to a dedicated component's identifier $i \in J$, and thus the set J is equivalent to the set of the dedicated components. Note that each component will be serviced for a finite lifetime T. We refer to the units of component i that are installed in the field, N_i, as the installed base of component i. In the remainder, we will use the terminology on the component level for the set J (the set of dedicated components) and for each element $i \in J$ (a dedicated component). Next to the notation for the dedicated components, we denote the common component by q, with $N_q = \sum_{i \in J} N_i$, and introduce the set $I = J \cup \{q\}$.

Next, we discuss the dynamics of a given component $i \in I$. At time $t = 0$, the OEM decides on the reliability level $\tau_i > 0$ in terms of the Mean Time Between Failures (MTBF) and the turnaround stock level s_i. We use the term reliability in the remainder instead of MTBF. At $t = 0$, $N_i + s_i$ units of component i are produced, at unit cost $\beta_i c(\tau_i)$. The function $c : \mathbb{R}_+ \to \mathbb{R}_+$[2] is identical for all components $i \in I$, and is a twice differentiable convex and strictly increasing function with $0 \leq \lim_{\tau \downarrow 0} c(\tau) < \infty$ and $0 \leq \lim_{\tau \downarrow 0} \frac{dc(\tau)}{d\tau} < \infty$. The parameter β_i enables us to differentiate the unit costs between the various components under identical reliability levels.

After the N_i units of component i have been installed in the field at $t = 0$, they operate independently with the same reliability τ_i. During operation, the units fail. We denote the failures during $[0, t]$ by $D_i(t, N_i, \tau_i)$ for any $t \geq 0$, and we assume that this failure process has independent and stationary increments. We also assume that $D_i(t, N_i, \tau_i)$ is normally distributed with mean $\mathbb{E}[D_i(t, N_i, \tau_i)] = \frac{N_i t}{\tau_i}$ and standard deviation $\sigma[D_i(t, N_i, \tau_i)] = \sqrt{\alpha \frac{N_i t}{\tau_i}}$, where the constant $\alpha > 0$ is the variance to mean ratio. Furthermore, we assume that the number of operating units N_i remains constant, even when a unit of the installed base fails. We make this assumption, because failed systems are down for a negligible amount of time in practical settings, and it is a common assumption in the literature, see e.g. Muckstadt (2005) and van Houtum and Kranenburg (2015). Our proposed failure process can approximate a Poisson process when $\mathbb{E}[D_i(t, N_i, \tau_i)]$ is sufficiently large and $\alpha = 1$. Kim et al. (2017) make the same assumption regarding the failure process to facilitate analysis. Nevertheless, we compare normally distributed spare parts demand to the more conventional Poisson distributed spare parts demand in Appendix J, and observe that deviations are small based on our testbed.

A failure of an installed unit of component $i \in I$ causes failure of the system in which it is built. After removal of the failed unit of component $i \in I$, the failed unit is sent to a repair shop. It

[2] $\mathbb{R}_+$ denotes the set of positive real numbers, i.e., $\mathbb{R}_+ = \{x \in \mathbb{R} \mid x > 0\}$.

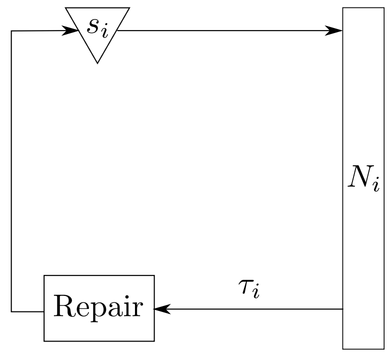


Figure 2 **Illustration of the failure and repair process of component $i \in I$.**

takes $L > 0$ time units to repair the failed unit, after which the unit returns to a spare parts stock point; see Figure 2. Note that this system corresponds to a system operating under a policy with a basestock level s_i and a leadtime L. We assume that the OEM pays the customer a cost $d \geq 0$ per failure of the system, and d is independent of the system. The cost d may include the penalty costs, as stated in the FSC, but also the cost of sending a maintenance engineer to the failed system's location etc. We will refer to a failure of the system as a downtime incident. The expected downtime costs during $(0, T]$ are derived from the failure process and are given by $d\mathbb{E}[D_i(T, N_i, \tau_i)] = d\frac{N_i T}{\tau_i}$. Immediately after the failure of an installed unit, a spare part is taken from stock if possible. In case such a spare part is not available, the replacement of the failed unit in the field cannot occur. Consequently, the system in which the failed unit was installed cannot operate until a new spare part is available again. The OEM pays a penalty b to the customer per unit time that the system cannot operate due to spare part unavailability. The parameter b represents the disruption costs per time unit due to a stockout situation, e.g. the downtime costs of a system per time unit. Note that the demand is thus backordered. Furthermore, we assume that the spare parts demand process is stationary during $(0, T]$. Then, the total expected backorder costs over $(0, T]$ are then given by $bT\mathbb{E}[(D_i(L, N_i, \tau_i) - s_i)^+]$, where $\mathbb{E}[(D_i(L, N_i, \tau_i) - s_i)^+]$ is the expected number of backorders at any given time instant.

Next, we assume that the costs per repair are a fraction $r \in (0, 1)$ of its unit costs, i.e., $r\beta_i c(\tau_i)$. The parameter r includes all costs incurred for one repair such as material and labor costs. The average number of repairs can be derived from our failure process. Thus, we have the following expression for the total expected repair costs: $r\beta_i c(\tau_i)\mathbb{E}[D_i(T,N_i,\tau_i)] = r\beta_i c(\tau_i)\frac{N_i T}{\tau_i}$.

The OEM owns all s_i turnaround units during $(0,T]$. Therefore, the OEM pays storage costs for all units, either in repair or in stock. The per time unit storage cost for one turnaround unit is a fraction $h \in (0,1)$ of its unit cost, i.e., $h\beta_i c(\tau_i)$. The parameter h includes all per time unit costs for a single turnaround unit, such as warehousing and insurance costs. The total spare parts storage costs over $(0,T]$ are given by $hs_i T\beta_i c(\tau_i)$.

We have explained the dynamics of a given component $i \in I$ in the foregoing. These dynamics are identical for all dedicated components $i \in J$, and also for the common component q. Therefore, we formulate a general LCC function for component $i \in I$:

$$\tilde{\pi}(\tau_i, s_i, N_i, \beta_i) = \beta_i c(\tau_i)(N_i + s_i) + hs_i T\beta_i c(\tau_i) + r\beta_i c(\tau_i)\frac{N_i T}{\tau_i} + d\frac{N_i T}{\tau_i} + bT\mathbb{E}[(D_i(L, N_i, \tau_i) - s_i)^+]. \tag{1}$$

We do not consider an index i for the LCC function $\tilde{\pi}(\tau_i, s_i, N_i, \beta_i)$, because each component $i \in I$ is fully characterized by its reliability level τ_i, turnaround stock level s_i, installed base size N_i, and its relative unit cost factor β_i. Hence, we propose a general parametrized LCC function, which we can later analyze for arbitrary component types more easily. Furthermore, note that our model allows $\beta_q \geq \beta_i$ for all $i \in J$ (as argued by e.g. van Mieghem 2004), but can also capture $\beta_q < \beta_i$ for one or more $i \in J$ (as discussed by e.g. Krishnan and Gupta 2001). Furthermore, we can set $\beta_i = 1$ for one particular component $i \in I$ without loss of generality.

3.1. Optimization problems

We use the LCC function from Eq. (1) to construct the optimization problems for the common and dedicated components. The first problem considers the common component, and is given by

$$(CP) \quad \min_{\tau_q \in \mathbb{R}_+, s_q \in \mathbb{R}} \left\{\tilde{\pi}(\tau_q, s_q, N_q, \beta_q)\right\}. \tag{2}$$

For the dedicated components problem, we consider the LCC – under reliability τ_i and turnaround stock level s_i – for each component $i \in J$, and sum these to obtain the total LCC. Hence, the dedicated component problem becomes

$$(DP) \qquad \min_{\boldsymbol{\tau}\in\mathbb{R}_+^{|J|}, \boldsymbol{s}\in\mathbb{R}^{|J|}} \left\{ \sum_{i\in J} \tilde{\pi}(\tau_i, s_i, N_i, \beta_i) \right\}, \tag{3}$$

with $\boldsymbol{\tau}$ and $\boldsymbol{s}$ denoting the vector of τ_i and s_i, $i \in J$, respectively. Note that (DP) is separable in the dedicated components i.

4. Asymptotics

In this section, we study the cost functions of our optimization problems from Section 3.1, i.e., $\tilde{\pi}(\tau_q, s_q, N_q, \beta_q)$ and $\sum_{i\in J} \tilde{\pi}(\tau_i, s_i, N_i, \beta_i)$. First, we analyze the cost functions with respect to the turnaround stock levels. We find an expression for the optimal turnaround stock level for each component $i \in I$, and we insert these into the cost functions of the Problems (2) and (3). However, the total LCC expressions – $\tilde{\pi}(\tau_q, s_q, N_q, \beta_q)$ and $\sum_{i\in J} \tilde{\pi}(\tau_i, s_i, N_i, \beta_i)$ – under the optimal turnaround stock level turn out to be intractable for further analysis. But, we observe that the backorder cost rate b is typically very high in environments where capital intensive systems are used. Therefore, we explore asymptotic behavior of our total LCC expressions, and formulate alternative total LCC functions that are asymptotically equivalent as b approaches infinity. Before we start our analysis, we first state a mild assumption that we use throughout this entire work. Furthermore, we would like to remark that omitted proofs can be found in the appendix.

ASSUMPTION 1. *For each component $i \in I$, we have $\tau_i \in (0, \overline{\tau}_i]$ and $bT > 2\beta_i c(\overline{\tau}_i)(1 + hT)$.*

Assumption 1 states the reliability of component $i \in I$ is bounded from above by $\overline{\tau}_i$ such that holding a backorder throughout $(0, T]$ is more than twice as expensive as producing a new unit and keeping this unit on stock throughout $(0, T]$. Such an assumption is typically satisfied in practice, particularly in a capital goods setting. Under this assumption, we are able to derive an expression for the optimal turnaround stock level of component $i \in I$. For the sake of notational brevity, we will omit the arguments N_i, β_i in the remainder of this section, unless indicated otherwise. Hence, we study $\tilde{\pi}(\tau_i, s_i) = \tilde{\pi}(\tau_i, s_i, N_i, \beta_i)$ for any component $i \in I$.

LEMMA 1. *For each component $i \in I$ and $\tau_i \in (0, \overline{\tau}_i]$, $\tilde{\pi}(\tau_i, s_i)$ is twice differentiable and strictly convex in s_i. $\tilde{\pi}(\tau_i, s_i)$ is minimized by a positive, unique, finite $s_i^*(\tau_i)$ that solves the first order condition, and is given by:*

$$s_i^*(\tau_i) = \mathbb{E}[D_i(L, N_i, \tau_i)] + \sigma[D_i(L, N_i, \tau_i)]\Phi^{-1}\left(\frac{bT - \beta_i c(\tau_i)(1 + hT)}{bT}\right), \tag{4}$$

where $\Phi^{-1}(\cdot)$ denotes the inverse of the standard normal distribution.

Our next step is to insert the expression for the optimal turnaround stock levels from Eq. (4) into Problems (2) and (3). However, this results in cost expressions that are not amenable for further analysis. To see this, let us consider $\tilde{\pi}(\tau_i, s_i^*(\tau_i))$ for an arbitrary component $i \in I$ and note that it corresponds to an expression in which we have the highly complex inclusion of $c(\tau_i)$ in the standard normal inverse function $\Phi^{-1}(\cdot)$. Therefore, we cannot find the optimal reliability levels easily, as convexity with respect to τ_i cannot be established. But at this point, we make the important observation that the cost of spare part unavailability is often high for the users of capital goods. This cost is typically in the order of thousands of US dollars per hour. As a consequence, the backorder cost rate b is typically very high. Therefore, we propose to study the asymptotic behavior of $\sum_{i \in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i))$ and $\tilde{\pi}(\tau_q, s_q^*(\tau_q))$ as b approaches infinity. Specifically, this means that we propose an alternative LCC function that can be analyzed and easily optimized, and furthermore is asymptotically equivalent to the original formulation as b tends to infinity.

We prove the asymptotic equivalence for the total LCC of the dedicated components $\sum_{i \in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i))$, but the same method also applies to the common component model (results will simplify because we have no summation). Our techniques are similar to Huh et al. (2009) and Bijvank et al. (2014), and we use some of their results to show the asymptotic equivalence. Moreover, we add b as an argument to our cost functions, since we study the limit behavior with respect to b, i.e. we use $\sum_{i \in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \,|\, b)$ because $s_i^*(\tau_i)$ also depends on b. Let the alternative total LCC function be defined by $\sum_{i \in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \,|\, b\beta_i c(\tau_i))$.

