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Abstract

We consider a situation where an original equipment manufacturer can either design a system component that is produced with traditional technology, or an alternative that is produced with additive manufacturing (AM). Designing either component requires a technology specific one-time investment and the components have different characteristics, notably in terms of production leadtime, production costs and component reliability. We support the design decision with a model that is based on evaluating the lifecycle costs of both components, covering design costs, maintenance and downtime costs, and performance benefits. We derive analytical properties of the required reliability and costs of the AM component such that its total lifecycle costs break even with that of its regular counterpart. Through our analysis, a numerical experiment and cases from two different companies, we find that component reliability is crucial to the success of AM components, while large AM component design costs or large AM component production costs can be overcome by generating performance benefits or using the short AM production leadtime to lower the after-sales logistical costs.

1 Introduction

Capital goods are complex technical systems that are essential to their users' business processes. Examples of such systems are airplanes, trains, military weapon platforms and semiconductor production machines. These systems are characterized by their high lifecycle costs, a large part of which is generated during their exploitation phase. For a large part, however, these exploitation costs are predetermined by decisions taken by the original equipment manufacturer (OEM) in the system design phase.

One interesting development, which may help reduce total lifecycle costs, is the development of additive manufacturing (AM) technology. AM, which is also sometimes called 3D printing, can be defined as parts fabrication by creation of successive cross-sectional layers of an object, usually based upon a three-dimensional solid model [1]. These parts can be plastic parts, but also many types of metal can be used. AM offers engineers greatly improved design freedom compared with traditional manufacturing technologies. From a performance point of view, this can offer large benefits, for instance by reducing fuel consumption for airplanes through reduced component weight. Furthermore, AM offers much shorter leadtimes for small production series, which can reduce required spare part investments and increase the responsiveness of after-sales service supply chains.

At the same time, AM components may currently require higher development costs compared to their regular counterparts. Due to AM's production process characteristics, the design of the regular component and its AM counterpart are almost never the same. While the regular component is usually an adapted version of a component installed in earlier systems, the AM component requires a whole new design, especially if there is a wish to capture any potential performance benefit by making use of AM's design freedom. Many engineers are still unfamiliar with the design rules that come with designing an AM component, leading to additional development costs. When a design has been decided upon, trial production runs are required to test product reliability and to fine-tune production parameter settings, such as laser intensity or layer thickness. Such investment costs can be very high, especially for more complex components or assemblies.

When evaluating the potential of AM, most product developers currently focus on weighing off the potential performance benefits against the required design investment. This means that potential cost reductions in the after-sales service supply chain, that are due to changes in reliability, production costs and production leadtime, are neglected. Assessing these potential savings is difficult, however, because AM is a rapidly developing technology that engineers have little experience with. In fact, a current limitation of AM technology is that there is often uncertainty concerning the mechanical properties of such parts [2]. On the other hand, the average production leadtime of AM is generally much shorter than the average production leadtime of traditional production technology. This is the key assumption that we make for our analytical model.

The question remains what properties the AM component must have in terms of reliability and production costs in order to be preferred over its regular counterpart. We introduce a model that compares the total lifecycle costs of the regular part with those of the AM part, taking into account design costs, performance benefits and all spare part related costs, including maintenance and downtime costs. This model is used to evaluate the break-even component production costs and the break-even component reliability, such that the total lifecycle costs of the regular part equal those of its AM counterpart. For these break-even characteristics we derive analytical properties. By examining the behavior of these properties related to parameters such as installed base size, estimated system lifetime and estimated design investment and potential performance benefits, we gain insight into the conditions under which an AM component outperforms a regular component.

The design decision that we consider takes place during the system design phase. The OEM can either design a regular component, that is usually based on a component that was used for earlier versions of the system, or he can design a completely new component that makes use of the capabilities of AM. Note that not all component types are suitable for AM, so a preselection of candidate components can be made based on top-down methods that evaluate AM suitability based on basic component characteristics (e.g. [3]). The break-even values that our model provides can be used by engineers, in cooperation with their AM service providers, to determine whether or not the required properties in terms of component costs and reliability can be achieved. If engineers estimate that break-even properties will be comfortably met, that means that AM is preferable. In cases where estimated properties are much worse than the break-even properties, it is better to opt for traditional production technology. If the estimated properties are similar to the break-even properties, more research must be done to more accurately determine the eventual AM component characteristics. After the design decision is taken, all systems are produced and the exploitation phase of the systems commences, which usually lasts for 20 to 30 years. Performance benefits are also generated during this time period. We assume that a stable installed base size is reached instantaneously at the start of the exploitation phase, which is a reasonable assumption given that the ramp-up phase of the installed base is generally short compared to the total time that the systems are in use.