LEMMA 2. *For each component $i \in I$, it holds that $\lim_{b \to \infty} \frac{b\mathbb{E}[(D_i(L, N_i, \tau_i) - s_i^*(\tau_i, b))^+]}{s_i^*(\tau_i, b)} = 0$ and $\lim_{b \to \infty} \frac{b\beta_i c(\tau_i)\mathbb{E}[(D_i(L, N_i, \tau_i) - s_i^*(\tau_i, b\beta_i c(\tau_i)))^+]}{s_i^*(\tau_i, b\beta_i c(\tau_i))} = 0$.*

We use Lemma 2 to prove the equivalence between the two LCC functions, see Theorem 1.

THEOREM 1. *For given $\tau_i \in (0,\infty)$, it holds that*

$$\lim_{b\to\infty} \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta c(\tau_i)) \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b)} = 1.$$

Proof. Fix all $\tau_i \in (0,\infty)$ and let $b \in (0,\infty)$ be sufficiently large, such that we satisfy Assumption 1. By definition of $s_i^*(\tau_i, b)$, we find the following bounds by inserting suboptimal turnaround stock levels:

$$\frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b\beta c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b)} \le \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b)} \le \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b)}.$$

We define $k = \arg\max_{i\in J}\{s_i^*(\tau_i, b\beta_i c(\tau_i))\}$, and we rewrite the lower bound to

$$\frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b)} = \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b\beta_i c(\tau_i))/s_k^*(\tau_k, b\beta_k c(\tau_k))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b)/s_k^*(\tau_k, b\beta_k c(\tau_k))}. \tag{5}$$

Taking the limit as $b \to \infty$ implies that $s_k^*(\tau_k, b\beta_k c(\tau_k)) \to \infty$, and that any $\tau_i \in (0,\infty)$ satisfies Assumption 1. Consequently, we are not further concerned with satisfying Assumption 1. Each cost term associated with component $i \in J$ in the numerator and denominator has a finite limit. To see this, we use Lemma 2 and conclude that $\lim_{b\to\infty} \frac{b\beta_i c(\tau_i)\mathbb{E}[(D(L,N_i,\tau_i) - s_i^*(\tau_i, b\beta_i c(\tau_i)))^+]}{s_k^*(\tau_k, b\beta_k c(\tau_k))} = 0$, because $s_k^*(\tau_k, b\beta_k c(\tau_k)) \ge s_i^*(\tau_i, b\beta_i c(\tau_i))$ for all components $i \in J$. Hence, we obtain

$$\lim_{b\to\infty} \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b)} = 1.$$

Now, we define $k = \arg\max_{i\in J}\{s_i^*(\tau_i, b)\}$ with a slight abuse of notation, and we rewrite the upper bound to

$$\frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b)} = \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b\beta_i c(\tau_i))/s_k^*(\tau_k, b)}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b)/s_k^*(\tau_k, b)}.$$

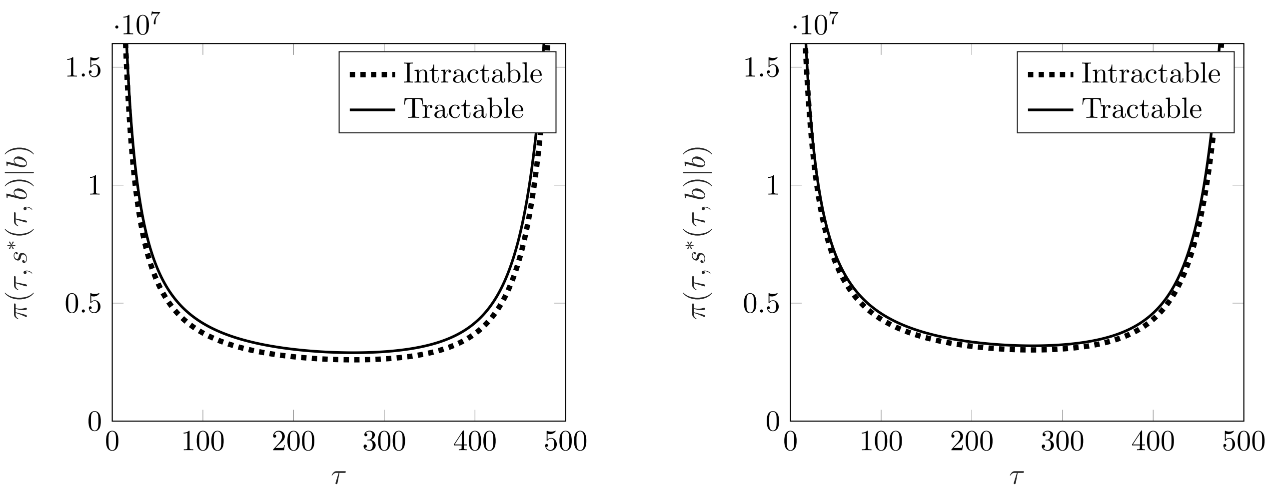
Again, taking the limit as $b \to \infty$ and applying Lemma 2, we obtain

$$\lim_{b\to\infty} \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b)} = 1.$$

By the sandwich theorem, it follows that $\lim_{b\to\infty} \frac{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b\beta_i c(\tau_i))}{\sum_{i\in J} \tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b)} = 1.$ $\square$

Theorem 1 shows that our original total LCC function $\sum_{i\in J}\tilde{\pi}(\tau_i, s_i^*(\tau_i, b) \mid b)$ is equivalent to $\sum_{i\in J}\tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b\beta_i c(\tau_i))$, as the backorder cost rate b tends to infinity. However, we also explore how well the alternative total LCC function represents the actual total LCC function for other values of the backorder cost rate b, see Figure 3 based on Example 1.

Example 1. Let $|J| = 1$, $N_1 = 200$ units, $h = 3$ % per item per month, $r = 20$ % per item, $L = 3$ months, $T = 360$ months, $\beta_1 = 1$, $\alpha = 1$ and $d = \$1 \cdot 10^3$ per downtime incident. Moreover, we use $c(\tau) = 5,000 + 1,000 \exp\left(\frac{\tau}{600-\tau}\right)$ in \$ per unit, where $\tau \in (0, 600)$. For further details, see Appendix J.



(a) $b = \$1 \cdot 10^4$ per month per backorder

(b) $b = \$1 \cdot 10^{10}$ per month per backorder

Figure 3 **Comparison between** $\tilde{\pi}(\tau, s^*(\tau, b) \mid b)$ **(dotted) and** $\tilde{\pi}(\tau, s^*(\tau, b\beta c(\tau)) \mid b\beta c(\tau))$ **(solid) for different** b**.**

In light of Theorem 1 and Figure 3, we propose to study $\sum_{i\in J}\tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b\beta_i c(\tau_i))$, which can be optimized easily and the satisfaction of Assumption 1 no longer depends on the reliability levels. But before we discuss the details of optimizing each reliability level τ_i, we first rewrite each $\tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b\beta_i c(\tau_i))$, with $\phi(\cdot)$ and Φ^{-1} denoting the standard normal pdf and the inverse of the standard normal cdf. Furthermore, we let $\pi(\tau_i, N_i, \beta_i) = \pi(\tau_i) = \tilde{\pi}(\tau_i, s_i^*(\tau_i, b\beta_i c(\tau_i)) \mid b\beta_i c(\tau_i))$.

Lemma 3. *Inserting $s_i^*(\tau_i, b\beta_i c(\tau_i))$ into Eq. (1) yields*

$$\pi(\tau_i, N_i, \beta_i) = \beta_i c(\tau_i)\left(1+\frac{rT+L(1+hT)}{\tau_i}\right)N_i + d\frac{N_i T}{\tau_i} + b\beta_i c(\tau_i)T\sqrt{\frac{\alpha N_i L}{\tau_i}}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right). \tag{6}$$

We use the expression in Lemma 3 to derive the following asymptotically equivalent (as $b\to\infty$) cost minimization problems for the common component and for the dedicated components:

$$(CP') \quad \min_{\tau_q\in\mathbb{R}}\left\{\pi(\tau_q, N_q, \beta_q)\right\}, \tag{7}$$

$$(DP') \quad \min_{\boldsymbol{\tau}\in\mathbb{R}^{|J|}}\left\{\sum_{i\in J}\pi(\tau_i, N_i, \beta_i)\right\} \tag{8}$$

We are interested in the reliability level that minimizes the costs for each of the components $i\in I$. Moreover, we are able to analyze the cost function for an arbitrary component $i\in I$ because the dedicated components problem (DP') is separable in the components $i\in J$. We will show that the cost function for an arbitrary component $\pi(\tau)$ is strictly convex, if $c(\tau)$ satisfies Assumption 2. As a consequence, we can determine the optimal reliability levels τ_i^* for all components $i\in I$ very easily by simple optimization techniques.

Assumption 2. *$c(\tau)$ satisfies the following: $\frac{c(\tau)}{\tau}$ is convex and $\frac{c(\tau)}{\sqrt{\tau}}$ is convex.*

We observe that there exists a large class of functions that satisfy Assumption 2: polynomial functions with one constant term and all others terms being at least second order, and exponential forms, e.g. Mettas (2000). If $c(\tau)$ satisfies Assumption 2, we can determine the optimal reliability levels τ_i^* by standard optimization techniques.

Lemma 4. *For each component $i\in I$, $\pi(\tau_i)$ is twice differentiable and strictly convex, and is minimized by a positive, unique, finite τ_i^*. This τ_i^* solves the first order condition.*

5. Analytical comparison

We will use the asymptotically equivalent models from Section 4 to answer our research questions (RQ1), (RQ2), and (RQ3) in Subsections 5.1, 5.2, and 5.3, respectively.

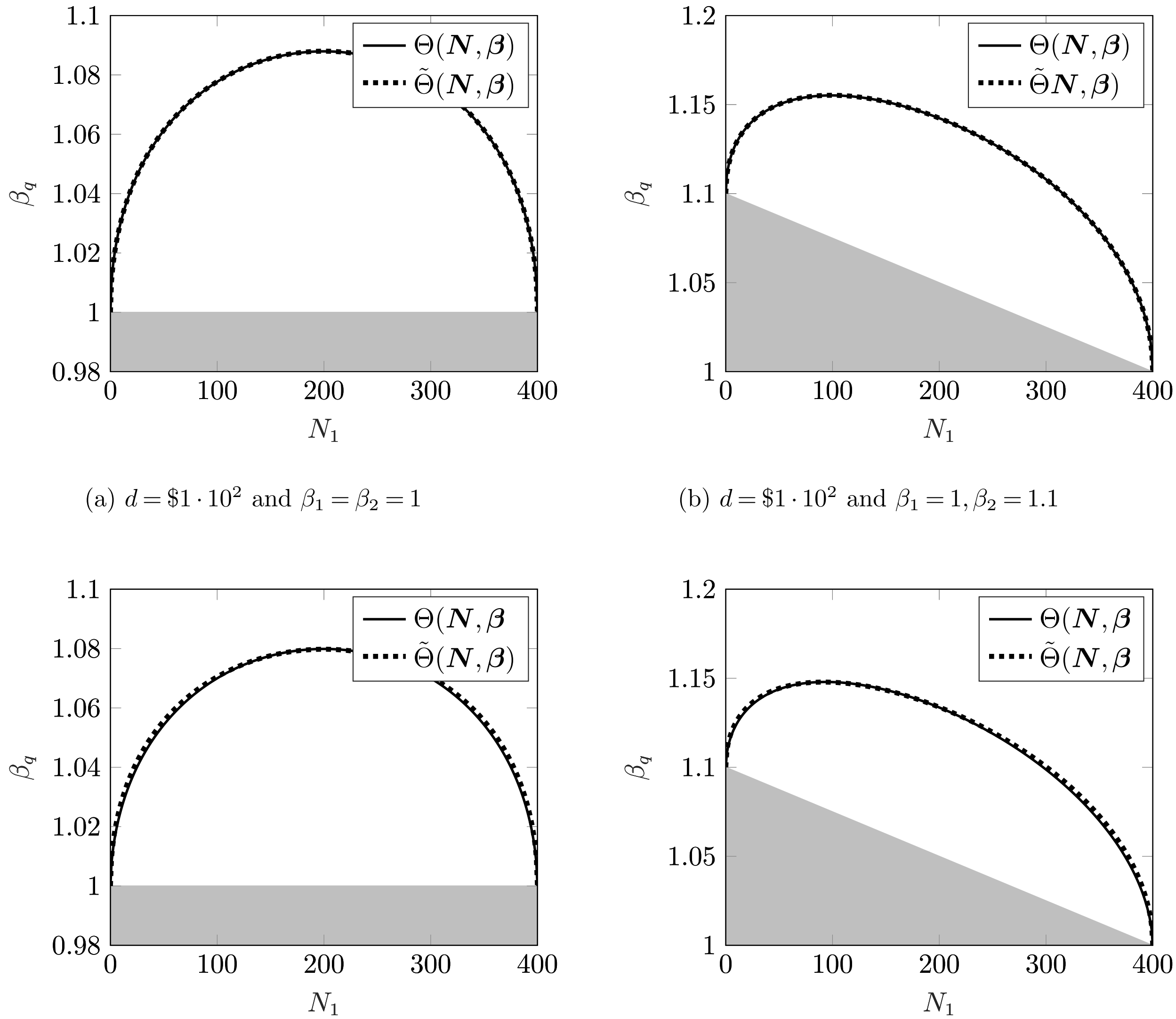
5.1. Attractiveness of commonality

We are interested in studying when commonality is more attractive than using dedicated components under the integrated LCC approach. In particular, we are concerned with the question under which conditions commonality results in lower LCC than dedicated components. To answer this question, we propose to study a measure that indicates how expensive a common component can be such that commonality still yields lower LCC than the alternative of dedicated components. We can numerically determine and study this measure, and provide an analytical underpinning that explains some phenomena observed. Lemma 5 characterizes the maximum costs of a common component in terms of the cost factor β_q, such that commonality yields lower LCC than dedicated components.

Lemma 5. *$\pi(\tau_q^*, N_q, \beta_q)$ is monotone increasing in β_q, where τ_q^* is the optimal reliability level under N_q and β_q. Furthermore, there exists a positive, finite, and unique value for β_q that satisfies $\pi(\tau_q^*, N_q, \beta_q) - \sum_{i\in J} \pi(\tau_i^*, N_i, \beta_i) = 0$, which we denote by $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$, where $\boldsymbol{N}$ and $\boldsymbol{\beta}$ correspond to the vectors of all N_i and β_i with $i \in J$.*

We illustrate $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ by using instances with the same parameter settings as Example 1, set $b = \$1 \cdot 10^7$ per month per backorder, and keep $\sum_{i\in J} N_i = 400$. Our numerical exploration is presented only for the cases in which we consider two ($|J| = 2$) or three ($|J| = 3$) dedicated components, because this allows for a visual representation, see Figures 4 and 5, where the latter depicts the contour lines. In our discussion, we will concentrate on two dedicated components, but all observations also hold for the case of three.

Upon studying Figure 4, we see that there exist a large number of settings (combinations of $\boldsymbol{N}$ and $\boldsymbol{\beta}$) that make commonality attractive. We see that – in many settings – the common component can be (substantially) more expensive than each of the dedicated components and still result in lower LCC than the alternative of dedicated components; i.e., $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ is larger than all β_i for many combinations $(\boldsymbol{N}, \boldsymbol{\beta})$. Therefore, we have that the benefits of component commonality are larger than previously known up to date. Component commonality does not only have the



(a) $d = \$1 \cdot 10^2$ and $\beta_1 = \beta_2 = 1$

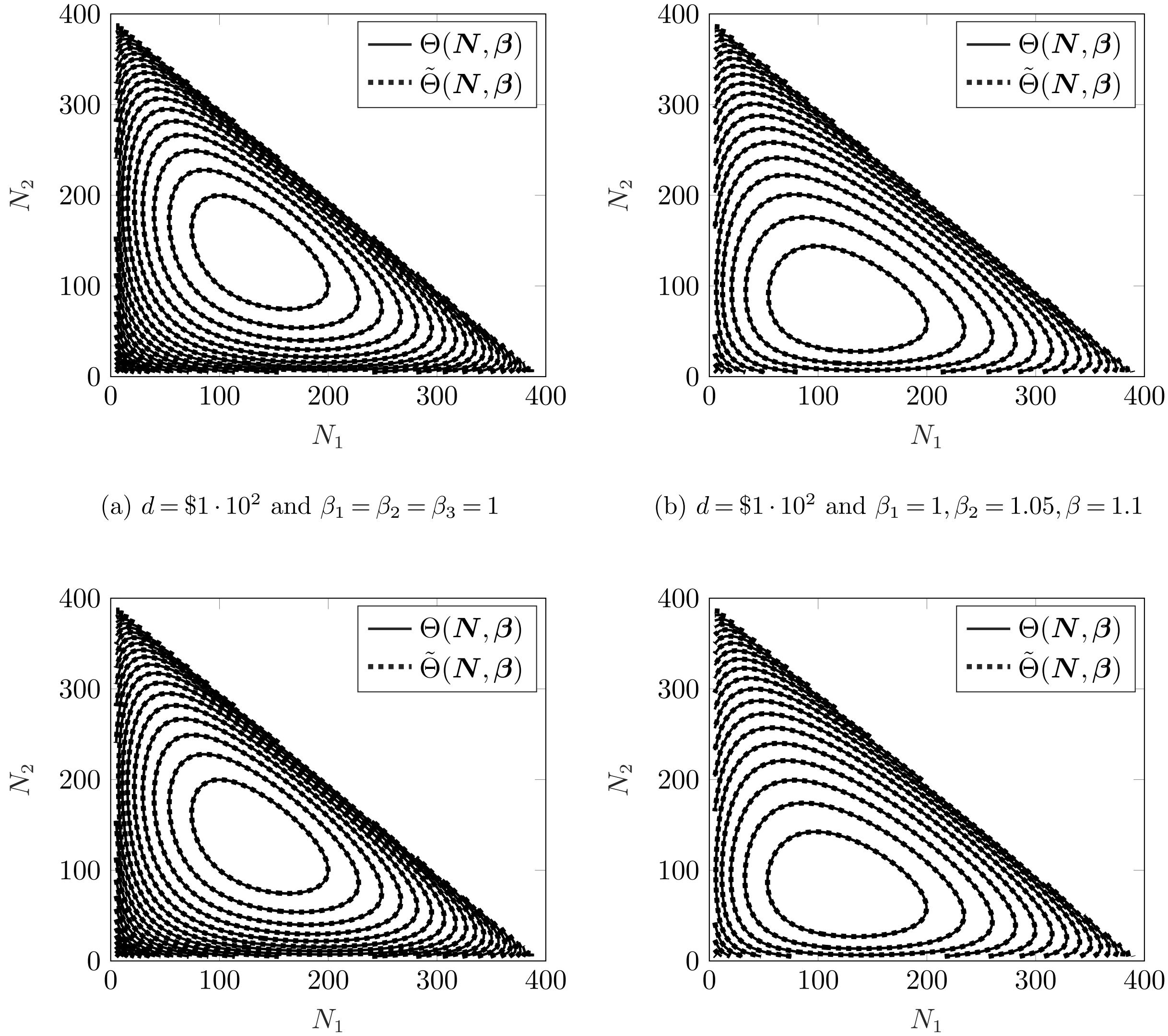
(b) $d = \$1 \cdot 10^2$ and $\beta_1 = 1, \beta_2 = 1.1$

(c) $d = \$1 \cdot 10^5$ and $\beta_1 = \beta_2 = 1$

(d) $d = \$1 \cdot 10^5$ and $\beta_1 = 1, \beta_2 = 1.1$

Figure 4 Numerical illustration of $\Theta(\boldsymbol{N},\boldsymbol{\beta})$ and $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$, for two dedicated components.

ability to offer a higher system variety without adversely affecting production costs, but even in case the production costs *are* affected adversely, component commonality still has the ability to yield lower LCC in many settings. The cost of after-sales services decreases when commonality is used, and this significantly impacts the LCC. The after-sales services allow for a cost reduction, because we are able to pool spare parts and still realize the same service levels, and as a result the total LCC decrease substantially. In particular, component commonality is (more) attractive for industries in which the after-sales services constitute a large portion of the LCC (e.g. aviation



(a) $d = \$1 \cdot 10^2$ and $\beta_1 = \beta_2 = \beta_3 = 1$

(b) $d = \$1 \cdot 10^2$ and $\beta_1 = 1, \beta_2 = 1.05, \beta = 1.1$

(c) $d = \$1 \cdot 10^5$ and $\beta_1 = \beta_2 = \beta_3 = 1$

(d) $d = \$1 \cdot 10^5$ and $\beta_1 = 1, \beta_2 = 1.05, \beta_3 = 1.1$

Figure 5 Numerical illustration of $\Theta(\boldsymbol{N},\boldsymbol{\beta})$ and $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$, for three dedicated components.

industry, semiconductor industry, healthcare industry (MRI), and material handling & operations), as the backorder cost rate is larger in these industries and consequently the effect of spare parts pooling increases, thereby increasing the attractiveness of commonality.

Next, we study the attractiveness of component commonality in somewhat more detail, and explore the settings in which component commonality is more (or even most) attractive, i.e., the maximum value of $\Theta(\boldsymbol{N},\boldsymbol{\beta})$. When there is one product with a large installed base and several products with a smaller installed base, one may expect that commonality is beneficial because the smaller products can use the economy of scale associated with the large product. However, the results in Figure 4 indicate that commonality is most attractive when there does *not* exist one very large installed base, but rather when the different installed bases have similar sizes. We explain this phenomenon by considering the spare parts safety stock of a common component. In case there exist multiple large installed bases, commonality allows for a larger reduction in the spare part safety stock levels, due to spare parts pooling. To illustrate this, let us consider Example 2 that uses the same values as Example 1 unless indicated otherwise.

EXAMPLE 2. Let $N_q = 400$ units, $\tau_1 = \tau_2 = \tau_q = 200$ months, $b = \$1 \cdot 10^7$, $\beta_1 = \beta_2 = \beta_q = 1$. If we have two different installed bases $N_1 = 399$ and $N_2 = 1$. Then, added value for safety stock pooling is small, i.e., $s_q^*(\tau_q) - (s_1^*(\tau_1) + s_2^*(\tau_2)) = -0.49$ spares are saved. However, when $N_1 = N_2 = 200$ the safety stock pooling effect becomes larger for commonality, i.e., in this case we have $s_q^*(\tau_q) - (s_1^*(\tau_1) + s_2^*(\tau_2)) = -4.15$ spare parts saved.

Example 2 illustrates our observation that commonality is more attractive when there exist multiple installed bases of similar size. More specifically, we observe that *(a)* commonality is most attractive for equally sized installed base in case the dedicated components are equally expensive, see Figures 4a and 4c. In case the cost of the dedicated components differ – cf. Figures 4b and 4d – we observe *(b)* that commonality is most attractive when we have a larger installed base of the more expensive component. We use the labels *(a)* and *(b)* in our subsequent proof that explains these labeled phenomena.

To provide support for the observations *(a)* and *(b)*, our objective is to analytically study the maximum costs of a common component in terms of $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ under various settings $(\boldsymbol{N}, \boldsymbol{\beta})$. However, this is highly cumbersome. To see this, let us rewrite the expression for $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$. We know (by Lemma 5) that this threshold satisfies $\pi(\tau_q^\Theta, N_q, \Theta(\boldsymbol{N}, \boldsymbol{\beta})) = \sum_{i \in J} \pi(\tau_i^*, N_i, \beta_i)$, where τ_q^Θ is the optimal reliability level under $\beta_q = \Theta(\boldsymbol{N}, \boldsymbol{\beta})$ and N_q. Let us rewrite $\pi(\tau_q^\Theta, N_q, \Theta(\boldsymbol{N}, \boldsymbol{\beta}))$ by Eq. (6):

$$\pi(\tau_q^\Theta, N_q, \Theta(\boldsymbol{N}, \boldsymbol{\beta})) = \Theta(\boldsymbol{N}, \boldsymbol{\beta}) \left[\pi(\tau_q^\Theta, N_q, 1) - dT \frac{\sum_{i \in J} N_i}{\tau_q^\Theta} \right] + dT \frac{\sum_{i \in J} N_i}{\tau_q^\Theta}.$$

Next, we want to determine $\Theta(\boldsymbol{N},\boldsymbol{\beta})$ such that $\pi(\tau_q^{\Theta}, N_q, \Theta(\boldsymbol{N},\boldsymbol{\beta})) = \sum_{i\in J}\pi(\tau_i^*, N_i, \beta_i)$, which yields:

$$\Theta(\boldsymbol{N},\boldsymbol{\beta}) = \frac{\sum_{i\in J}\pi(\tau_i^*, N_i, \beta_i) - dT\frac{\sum_{i\in J} N_i}{\tau_q^{\Theta}}}{\pi(\tau_q^{\Theta}, N_q, 1) - dT\frac{\sum_{i\in J} N_i}{\tau_q^{\Theta}}} \tag{9}$$

However, deriving insights for $\Theta(\boldsymbol{N},\boldsymbol{\beta})$ is intractable, as τ_q^{Θ} is derived for a given $\Theta(\boldsymbol{N},\boldsymbol{\beta})$, and perturbations in N_i or β_i induce complex changes in the optimal reliability levels τ_i^* for all components $i \in J$. Hence, we propose to study an approximation for $\Theta(\boldsymbol{N},\boldsymbol{\beta})$, with equal τ_i^* for all components $i \in I$. We denote this approximation of $\Theta(\boldsymbol{N},\boldsymbol{\beta})$ by

$$\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta}) = \frac{\sum_{i\in J}\pi(\tau_q^1, N_i, \beta_i) - dT\frac{\sum_{i\in J} N_i}{\tau_q^1}}{\pi(\tau_q^1, N_q, 1) - dT\frac{\sum_{i\in J} N_i}{\tau_q^1}},$$

where τ_q^1 corresponds to the LCC minimizer of the common component under $\beta_q = 1$. Recall that the minimum of $\beta_i = 1$ without loss of generality. Note that $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ is an upper bound to $\Theta(\boldsymbol{N},\boldsymbol{\beta})$ when $d = 0$. Before we study $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$, we test how well it approximates $\Theta(\boldsymbol{N},\boldsymbol{\beta})$, and in particular how the maximums compare. We depict this numerical comparison for two and three dedicated components in Figures 4 and 5. Figures 4 and 5 illustrate that $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ approximates $\Theta(\boldsymbol{N},\boldsymbol{\beta})$ very well, and in particular at the maximums. Therefore, we propose to study $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ further to provide us with analytical insights that support observations *(a)* and *(b)*.

THEOREM 2.

(a) *If $\beta_i = \beta_j$ for all components $i, j \in J$, $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ increases when the difference in installed base sizes decreases. That is, when $\beta_i = \beta_j$, $i, j \in J$ and $\sum_{i\in J} N_i = \overline{N}$ for some $\overline{N} \in \mathbb{N}$, if there exist $j, k \in J$ such that $N_j - N_k > 1$ then $\tilde{\Theta}(\boldsymbol{N} - e_j + e_k, \boldsymbol{\beta}) \geq \tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$, where e_j denotes the indicator vector of component $j \in J$.*

(b) *For any β_i and relaxed integrality of N_i for all components $i \in J$, $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ increases when the installed base sizes are ordered the same way as the relative unit cost factors. That is, if $l = |J|$ and $\beta_1 \leq \beta_2 \leq \ldots \leq \beta_l$, then the vector $\boldsymbol{N}^* \in \mathbb{R}^l$ that maximizes $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ such that $\sum_{i\in J} N_i^* = \overline{N}$ for some $\overline{N} \in \mathbb{N}$ satisfies $N_1^* \leq N_2^* \leq \ldots \leq N_l^*$.*

Proof. The denominator of $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ is constant, since $\sum_{i\in J} N_i = \overline{N}$ is constant for any $\overline{N} \in \mathbb{N}$.

(a) Let us recall the numerator of $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$:

$$\sum_{i\in J}\beta_i c(\tau_q^1)\left(1+\frac{rT+L(1+hT)}{\tau_q^1}\right)N_i+\sum_{i\in J}\beta_i c(\tau_q^1)bT\sqrt{\frac{\alpha N_i L}{\tau_q^1}}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right).$$

From the above, we see that the first summation of the numerator is constant, as $\beta_i=\beta_j$, $i,j\in J$. Hence, we focus on the numerator's second summation. Suppose that $\boldsymbol{N}$ is such that there exists $j,k\in J$ such that $N_j-N_k>1$, then a swap from one unit of j to k increases the second term of the numerator, and therefore we find that $\tilde{\Theta}(\boldsymbol{N}-e_j+e_k,\boldsymbol{\beta})\geq\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$. Indeed, note that

$$\begin{aligned}
&\sqrt{N_j-1}+\sqrt{N_k+1}+\sum_{i\in J\setminus\{j,k\}}\sqrt{N_i}-\sum_{i\in J}\sqrt{N_i}+\sum_{i\in J}\sqrt{N_i}\\
&\quad=\sqrt{N_j-1}+\sqrt{N_k+1}-\sqrt{N_j}-\sqrt{N_k}+\sum_{i\in J}\sqrt{N_i}\\
&\quad>\sqrt{N_k}+\sqrt{N_k+1}-\sqrt{N_k+1}-\sqrt{N_k}+\sum_{i\in J}\sqrt{N_i}=\sum_{i\in J}\sqrt{N_i},
\end{aligned}$$

where the inequality follows from the assumption that $N_j-N_k>1$.

(b) To prove the assertion, we relax the integrality of N_i. Furthermore, let us define $A=c(\tau_q^1)\left(1+\frac{rT+L(1+hT)}{\tau_q^1}\right)$, $B=c(\tau_q^1)bT\sqrt{\frac{\alpha L}{\tau_q^1}}\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)$, and $\overline{N}=\sum_{i\in J}N_i$. As the denominator of $\tilde{\Theta}(\boldsymbol{N},\boldsymbol{\beta})$ is constant, we are interested in maximizing the numerator:

$$\boldsymbol{N}^*=\arg\max_{\boldsymbol{N}}\left\{A\sum_{i\in J}N_i\beta_i+B\sum_{i\in J}\sqrt{N_i}\beta_i\,:\,\sum_{i\in J}N_i=\overline{N},N_i\geq 0\right\},$$

which is equivalent to

$$(\boldsymbol{N}^*,\boldsymbol{v})=\arg\max_{\boldsymbol{N},\boldsymbol{v}}\left\{A\sum_{i\in J}N_i\beta_i+B\sum_{i\in J}\sqrt{N_i}\beta_i\,:\,\sum_{i\in J}N_i=\overline{N},N_i-v_i^2=0\right\},$$

where $\boldsymbol{N}$ and $\boldsymbol{v}$ are the vectors of all N_i and v_i, $i\in J$. We square v_i to enforce non-negativity for any value of $v_i\in\mathbb{R}$. Consequently, the Lagrangian of the above problem is

$$\mathcal{L}(\boldsymbol{N},\boldsymbol{v},\lambda,\boldsymbol{\mu})=A\sum_{i\in J}N_i\beta_i+B\sum_{i\in J}\sqrt{N_i}\beta_i-\sum_{i\in J}\lambda(N_i-\overline{N})-\sum_{i\in J}\mu_i(N_i-v_i^2),$$

with $\boldsymbol{v}$, and $\boldsymbol{\mu}$ denoting the vectors of all v_i, $i \in J$ and μ_i, $i \in J$, respectively. The Lagrange multipliers are λ and $\boldsymbol{\mu}$. The first order conditions, required to maximize the Lagrangian, are given by:

$$\frac{\partial \mathcal{L}}{\partial N_i} = A\beta_i + \frac{\beta_i B}{2\sqrt{N_i}} - \lambda - \mu_i = 0, \quad \forall i \in J \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_i} = N_i - v_i^2 = 0, \quad \forall i \in J \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial v_i} = 2\mu_i v_i = 0, \quad \forall i \in J \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \sum_{i \in J} N_i - \overline{N} = 0. \tag{13}$$

$v_i = 0$ cannot occur, as Eq. (11) implies $N_i = 0$, which violates feasibility in Eq. (10). Thus, we have that $v_i > 0$ or $v_i < 0$. From Eq. (12) we have that $\mu_i = 0$, and from Eq. (11) we know that $N_i = v_i^2$. We use Eq. (10) to determine the optimal size of the installed base:

$$\sqrt{N_i^*} = \sqrt{v_i^{*2}} = \frac{B}{2}\frac{\beta_i}{\lambda - A\beta_i}.$$

Since we square v_i^*, we have that $\sqrt{v_i^{*2}} > 0$ for $v_i > 0$ and for $v_i < 0$, implying that $\lambda > A\beta_i$, because $A, B, \beta_i \geq 0$. The three cases for v_i show that $\mu_i = 0$ and $\lambda > A\beta_i$, for all $i \in J$ must hold in order to have a feasible solution. Thus, we have for the optimal installed base size:

$$N_i^* = \frac{B^2}{2}\left[\frac{\beta_i}{\lambda - A\beta_i}\right]^2.$$

Since $\lambda > A\beta_i$, for all $i \in J$, we have that N_i^* increases with increasing β_i. Therefore, if $\beta_i \leq \beta_j$ then $N_i^* \leq N_j^*$ for all $j \in J$. □

5.2. Benefits of the integrated LCC approach

Let us first describe the sequential approach in more detail. Typically, the engineering department of an OEM only makes reliability and commonality decisions based on production costs subject to some minimum reliability level $\underline{\tau}$. In particular, this corresponds to solving the optimization problems $\min_{\tau_q} \{\beta_q c(\tau_q) N_q : \tau_q \geq \underline{\tau}\}$ and $\min_{\boldsymbol{\tau}} \{\sum_{i \in J} \beta_i c(\tau_i) N_i : \tau_i \geq \underline{\tau}\}$ for the common component and dedicated components, respectively. The minimum costs under the sequential approach are denoted by $\pi_S(\tau_q^*, N_q, \beta_q)$ and $\sum_{i\in J} \pi_S(\tau_i^*, N_i, \beta_i)$ for the common and dedicated components model, respectively. Subsequently, the OEM selects the alternative with minimum costs. Note that the optimal reliability levels will satisfy $\tau_i^* = \underline{\tau}$ for all components $i \in I$, since $c(\tau)$ is convex and increasing in τ.

In case of the sequential approach, we find that commonality is only chosen if the cost of the common component is less than the weighted average of the costs of the dedicated components, see Proposition 1. All these instances correspond to the gray areas in Figures 4.

Proposition 1. *The sequential approach chooses commonality if and only if β_q is smaller than or equal to the weighted average of all β_i. That is, $\pi_S(\tau_q^*, N_q, \beta_q) \leq \sum_{i\in J} \pi_S(\tau_i^*, N_i, \beta_i)$ if and only if $\beta_q \leq \sum_{i\in J} \frac{N_i\beta_i}{N_q}$.*

Under the integrated LCC approach, we use the decision rule based on $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$, and consequently decide strictly more for commonality than the decision rule under the sequential approach, see Theorem 3. We also see this in Figure 4, because the area that favors commonality (under the curve) is strictly larger than the gray area, which corresponds to the decision rule of the sequential approach. Therefore, if we consider the sequential approach instead of the integrated LCC approach, we underestimate the attractiveness of commonality and take decisions that do not optimize the LCC in many instances.

Theorem 3. *If β_q is smaller or equal to the weighted average of all β_i, then commonality yields lower minimum LCC than dedicated components. That is, if $\beta_q \leq \sum_{i\in J} \frac{N_i\beta_i}{N_q}$, then $\pi(\tau_q^*, N_q, \beta_q) < \sum_{i\in J} \pi(\tau_i^*, N_i, \beta_i)$ and thus $\Theta(\boldsymbol{N}, \boldsymbol{\beta}) > \sum_{i\in J} \frac{N_i\beta_i}{N_q}$.*

The combined results from Proposition 1 and Theorem 3 explain why commonality is *always* more attractive than the alternative of dedicated components, if the dedicated components are equally expensive, cf. Figures 4a and 4d. In this case, commonality always lowers the LCC because it pools the spare parts of all dedicated components and thus reduces costs. As a result, the common component can be more expensive than all the dedicated components, and still yield lower LCC.

Although the integrated LCC approach strictly decides more for commonality compared to the sequential approach, we do not yet have an indication of the relative cost difference between both approaches. We define the relative difference between the sequential and integrated LCC approach by

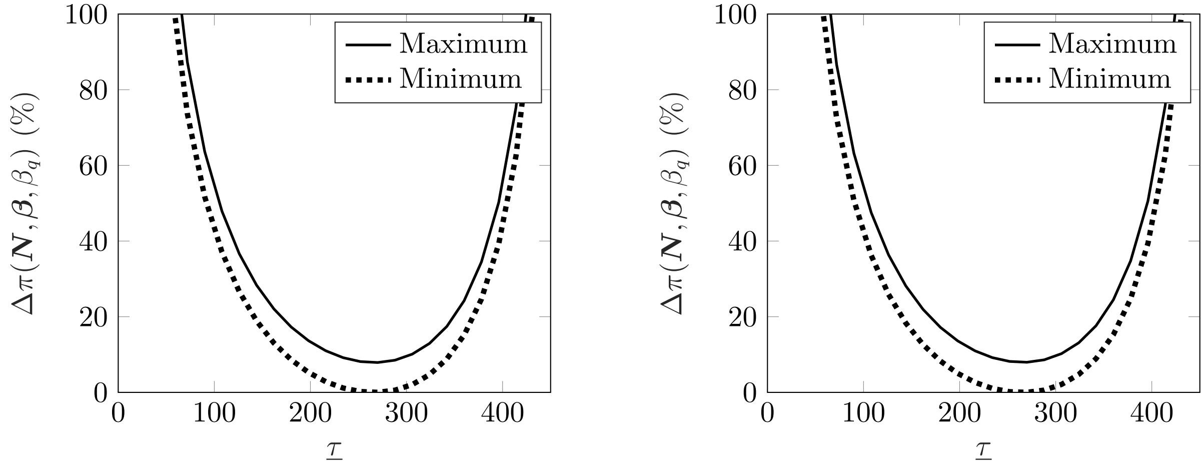
$$\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q) = \left(\frac{\gamma\pi(\underline{\tau}, N_q, \beta_q) + (1-\gamma)\sum_{i\in J}\pi(\underline{\tau}, N_i, \beta)}{\min\{\pi(\tau_q^*, N_q, \beta_q), \sum_{i\in J}\pi(\tau_i^*, N_i, \beta_i)\}} - 1 \right) \times (100\%),$$

with $\gamma = 1$ if $\pi_S(\underline{\tau}, N_q, \beta_q) \leq \sum_{i\in J}\pi_S(\underline{\tau}, N_i, \beta_i)$, and $\gamma = 0$ otherwise. We have that $\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q)$ is always non-negative, see Proposition 2. Furthermore, the relative cost difference between the sequential and the integrated LCC approach can be arbitrarily large, dependent on the minimum reliability level $\underline{\tau}$. To see this, recall that each optimal reliability level equals the minimum reliability level under the sequential approach ($\tau_i^* = \underline{\tau}$). Thus, we can increase the minimum LCC under the sequential approach to arbitrary values resulting in arbitrary large cost differences $\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q)$.

Next, we illustrate the cost difference $\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q)$ by considering the same numerical setting as Example 1, unless indicated otherwise. We set $b = \$1\cdot 10^7$ per backorder per month and $d = \$1\cdot 10^3$ per downtime incident. Furthermore, our instances are characterized by the tuple (N_1, β_q) such that $N_1 + N_2 = 400$ and $\beta_q \in [1, 1.2]$. For a given minimum reliability level $\underline{\tau}$, we explore the maximum and minimum cost difference over all possible (N_1, β_q). We have illustrated these in Figure 6 for two settings of (β_1, β_2). We observe that a substantial cost differences may exist (8%), even in the best case of the minimum reliability level.

Proposition 2. *The relative cost difference* $\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q)$ *is non-negative for all* $\boldsymbol{N}$, $\boldsymbol{\beta}$ *and* β_q.

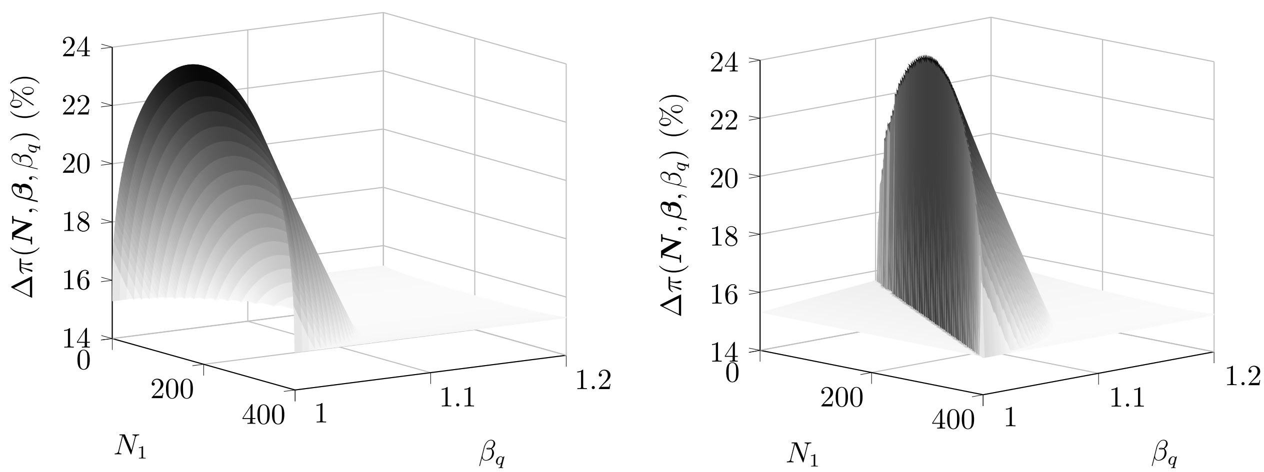
Next, we study the cost differences in more detail implying that we explicitly consider the installed base size under a given reliability level constraint, see Figure 7. We have used the same parameter values as in Example 1 with $b = \$1\cdot 10^7$ per month backorder, $d = \$1\cdot 10^3$ per downtime incident, $N_1 + N_2 = 400$, and we set the minimum reliability level to $\underline{\tau} = T$ months. Our first observation is that $\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q)$ is smaller when we have one relatively large installed base, see Figure 7 at the extremes of the N_1 axis. Therefore, if there exists one large installed base, we may



(a) $\beta_1 = \beta_2 = 1$ (b) $\beta_1 = 1, \beta_2 = 1.1$

Figure 6 Relative cost difference for different minimum reliability levels.

be able to make reasonable decisions under the sequential approach. However, this also depends on the value of the minimum reliability level.



(a) $\beta_1 = \beta_2 = 1$ (b) $\beta_1 = 1, \beta_2 = 1.1$

Figure 7 Numerical illustration of $\Delta\pi(\boldsymbol{N},\boldsymbol{\beta},\beta_q)$, for two dedicated components.

Hence, the sequential approach may only perform reasonable in case we have one relatively large installed base size and the minimum reliability level is set to the right value, which is extremely challenging to determine a priori. Therefore, we conclude that the sequential approach typically results in substantially higher LCC compared to the integrated LCC approach, and it underestimates the attractiveness of commonality largely. The OEM should implement the integrated LCC approach instead of the sequential approach, as the former better estimates the attractiveness of commonality and yields lower LCC, at the expense of very little implementation complexity.

5.3. Optimal reliability analysis

Finally, we study whether a common component is more or less reliable than the dedicated components at optimality, and we shed some light on this question by considering two practical cases. The first occurs when the replacement of a unit of component $i \in I$ does not induce an additional system down, e.g. for components subject to frequent inspections and preventive maintenance, or components that have redundancy to increase reliability. This corresponds to a situation in which we have a negligible cost per downtime incident, i.e., $d = 0$. The second case occurs when all components $i \in I$ are very similar to each other, e.g. the components only differ because they have a different serial number, but their functionality and designs are identical or similar. This is captured when $\beta_i = \beta_j$ for all components $i, j \in I$. Another example where $\beta_i = \beta_j$ may hold, can come from a sourcing environment where very similar components are sourced from different suppliers, e.g. aviation rotor blades with certain specifications from supplier X and rotor blades with the same specifications from supplier Y.

Before we start our analysis on the optimal reliability level of a common component compared to the optimal reliability levels of the dedicated components, we explore our cost function and derive a certain threshold $\tilde{\tau}$. This threshold corresponds to the optimal reliability level when we only consider the spare parts safety stock. In other words, when the reliability level is lower than the threshold, it is beneficial from a spare parts safety stock perspective to increase the reliability; and, if the reliability level is higher than the threshold, then spare parts are too expensive and one can better lower the reliability.

THEOREM 4. *There exists a positive, unique, and finite $\tilde{\tau}$ which is the solution to $c'(\tau) - \frac{c(\tau)}{2\tau} = 0$. Suppose $0 < N_1 \leq N_2 \leq \ldots \leq N_{|J|} < \infty$,* *(i) When* $d = 0$ *and* $\tilde{\tau} \geq rT + L(1 + hT)$*, then a larger installed base implies a lower optimal reliability level, i.e.,* $\infty > \tilde{\tau} \geq \tau_1^* \geq \tau_2^* \geq \ldots \geq \tau_q^* > 0$*.*

(ii) When $d = 0$ *and* $\tilde{\tau} < rT + L(1 + hT)$*, then a larger installed base implies a lower optimal reliability level, i.e.,* $0 < \tilde{\tau} < \tau_1^* < \tau_2^* < \ldots < \tau_q^* < \infty$*.*

(iii) When all β_i *are identical for all* $i \in J$ *and* $c(\tilde{\tau})[\tilde{\tau} - rT - L(1 + hT)] \geq 2dT$*, then a larger installed base implies a lower optimal reliability level, i.e.,* $\infty > \tilde{\tau} \geq \tau_1^* \geq \tau_2^* \geq \ldots \geq \tau_q^* > 0$*.*

(iv) When all β_i *are identical for all* $i \in J$ *and* $c(\tilde{\tau})[\tilde{\tau} - rT - L(1 + hT)] < 2dT$*, then a larger installed base implies a higher optimal reliability level, i.e.,* $0 < \tilde{\tau} < \tau_1^* < \tau_2^* < \ldots < \tau_q^* < \infty$*.*

For the cases considered, we have a simple condition that specifies whether a common component is more or less reliable than the dedicated components. The conditions denote whether the derivative of the LCC with respect to the reliability level is positive or negative, and consequently determines the behavior of the optimal reliability levels for increasing installed base sizes. Each condition's interpretation can be seen after rewriting the conditions such that we obtain the LCC derivative at $\tilde{\tau}$, cf. the proof of Theorem 4. In case the production costs dominate the after-sales services costs in determining the optimal reliability level, larger installed bases have a lower optimal reliability level. In other words, if the derivative of the production costs dominates the cost derivative of the after-sales services, larger installed base sizes will have less reliable components at optimality. In this case, it is too expensive to further invest in reliability as the investment cannot be earned back throughout the component's life cycle. Similarly, components with a larger installed base are more reliable if extra reliability investments at production *can* be earned back throughout the component's life cycle.

Although we have been able to characterize cases in which we can easily determine whether the common component is more (less) reliable than all dedicated components, we cannot say that such a simple condition exists for arbitrary instances. Therefore, managerial care is needed in answering the reliability question when one encounters an instance other than the ones considered in Theorem 4.

6. Conclusion

We have considered an the choice of an OEM to make components common and studied how the LCC are affected by this decision. We have further studied how optimal reliability levels of components are affected by the commonality decision. In particular, we have formulated two stylized LCC models: one commonality model, and one model for the alternative of dedicated components. We proposed asymptotically equivalent LCC models that facilitate analytical comparison of decisions.

We found that commonality is even more attractive when taking the LCC perspective, than it is when only considering production costs. Unfortunately, the LCC is often not used in practice because the related decision of commonality, reliability, and spare parts stocking are made sequentially by different departments within an OEM. such a sequential decision making approach underestimates the the attractiveness of commonality and can lead to arbitrarily higher LCC compared to the optimal approach. OEMs that sell full service contracts together with their equipment, can make much better decisions by explicitly incorporating the after-service costs in their optimizations.

Finally, we found that – for two practical cases – a common component is more (less) reliable than all dedicated components, only if the unit costs for initial production can (cannot) be earned back during the time the unit of a component is used.

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Appendix A: Proof of Lemma 1

Let us first write:

$$\begin{aligned}\tilde{\pi}(\tau_i, s_i) &= \beta_i c(\tau_i)(N_i + s_i) + hs_iT\beta_i c(\tau_i) + (r\beta_i c(\tau_i) + d)\frac{N_iT}{\tau_i} + bT\int_{s_i}^{\infty}(x - s_i)f_i(x)dx \\ &= \beta_i c(\tau_i)(N_i + s_i) + hs_iT\beta_i c(\tau_i) + (r\beta_i c(\tau_i) + d)\frac{N_iT}{\tau_i} + bT\mathbb{E}[D_i(L, N_i, \tau_i)] - bTs_i + bT\int_0^{s_i}(s_i - x)f_i(x)dx,\end{aligned} \tag{14}$$

with $f_i(x)$ the pdf of $D_i(L, N_i, \tau_i)$. From Leibniz' rule, we obtain the first order derivative: $\frac{\partial \tilde{\pi}(\tau_i, s_i)}{\partial s_i} = \beta_i c(\tau_i) + \beta_i c(\tau_i)hT - bT + bT\int_0^{s_i} f_i(x)dx = \beta_i c(\tau_i) + \beta_i c(\tau_i)hT - bT + bTF(s_i)$, with $F(s_i)$ the cdf of $D_i(L, N_i, \tau_i)$. Applying Leibniz' rule again, we get the second order derivative $\frac{\partial^2 \tilde{\pi}(\tau_i, s_i)}{\partial {s_i}^2} = bTf_i(s_i) > 0$, as $b, T > 0$, and $f_i(s_i) > 0$ by definition. Hence, $\tilde{\pi}(\tau_i, s_i)$ is twice differentiable and strictly convex in s_i. Next, we prove the existence of a positive, unique, finite $s_i^*(\tau_i)$ that solves the first order condition.

(i) First, we prove that $\tilde{\pi}(\tau_i, s_i)$ strictly decreasing at $s_i = 0$. Consider the derivative $\frac{\partial \tilde{\pi}(\tau_i, s_i)}{\partial s_i}$ as $s_i = 0$:

$$\begin{aligned}\left.\frac{\partial \tilde{\pi}(\tau_i, s_i)}{\partial s_i}\right|_{s_i=0} &= \beta_i c(\tau_i) + \beta_i c(\tau_i)hT - bT + bTF_i(s_i) = \beta_i c(\tau_i) + \beta_i c(\tau_i)hT - bT + bT\Phi\left(-\frac{\mathbb{E}[D_i(L, N_i, \tau_i)]}{\sigma[D_i(L, N_i, \tau_i)]}\right) \\ &\leq \beta_i c(\tau_i) + \beta c(\tau_i)hT - \frac{bT}{2} < 0,\end{aligned}$$

where the second equality follows because we consider normally distributed demand during L, and the first inequality follows because $\Phi\left(-\frac{\mathbb{E}[D_i(L,N_i,\tau_i)]}{\sigma[D_i(L,N_i,\tau_i)]}\right) \leq 1/2$, with $\Phi(\cdot)$ denoting the standard normal cdf. The final inequality follows from Assumption 1. Hence, we conclude that $\tilde{\pi}(\tau_i, s_i)$ is strictly decreasing at $s_i = 0$.

(ii) Let us now prove that $\tilde{\pi}(\tau_i, s_i)$ is strictly increasing as s_i tends to infinity. Consider the derivative $\frac{\partial \tilde{\pi}(\tau_i, s_i)}{\partial s_i}$ as $s_i \to \infty$, i.e., $\lim_{s_i \to \infty} \frac{\partial \tilde{\pi}(\tau_i, s_i)}{\partial s_i} = \lim_{s_i \to \infty} \{\beta_i c(\tau_i) + \beta_i c(\tau_i)hT - bT + bTF_i(s_i)\} = \beta_i c(\tau_i)(1 + hT) > 0$, where the second equality follows from the definition of $F_i(s_i)$, i.e., $\lim_{s_i \to \infty} F_i(s_i) = 1$. The inequality follows from $c(\tau_i) > 0$ for all $\tau_i \in (0, \infty)$, and $\beta_i, h, T > 0$. Thus, $\tilde{\pi}(\tau_i, s_i)$ is strictly increasing as s_i tends to infinity.

By combining *(i)*, *(ii)* and the strict convexity of $\pi(\tau_i, s_i)$, we know that there exists a positive, unique, finite optimum $s_i^*(\tau_i)$ that solves the first order condition. Exploiting the properties of the normal distribution, and standardization yields

$$s_i^*(\tau_i) = \mathbb{E}[D_i(L, N_i, \tau_i)] + \sigma[D_i(L, N_i, \tau_i)]\Phi^{-1}\left(\frac{bT - \beta_i c(\tau_i)(1 + hT)}{bT}\right).$$

□

Appendix B: Proof of Lemma 2

For given $\tau_i \in (0, \infty)$, we let b be such that we satisfy Assumption 1. For each $i \in I$ we have

$$\begin{aligned}0 \leq \frac{b\mathbb{E}[(D_i(L, N_i, \tau_i) - s_i^*(\tau_i, b))^+]}{s_i^*(\tau_i, b)} &= \frac{b\mathbb{P}\left[D_i(L, N_i, \tau_i) > s_i^*(\tau_i, b)\right]\mathbb{E}\left[D_i(L, N_i, \tau_i) - s_i^*(\tau_i, b) \mid D_i(L, N_i, \tau_i) > s_i^*(\tau_i, b)\right]}{s_i^*(\tau_i, b)} \\ &= \frac{\beta c(\tau_i)(1 + hT)}{T} \times \frac{\mathbb{E}\left[D_i(L, N_i, \tau_i) - s_i^*(\tau_i, b) \mid D_i(L, N_i, \tau_i) > s_i^*(\tau_i, b)\right]}{s_i^*(\tau_i, b)},\end{aligned}$$

where the first inequality follows from Eq. (14) in the proof of Lemma 1. The second inequality follows from the definition of the optimal turnaround stock level. That is, we know that $s_i^*(\tau_i,b)$ satisfies $\frac{\partial\tilde{\pi}(\tau_i,s_i)}{\partial s_i}=0$, which implies $\mathbb{P}[D_i(L,N_i,\tau_i)>s_i^*(\tau_i,b)]=\frac{\beta c(\tau_i)(1+hT)}{bT}$ by the right continuity of the distribution function (Huh et al. 2009, p. 409). Then, for the limit of $b\to\infty$, Assumption 1 is satisfied for any finite $\tau_i\in(0,\infty)$ and thus

$$
\begin{aligned}
0 &\le \lim_{b\to\infty}\frac{b\mathbb{E}[(D_i(L,N_i,\tau_i)-s_i^*(\tau_i,b))^+]}{s_i^*(\tau_i,b)}\\
&\le \lim_{b\to\infty}\frac{\beta c(\tau_i)(1+hT)}{T}\times\frac{\mathbb{E}\left[D_i(L,N_i,\tau_i)-s_i^*(\tau_i,b)\mid D_i(L,N,\tau_i)>s_i^*(\tau_i,b)\right]}{s_i^*(\tau_i,b)}=0.
\end{aligned}
$$

The equality follows from the fact that $s_i^*(\tau_i,b)\to\infty$ as $b\to\infty$ and $\frac{\mathbb{E}[D_i(L,N_i,\tau_i)-s_i^*(\tau_i,b)\mid D_i(L,N_i,\tau_i)>s_i^*(\tau_i,b)]}{s_i^*(\tau_i,b)}\to 0$ as $b\to\infty$ due to the increasing failure rate of a normal distribution, see Huh et al. (2009, p. 409). $\square$

Appendix C: Proof of Lemma 3

Let us write $\pi(\tau_i)$ in terms of the normalized loss function of the normal distribution, with $\hat{s}_i=s_i^*(\tau_i,b\beta c(\tau_i))$.

$$
\begin{aligned}
\pi(\tau_i)=&\beta_i c(\tau_i)(N_i+\hat{s}_i)+h\hat{s}_iT\beta_i c(\tau_i)+(r\beta_i c(\tau_i)+d)\frac{N_iT}{\tau}+b\beta_i c(\tau_i)T\sigma[D_i(L,N_i,\tau_i)]\\
&\times\left\{\phi\left(\frac{\hat{s}_i-\mathbb{E}[D_i(L,N_i,\tau_i)]}{\sigma[D_i(L,N_i,\tau_i)]}\right)-\frac{\hat{s}_i-\mathbb{E}[D_i(L,N_i,\tau_i)]}{\sigma[D_i(L,N_i,\tau_i)]}\left(1-\Phi\left(\frac{\hat{s}_i-\mathbb{E}[D_i(L,N_i,\tau_i)]}{\sigma[D_i(L,N_i,\tau_i)]}\right)\right)\right\},
\end{aligned}
$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal pdf and cdf, respectively. Simplification yields the desired result. $\square$

Appendix D: Proof of Lemma 4

All terms are twice differentiable by assumption, and thus $\pi(\tau_i)$ is twice differentiable. We have

$$
\begin{aligned}
\frac{d^2\pi(\tau_i)}{d\tau_i^2}=&c''(\tau_i)N_i\beta_i+\beta_i(rT+L(1+hT))N_i\left(\frac{c''(\tau_i)}{\tau_i}-2\frac{c'(\tau_i)}{\tau_i^2}+2\frac{c(\tau_i)}{\tau_i^3}\right)+2d\frac{N_iT}{\tau_i^2}\\
&+\left(c''(\tau_i)\tau_i^{-1/2}-c'(\tau_i)\tau_i^{-3/2}+\frac{3}{4}c(\tau_i)\tau_i^{-5/2}\right)\beta_ibT\phi\left(\Phi^{-1}\left(\frac{bT-1-hT)}{bT}\right)\right)\sqrt{\alpha N_iL}\\
=&c''(\tau_i)N_i\beta_i+\beta_i(rT+L(1+hT))\frac{N_i}{\tau_i^3}\left(\tau_i^2c''(\tau_i)-2\tau_ic'(\tau_i)+2c(\tau_i)\right)+2d\frac{N_iT}{\tau_i^2}\\
&+\tau_i^{-5/2}\left(\tau_i^2c''(\tau_i)-\tau_ic'(\tau_i)+\frac{3}{4}c(\tau_i)\right)\beta_ibT\phi\left(\Phi^{-1}\left(\frac{bT-1-hT)}{bT}\right)\right)\sqrt{\alpha N_iL}>0,
\end{aligned}
$$

since $c''(\tau_i)>0$ (by assumption) and $c(\tau_i)$ satisfying Assumption 2. The latter can be seen by the second order derivative test applied to the conditions in Assumption 2. Hence, $\pi(\tau_i)$ is twice differentiable and strictly convex in τ_i. The next step is to show that there exists a positive, unique, finite τ_i^* that minimizes $\pi(\tau_i)$, and that this τ_i^* solves the first order condition.

(i) First, let us prove that $\pi(\tau_i)$ is strictly decreasing for $\tau_i \downarrow 0$. The derivative of $\pi(\tau_i)$ is given by

$$\frac{d\pi(\tau_i)}{d\tau_i} = c'(\tau_i)N_i\beta_i + (rT + L(1+hT))N_i\beta_i\left(\frac{c'(\tau_i)}{\tau_i} - \frac{c(\tau_i)}{\tau_i^2}\right) - d\frac{N_iT}{\tau_i}$$
$$+\left(c'(\tau_i)\tau_i^{-1/2} - \frac{1}{2}c(\tau_i)\tau_i^{-3/2}\right)\beta_i bT\phi\left(\Phi^{-1}\left(\frac{bT-1-hT)}{bT}\right)\right)\sqrt{\alpha N_i L}.$$

Rewriting $\frac{c'(\tau_i)}{\tau_i} - \frac{c(\tau_i)}{\tau_i^2}$ and $c'(\tau_i)\tau_i^{-1/2} - \frac{1}{2}c(\tau_i)\tau_i^{-3/2}$, and subsequently taking the limit $\tau_i \downarrow 0$ of $\frac{d\pi(\tau_i)}{d\tau_i}$, yields

$$\lim_{\tau_i\downarrow 0}\frac{d\pi(\tau_i)}{d\tau_i} = \lim_{\tau_i\downarrow 0}\{c'(\tau_i)N_i\beta_i\} + (rT + L(1+hT))N_i\beta_i \lim_{\tau_i\downarrow 0}\left\{\frac{\tau_i c'(\tau_i) - c(\tau_i)}{\tau_i^2}\right\} - dN_iT\lim_{\tau_i\downarrow 0}\left\{\frac{1}{\tau_i}\right\}$$
$$+\beta_i bT\phi\left(\Phi^{-1}\left(\frac{bT-1-hT)}{bT}\right)\right)\sqrt{\alpha N_i L}\lim_{\tau_i\downarrow 0}\left\{\frac{\tau_i c'(\tau_i) - \frac{1}{2}c(\tau_i)}{\tau_i\sqrt{\tau_i}}\right\}.$$

Since $0 \le \lim_{\tau_i\downarrow 0} c(\tau_i) < \infty$ and $0 \le \lim_{\tau_i\downarrow 0} c'(\tau_i) < \infty$, we have $\lim_{\tau_i\downarrow 0}\left\{\frac{\tau_i c'(\tau_i) - c(\tau_i)}{\tau_i^2}\right\} = -\infty$ and $\lim_{\tau_i\downarrow 0}\left\{\frac{\tau_i c'(\tau_i) - \frac{1}{2}c(\tau_i)}{\tau_i\sqrt{\tau_i}}\right\} = -\infty$. Hence, $\lim_{\tau_i\downarrow 0}\frac{d\pi(\tau_i)}{d\tau_i} = -\infty$, and $\pi(\tau_i)$ is strictly decreases as $\tau_i \downarrow 0$.

(ii) Let us now prove that $\pi(\tau_i)$ is strictly increasing as τ_i tends to infinity. Consider the derivative $\frac{d\pi(\tau_i)}{d\tau_i}$ as $\tau_i \to \infty$:

$$\lim_{\tau_i\to\infty}\frac{d\pi(\tau_i)}{d\tau_i} = \lim_{\tau_i\to\infty}\{c'(\tau_i)N_i\beta_i\} + (rT + L(1+hT))N_i\beta_i \lim_{\tau_i\to\infty}\left\{\frac{\tau_i c'(\tau_i) - c(\tau_i)}{\tau_i^2}\right\} - dN_iT\lim_{\tau_i\to\infty}\left\{\frac{1}{\tau_i}\right\}$$
$$+\beta_i bT\phi\left(\Phi^{-1}\left(\frac{bT-1-hT)}{bT}\right)\right)\sqrt{\alpha N_i L}\lim_{\tau_i\to\infty}\left\{\frac{\tau_i c'(\tau_i) - \frac{1}{2}c(\tau_i)}{\tau_i\sqrt{\tau_i}}\right\}$$

We know that $\lim_{\tau_i\to\infty}\{c'(\tau_i)N_i\beta_i\} = +\infty$. Furthermore,

$$\lim_{\tau_i\to\infty}\left\{\frac{\tau_i c'(\tau_i) - c(\tau_i)}{\tau_i^2}\right\} = \lim_{\tau_i\to\infty}\left\{\frac{1}{\tau_i}\left(c'(\tau_i) - \frac{c(\tau_i)}{\tau_i}\right)\right\}$$
$$> \lim_{\tau_i\to\infty}\left\{\frac{1}{\tau_i}\left(\frac{c(\tau_i)}{\tau_i} - \frac{\lim_{\widehat{\tau}_i\downarrow 0}[c(\widehat{\tau}_i)]}{\tau_i} - \frac{c(\tau_i)}{\tau_i}\right)\right\} = \lim_{\tau_i\to\infty}\left\{-\frac{1}{\tau_i^2}\lim_{\widehat{\tau}_i\downarrow 0}[c(\widehat{\tau}_i)]\right\} = 0.$$

where the inequality follows from the strict convexity of $c(\tau_i)$, which implies $c'(\tau_i) > \frac{c(\tau_i) - \lim_{\widehat{\tau}_i\downarrow 0}[c(\widehat{\tau}_i)]}{\tau_i}$. Moreover, it holds by assumption that $\lim_{\widehat{\tau}_i\downarrow 0} c(\widehat{\tau}_i) < \infty$. Similarly, we obtain

$$\lim_{\tau_i\to\infty}\left\{\frac{\tau_i c'(\tau_i) - \frac{1}{2}c(\tau_i)}{\tau_i\sqrt{\tau_i}}\right\} > \lim_{\tau_i\to\infty}\left\{\frac{\frac{1}{2}c(\tau_i) - \lim_{\widehat{\tau}_i\downarrow 0}[c(\widehat{\tau}_i)]}{\tau_i\sqrt{\tau_i}}\right\} > \lim_{\tau_i\to\infty}\left\{-\frac{1}{\tau_i\sqrt{\tau_i}}\lim_{\widehat{\tau}_i\downarrow 0}[c(\widehat{\tau}_i)]\right\} = 0,$$

where the first inequality follows from the strict convexity of $c(\tau_i)$, i.e., $c'(\tau_i) > \frac{c(\tau_i) - \lim_{\widehat{\tau}_i\downarrow 0}[c(\widehat{\tau}_i)]}{\tau_i}$, and the second inequality follows from $c(\tau_i) > 0$ for all $\tau_i \in (0,\infty)$. Hence, we have $\lim_{\tau_i\to\infty}\frac{d\pi(\tau_i)}{d\tau_i} = +\infty$ and $\pi(\tau_i)$ is strictly increasing as $\tau_i \to \infty$. By combining *(i)* and *(ii)* with the strict convexity of $\pi(\tau_i)$, we obtain the desired result. □

Appendix E: Proof of Lemma 5

Take any $\beta_q > 0$ and $\tilde{\beta}_q > \beta_q$. Let us denote τ_q^* and $\tilde{\tau}_q^*$ as the optimal reliability levels under β_q and $\tilde{\beta}_q$, respectively. Then, we have $\pi(\tau_q^*, N_q, \beta_q) \le \pi(\tilde{\tau}_q^*, N_q, \beta_q) < \pi(\tilde{\tau}_q^*, N_q, \tilde{\beta}_q)$, where the first inequality follows from optimality, and the last inequality follows from the linear dependency of $\pi(\tau_q^*, N_q, \beta_q)$ on β_q, combined with $\tilde{\beta}_q > \beta_q$. Thus $\pi(\tau_q^*, N_q, \beta_q)$ is monotone increasing in β_q. Next, we show that there exists a positive, finite and unique β_q that satisfies $\pi(\tau_q^*, N_q, \beta_q) - \sum_{i\in J} \pi(\tau_i^*, N_i, \beta_i) = 0$. Let us first consider β_q approaching 0, i.e., $\lim_{\beta_q \downarrow 0} \pi(\tau_q^*, N_q, \beta_q) = \lim_{\beta_q \downarrow 0} dT \frac{N_q}{\tau_q^*} = 0$, where the final equality follows from the fact that $\tau_q^* \to \infty$ when $\beta_q \downarrow 0$. Secondly, we consider β_q approaching infinity, i.e., $\lim_{\beta_q \to \infty} \pi(\tau_q^*, N_q, \beta_q) = \infty$, where the final equality holds because τ_q^* is a finite solution (Lemma 4) for any β_q. Hence, there exists a positive, finite and unique β_q such that $\pi(\tau_q^*, N_q, \beta_q) - \sum_{i\in J} \pi(\tau_i^*, N_i, \beta_i) = 0$. □

Appendix F: Proof of Proposition 1

($\Rightarrow$) Let $\beta_q \leq \sum_{i\in J} \frac{N_i \beta_i}{N_q}$. Then, we have for the costs of the common component:

$$\pi_S(\tau_q^*, N_q, \beta_q) \leq \sum_{i\in J} \frac{N_i \beta_i}{N_q} N_q c(\tau_q^*) = \sum_{i\in J} \beta_i c(\underline{\tau}) N_i = \sum_{i\in J} \pi_S(\tau_i^*, N_i, \beta_i),$$

where the inequality follows from inserting $\beta_q \leq \sum_{i\in J} \frac{N_i \beta_i}{N_q}$, and the first equality follows from $\tau_i^* = \underline{\tau}$.

($\Leftarrow$) Let $\pi_S(\tau_q^*, N_q, \beta_q) \leq \sum_{i\in J} \pi_m(\tau_i^*, N_i, \beta_i)$. We have,

$$\pi_S(\tau_q^*, N_q, \beta_q) = c(\underline{\tau})\beta_q N_q \leq \sum_{i\in J} N_i \beta_i c(\tau_i^*) = \sum_{i\in J} N_i \beta_i c(\underline{\tau}),$$

where the first and final equality follow from $\tau_i^* = \underline{\tau}$. The inequality follows by assumption. Rewriting the above yields $\beta_q \leq \sum_{i\in J} \frac{N_i \beta_i}{N_q}$. □

Appendix G: Proof of Theorem 3

We will consider $\beta_q = \sum_{i\in J} \frac{N_i \beta_i}{N_q}$, because $\pi(\tau_q^*, N_q, \beta_q)$ is increasing in β_q, see Lemma 5. We rewrite the cost function to obtain

$$\begin{aligned}
&\pi(\tau_q^*, N_q, \beta_q) \\
&= \sum_{i\in J} \frac{N_i}{N_q} \beta_i c(\tau_q^*) \left(1 + \frac{rT + L(1+hT)}{\tau_q^*}\right) N_q + \sum_{i\in J} \frac{N_i}{N_q} d \frac{N_q T}{\tau_q^*} + \sum_{i\in J} \frac{N_i}{N_q} b\beta_i c(\tau_q^*) T \sqrt{\frac{\alpha N_q L}{\tau_q^*}} \phi\left(\Phi^{-1}\left(\frac{bT - 1 - hT}{bT}\right)\right) \\
&\leq \sum_{i\in J} \frac{N_i}{N_q} \beta_i c(\tau_i^*) \left(1 + \frac{rT + L(1+hT)}{\tau_i^*}\right) N_q + \sum_{i\in J} \frac{N_i}{N_q} d \frac{N_q T}{\tau_i^*} + \sum_{i\in J} \frac{N_i}{N_q} b\beta_i c(\tau_i^*) T \sqrt{\frac{\alpha N_q L}{\tau_i^*}} \phi\left(\Phi^{-1}\left(\frac{bT - 1 - hT}{bT}\right)\right) \\
&= \sum_{i\in J} \beta_i c(\tau_i^*) \left(1 + \frac{rT + L(1+hT)}{\tau_i^*}\right) N_i + \sum_{i\in J} d \frac{N_i T}{\tau_i^*} + \sum_{i\in J} \frac{N_i}{N_q} b\beta_i c(\tau_i^*) T \sqrt{\frac{\alpha N_q L}{\tau_i^*}} \phi\left(\Phi^{-1}\left(\frac{bT - 1 - hT}{bT}\right)\right) \\
&< \sum_{i\in J} \beta_i c(\tau_i^*) \left(1 + \frac{rT + L(1+hT)}{\tau_i^*}\right) N_i + \sum_{i\in J} d \frac{N_i T}{\tau_i^*} + \sum_{i\in J} b\beta_i c(\tau_i^*) T \sqrt{\frac{\alpha N_i L}{\tau_i^*}} \phi\left(\Phi^{-1}\left(\frac{bT - 1 - hT}{bT}\right)\right) \\
&= \sum_{i\in J} \pi(\tau_i^*, N_i, \beta_i),
\end{aligned}$$

where the first inequality follows from inserting sub-optimal values τ_i^* instead of the optimal τ_q^*. The last inequality follows from the fact that $\frac{N_i}{N_q}\sqrt{N_q} = \sqrt{N_i}\sqrt{\frac{N_i}{N_q}} < \sqrt{N_i}$ for all $i \in J$. Hence, we conclude that $\Theta(\boldsymbol{N}, \boldsymbol{\beta}) > \sum_{i\in J} \frac{N_i \beta_i}{N_q}$ by definition of $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$. □

Appendix H: Proof of Proposition 2

Let $\boldsymbol{\beta}$ be the vector of all relative unit cost factors β_i for all $i \in J$. For any $\boldsymbol{N}$ and β_q, we can have three cases for optimal solutions: (i) both approaches' optimal solution is commonality; (ii) the optimal solution for both is dedicated; and (iii) the optimal decision of the LCC approach is commonality, while the sequential approach prefers dedicated. In the first two cases, we have that LCC approach yields lower costs. To see this, let $\tau_q^* = \underline{\tau}$ ($\tau_i^* = \underline{\tau}$) for the first (second) case. Hence, we have that the LCC approach yields lower LCC costs by sub-optimality of $\underline{\tau}$, and consequently $\Delta\pi(\boldsymbol{N}, \boldsymbol{\beta}, \beta_q) \geq 0$ in these cases. The third case is proven by considering the sub-optimal decision of the LCC approach to choose dedicated. In this case, we obtain case (ii) for which we have proven the assertion. $\square$

Appendix I: Proof of Theorem 4

We start our proof by showing that there exists a positive, unique, and finite $\tilde{\tau}$ which is the solution to $c'(\tau) - \frac{c(\tau)}{2\tau} = 0$. Subsequently, we will prove the existence of the two cases.

We first show that $c'(\tau) - \frac{c(\tau)}{2\tau}$ is monotonically increasing in τ. Since $c(\tau)$ is twice differentiable in τ, we obtain the derivative of $c'(\tau) - \frac{c(\tau)}{2\tau}$:

$$c''(\tau) + \frac{c(\tau)}{2\tau^2} - \frac{c'(\tau)}{2\tau} = \frac{1}{4\tau^2}\left(\tau^2 c''(\tau) - 2\tau c'(\tau) + 2c(\tau) + 3\tau^2 c''(\tau)\right) \geq 0,$$

where the inequality follows from Assumption 2 (via the second order derivative test) combined with the convexity of $c(\tau)$ that implies $3\tau^2 c''(\tau) \geq 0$. Hence, the derivative of $c'(\tau) - \frac{c(\tau)}{2\tau}$ is non-negative, implying that $c'(\tau) - \frac{c(\tau)}{2\tau}$ is a monotone increasing function. Next, we show that $\lim_{\tau\downarrow 0} c'(\tau) - \frac{c(\tau)}{2\tau} = -\infty$, and that $\lim_{\tau\to\infty} c'(\tau) - \frac{c(\tau)}{2\tau} > 0$. First, consider the behavior when $\tau \downarrow 0$. That is, $\lim_{\tau\downarrow 0}\left\{c'(\tau) - \frac{c(\tau)}{2\tau}\right\} = \lim_{\tau\downarrow 0} c'(\tau) - \lim_{\tau\downarrow 0}\frac{c(\tau)}{2\tau} = -\infty$, where the last equality follows from the assumption that $0 \leq \lim_{\tau\downarrow 0} c'(\tau) < \infty$, and $0 \leq \lim_{\tau\downarrow 0} c(\tau) < \infty$. Secondly, we get the following as τ tends to infinity: $\lim_{\tau\to\infty}\left\{c'(\tau) - \frac{c(\tau)}{2\tau}\right\} > \lim_{\tau\to\infty}\left\{\frac{c(\tau)}{\tau} - \frac{\lim_{\hat{\tau}\downarrow 0}[c(\hat{\tau})]}{\tau} - \frac{c(\tau)}{2\tau}\right\} = \lim_{\tau\to\infty}\left\{\frac{c(\tau)}{2\tau}\right\} - \lim_{\tau\to\infty}\left\{\frac{\lim_{\hat{\tau}\downarrow 0}[c(\hat{\tau})]}{\tau}\right\} = \lim_{\tau\to\infty}\left\{\frac{c(\tau)}{2\tau}\right\} \geq 0$, where the first inequality follows from the convexity of $c(\tau)$, and the second equality follows from the assumption $0 \leq \lim_{\tau\downarrow 0} c(\tau) < \infty$. Hence, we conclude that there exists a unique, positive and finite $\tilde{\tau}$ which is the solution to $c'(\tau) - \frac{c(\tau)}{2\tau} = 0$.

Next, we prove all cases of Theorem 4 by induction. The proof of each case starts by deriving the induction hypothesis from the condition as stated in Theorem 4.

(i) Let $\tilde{\tau} \geq rT + L(1+hT)$. Rewriting this condition yields the equivalent formulation: $1 - \frac{rT+L(1+hT)}{\tilde{\tau}} \geq 0$. This implies for component 1 that $N_1\beta_1 \left\{ \frac{c(\tilde{\tau})}{2\tilde{\tau}} \left(1 - \frac{rT+L(1+hT)}{\tilde{\tau}}\right)\right\} \geq 0$ since $c(\tilde{\tau}), \tilde{\tau}, N_1, \beta_1 > 0$. Then,

$$\begin{aligned}
&N_1\beta_1 \left\{ \frac{c(\tilde{\tau})}{2\tilde{\tau}} \left(1 - \frac{rT+L(1+hT)}{\tilde{\tau}}\right)\right\} = N_1\beta_1 \left\{ c'(\tilde{\tau}) - \frac{rT+L(1+hT)}{\tilde{\tau}} \frac{c(\tilde{\tau})}{2\tilde{\tau}} \right\} \\
&\quad = N_1\beta_1 \left\{ c'(\tilde{\tau}) + \frac{rT+L(1+hT)}{\tilde{\tau}} \left(\frac{c(\tilde{\tau})}{2\tilde{\tau}} - \frac{c(\tilde{\tau})}{\tilde{\tau}} \right)\right\} = N_1\beta_1 \left\{ c'(\tilde{\tau}) + \frac{rT+L(1+hT)}{\tilde{\tau}} \left(c'(\tilde{\tau}) - \frac{c(\tilde{\tau})}{\tilde{\tau}} \right)\right\} \\
&\quad = N_1\beta_1 \left\{ c'(\tilde{\tau}) + \frac{rT+L(1+hT)}{\tilde{\tau}} \left(c'(\tilde{\tau}) - \frac{c(\tilde{\tau})}{\tilde{\tau}} \right) + \left(c'(\tilde{\tau}) - \frac{c(\tilde{\tau})}{2\tilde{\tau}} \right) bT\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)\sqrt{\frac{\alpha L}{N_1\tilde{\tau}}} \right\} \\
&\quad = \left.\frac{\partial \pi(\tau_1, N_1, \beta_1)}{\partial \tau_1}\right|_{\tau_1=\tilde{\tau}},
\end{aligned}$$

where the first, third, and fourth equality hold by the existence of $\tilde{\tau}$. Hence, we have $\left.\frac{\partial \pi(\tau_1,N_1,\beta_1)}{\partial \tau_1}\right|_{\tau_1=\tilde{\tau}} = N_1\beta_1 \left\{ \frac{c(\tilde{\tau})}{2\tilde{\tau}} \left(1 - \frac{rT+L(1+hT)}{\tilde{\tau}}\right)\right\} \geq 0$. Furthermore, we know that $\left.\frac{\partial \pi(\tau_1,N_1,\beta_1)}{\partial \tau_1}\right|_{\tau_1=\tau_1^*} = 0$ by optimality of τ_1^*, implying that $\left.\frac{\partial \pi(\tau_1,N_1,\beta_1)}{\partial \tau_1}\right|_{\tau_1=\tilde{\tau}} \geq \left.\frac{\partial \pi(\tau_1,N_1,\beta_1)}{\partial \tau_1}\right|_{\tau_1=\tau_1^*} = 0$. Since the derivative of the cost function in $\tau_1 = \tilde{\tau}$, under N_1 and β_1, is larger than 0, we know by the convexity of $\pi(\tau_1, N_1, \beta_1)$ (Lemma 4) that $\tau_1^* \leq \tilde{\tau}$.

In the next part of the proof, we will show the monotonicity of τ_i^* via induction. That is, we show that $\tau_i^* \leq \tau_{i-1}^*$, for all $i \in I$. Let $\tau_0^* := \tilde{\tau}$. We know that $\tau_1^* \leq \tilde{\tau} = \tau_0^*$. Let $0 < N_{i-1} \leq N_i < \infty$, for all $i \in I$. For the induction hypothesis, suppose that $\tau_i^* \leq \tau_{i-1}^*$. We know that

$$\begin{aligned}
\left.\frac{\partial \pi(\tau_i, N_i, \beta_i)}{\partial \tau_i}\right|_{\tau_i=\tau_i^*} &= N_i\beta_i \left\{ c'(\tau_i^*) + \frac{rT+L(1+hT)}{\tau_i^*} \left(c'(\tau_i^*) - \frac{c(\tau_i^*)}{\tau_i^*} \right) \right. \\
&\quad \left. + \left(c'(\tau_i^*) - \frac{c(\tau_i^*)}{2\tau_i^*} \right) bT\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)\sqrt{\frac{\alpha L}{N_i\tau_i^*}} \right\} = 0, \qquad (15)
\end{aligned}$$

which implies that

$$c'(\tau_i^*) + \frac{rT+L(1+hT)}{\tau_i^*} \left(c'(\tau_i^*) - \frac{c(\tau_i^*)}{\tau_i^*} \right) + \left(c'(\tau_i^*) - \frac{c(\tau_i^*)}{2\tau_i^*} \right) bT\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)\sqrt{\frac{\alpha L}{N_i\tau_i^*}} = 0,$$

since $\beta_i, N_i > 0$. We know that $c'(\tau_i^*) - \frac{c(\tau_i^*)}{2\tau_i^*} \leq 0$, by the monotonicity of $c'(\tau) - \frac{c(\tau)}{2\tau}$ (by the existence of $\tilde{\tau}$) and by $\tau_i^* \leq \tilde{\tau}$. The latter expressions is implied by the induction hypothesis. Hence, we find

$$c'(\tau_i^*) + \frac{rT+L(1+hT)}{\tau_i^*} \left(c'(\tau_i^*) - \frac{c(\tau_i^*)}{\tau_i^*} \right) + \left(c'(\tau_i^*) - \frac{c(\tau_i^*)}{2\tau_i^*} \right) bT\phi\left(\Phi^{-1}\left(\frac{bT-1-hT}{bT}\right)\right)\sqrt{\frac{\alpha L}{N_{i+1}\tau_i^*}} \geq 0,$$

as $0 < N_i \leq N_{i+1} < \infty$. This implies that $\left.\frac{\partial \pi(\tau_i,N_{i+1},\beta_i)}{\partial \tau_i}\right|_{\tau_i=\tau_i^*} \geq 0$, and observe that $\left.\frac{\partial \pi(\tau_i,N_i,\beta_i)}{\partial \tau_i}\right|_{\tau_i=\tau_i^*} = 0$ (Lemma 4). Hence, $\left.\frac{\partial \pi(\tau_i,N_{i+1},\beta_{i+1})}{\partial \tau_i}\right|_{\tau_i=\tau_i^*} \geq \left.\frac{\partial \pi(\tau_i,N_i,\beta_i)}{\partial \tau_i}\right|_{\tau_i=\tau_i^*} = 0$. Since $0 < N_i \leq N_{i+1} < \infty$ and the derivative, under N_{i+1}, is larger or equal than 0 in τ_i^*, we know that $\tau_{i+1}^* \leq \tau_i^*$ by the convexity of $\pi(\tau, N_{i+1}, \beta_i)$ (Lemma 4). Hence, we conclude that when $\tilde{\tau} \geq rT + L(1+hT)$ and $0 < N_1 \leq N_2 \leq \ldots \leq N_{|J|} < \infty$, then $\infty > \tilde{\tau} \geq \tau_1^* \geq \tau_2^* \geq \ldots \geq \tau_{|J|}^* > \tau_q^* > 0$.

(ii) The proof is similar to (i), but with opposite signs.

(iii) The proof is similar to (i), but with a different reliability threshold and a slightly different derivative. Furthermore, we exploit the fact that we can set $\beta_i = 1$ for any component $i \in J$, and consequently we have $\beta_i = \beta_j = 1$ for all components $i, j \in I$.

(iv) The proof is similar to (iii), but with opposite signs. □

Appendix J: Poisson Distributed Demand

We explore the impact of studying an asymptotically equivalent LCC function that assumes normally distributed demand during L, in comparison with the original LCC function under Poisson distributed demand during L. Let us denote the LCC of component $i \in I$, under Poisson distributed demand during L by $\pi(\tau_i^p, s_i^p, N_i, \beta_i) = \pi^p(\tau^p, s^p)$, where τ_i^p and s_i^p correspond to the reliability and the turnaround stock level under Poisson demand, respectively. We define the threshold for β_q under Poisson demand as

$$\Theta^p(\boldsymbol{N}, \boldsymbol{\beta}) = \max\left\{\beta_q \,:\, \pi^p(\tau_q^{p*}, s_q^{p*}, N_q, \beta_q) \leq \sum_{i \in J} \pi^p(\tau_i^{p*}, s_i^{p*}, N_i, \beta_i)\right\},$$

and compare it to $\Theta(\boldsymbol{N}, \boldsymbol{\beta})$ by considering $\delta(\boldsymbol{N}, \boldsymbol{\beta}) = \frac{\Theta(\boldsymbol{N},\boldsymbol{\beta}) - \Theta^p(\boldsymbol{N},\boldsymbol{\beta})}{\Theta^p(\boldsymbol{N},\boldsymbol{\beta})} \times (100\%)$, where $\delta(\boldsymbol{N}, \boldsymbol{\beta})$ is the relative difference between both thresholds. We generate a large testbed consisting of 6,656 instances. We vary various parameters on two levels, see Table 2.

Table 2 Parameter values.

	h	r	d	b	T	L
Low	0.015	0.2	10,000	1,000,000	180	3
High	0.03	0.3	100,000	10,000,000	360	4

Next to the parameters in Table 2, we also vary the relative unit cost factors and the installed base sizes of the dedicated components $i \in J$. We consider three dedicated components with $\beta_1 = 1$, $(\beta_2, \beta_3) \in \{(1,1), (1.05, 1.15), (1.1, 1.15), (1.2, 1.3), (1.25, 1.3)\}$. For N_i, we let $\sum_{i \in J} N_i = N_1 + N_2 + N_3 = 500$ for all instances, and vary two installed base sizes on two levels: for each $i, j \in J$ and $i \neq j$, have that $N_i, N_j \in \{100, 200\}$, and $l \neq i, j$, $l \in J$ we have $N_l = 500 - N_i - N_j$. This yields $2^2 \times 3 \times 2^2 + 4 = 52$ possible instances (due to duplicates when $\beta_1 = \beta_2 = \beta_3$).

Finally, we also consider a unit cost function for $c(\tau)$ with parameter values such that we satisfy Assumption 2. We use a modified version of a well-established unit cost function, see Mettas (2000) and Öner et al. (2010)

$$c(\tau) = p_1 + p_2 \exp\left(k \frac{\tau}{\overline{\tau} - \tau}\right), \quad p_1, p_2, k > 0, \quad \tau < \overline{\tau}$$

We set $p_1 = 5,000$, $p_2 = 1,000$, $\overline{\tau} = 600$. For for Sections 4 and 5 we set $k = 1.0$. For the comparison in this section, we consider $k = 1.0$ and $k = 2.0$. Care is needed when defining $c(\tau)$ on a finite support $(0, \overline{\tau})$. We need to ensure that $\pi^p(\tau^p, s^p)$ and $\pi(\tau)$ are increasing when τ or τ^p approach $\overline{\tau}$. We can verify, under our definition of $c(\tau)$, that $\lim_{\tau \to \overline{\tau}} \pi(\tau) = +\infty$ and for any $s^p \in \mathbb{N}_0$ $\lim_{\tau^p \to \overline{\tau}} \pi^p(\tau^p, s^p) = +\infty$. Hence, for any $s^p \in \mathbb{N}_0$, the minimizer of $\pi^p(\tau^p, s^p)$ lies in the interval $(0, \overline{\tau})$, and the minimizer of $\pi(\tau)$ lies in $(0, \overline{\tau})$.

Let us define the optimal reliability level and optimal turnaround stock level, that minimizes $\pi^p(\tau_i^p, s_i^p, N_i, \beta_i)$ by (τ_i^{p*}, s_i^{p*}). To determine τ_i^{p*} and s_i^{p*}, we observe that the first and second order derivative of the expected backorders are negative and positive, respectively. Hence, $\mathbb{E}\left[(D_i(L, N_i, \tau_i) - s_i)^+\right]$ is convex and decreasing in τ_i for a given s_i. We use this property to find (τ_i^{p*}, s_i^{p*}). We enumerate $s_i^p = 1, 2, \ldots, 100$ and find $\tau_i^{p*} | s_i^p$ by standard optimization techniques. Subsequently, we determine τ_i^{p*} and s_i^{p*}. We follow the procedure from our paper to determine τ_i^* for all $i \in I$.

We find that the differences $\delta(\boldsymbol{N}, \boldsymbol{\beta})$ are small, see the histogram of in Figure 8. On average, the normally distributed demand during L underestimates the threshold by -0.89%, with a standard deviation of 0.66%. Furthermore, we have that 95% of the instances have an absolute error of less than 2.19%, and 99% of the instances an absolute error less than 2.47%.

Figure 8 **Histogram of $\delta(\boldsymbol{N}, \boldsymbol{\beta})$ of our testbed (6,656 instances).**