Alternatively, our model can also be applied on redesign decisions that are taken during the exploitation phase. In the case of such a redesign, we require a negligible transition period in replacing the old component with new versions, to avoid a period of the lifecycle where two component types are operating in the field simultaneously. Such fast transitions can occur when there is a large performance benefit that we wish to exploit, in combination with ample opportunity to upgrade to the AM part, for example in the aviation industry when lighter AM components become available. Another application of our model during the exploitation phase, is for cases where the OEM must redesign a poorly designed component. The OEM again can decide to design a regular upgraded version or an AM version. Earlier studies have shown that in such an upgrade situation, it is often advantageous to preventively replace all components directly after redesign, instead of replacing them one-for-one at the time of failure (e.g., [4]; [5]).

Our model calculates lifecycle costs that are generated by all parties in the supply chain, from the OEM to the end user, aiming to identify when an AM component is preferable over a regular component. If the AM component is preferable due to lower lifecycle costs, and there are multiple parties involved in generating the lifecycle costs, including potential benefits, then a method is required to determine in which way each party benefits, for example via game-theoretical methods. Developing such a framework, however, is beyond the scope of this paper. If the level of cooperation in the supply chain is limited, our model can still be used by individual parties, who must then recognize which parts of the lifecycle costs and potential performance benefits apply to their situation.

In summary, our contribution is as follows:

1. We develop an original model for a component design decision, based on the evaluation of the total lifecycle costs of two competing types of components, one produced with traditional technology and one produced via additive manufacturing. We take into account design costs, performance benefits and after-sales service logistics costs.

2. We generate analytical insights into the relationship between design costs, performance benefits and the minimally required AM component characteristics. We conduct a numerical experiment to generate additional insight into situations where AM can likely be successfully applied to component design.

3. Two case studies are conducted to test the current applicability of AM in a component design setting.

The remainder of this paper is organized as follows. In Section 2, we survey the literature on related system design problems and on spare parts related to AM. Next, Section 3 contains the basic model formulation. Section 4 contains the analysis of our model, and Section 5 contains a numerical experiment that is used to generate managerial insight into the potential of AM for spare part supply. In Section 6 we present a set of case studies and in Section 7 we provide our conclusions as well as directions for further research.

2 Literature review

The decision to opt for either a traditionally produced part or a part produced via AM is a design decision, which we evaluate based on its effect on total lifecycle costs. In this section, we first review literature related to such design decisions in reliability and redundancy allocation problems and then in warranty problems. Our total lifecycle cost model also includes a spare part inventory system, so we also briefly review literature on spare parts management in relation to additive manufacturing. Finally we review a case study on AM component redesign.

In the literature on reliability or redundancy allocation problems, typically, the dependency is modeled between a design decision, which is to select a certain component, or a certain reliability level, and total lifecycle costs. Reliability allocation literature deals with selecting an optimal reliability level for a particular component. In the case of spare parts, the analysis also requires modeling an inventory system. The lifecycle costs then consist of design, production, inventory holding and repair/downtime costs. A typical objective is to maximize system availability given a budget constraint. Or alternatively, to minimize total lifecycle costs under a system availability constraint. Two articles that deal with reliability allocation and spare parts are [6] and [7]. Literature on redundancy allocation decisions sometimes also follows this approach. For example, [8] models a redundancy allocation decision and minimizes the lifecycle costs related to spare parts, aqcuisition and repair for a system under an availability constraint. For a general review on redundancy allocation problems, see [9].

Our work is different from reliability and redundancy allocation problems in terms of the approach that we follow. Due to the practical motivation of our work, we incorporate fixed development costs and a performance benefit that one component may have over the other. We deploy a model similar to [6], but we apply it differently to find a break-even point where the total lifecycle costs of the regular part and its AM counterpart are equal. This requires a different solution approach than traditional cost function minimization.

Furthermore, in the reliability allocation literature, it is common to make unit production costs dependent on the product's reliability (e.g., [6]; [7]). We do not model this dependency, due to the characteristics of AM. Greater reliability often requires introducing additional complexity to a component. For traditional production technology, it is generally so that more complexity results in higher unit production costs, for instance due to added production steps. For AM, on the other hand, we find that additional complexity does not influence unit production costs, but instead requires higher investment costs, due to more thorough testing and fine-tuning of parameters, or more sophisticated design alterations. Once these investments have been made, efficient unit production can commence.

Another literature stream that deals with evaluation/optimization of total lifecycle costs through design decisions, are warranty models. These often have a separate design decision related to the warranty type or warranty period length, which impacts total lifecycle costs. In the warranty literature, however, it is common not to take into account spare part holding costs and system downtime, which is essential for our evaluation, since AM has the potential to significantly reduce these cost components. Several examples of warranty models that also deal with reliability allocation, and that are, therefore, the most related to our work within the warranty literature stream, are [10], [11], [12] and [13].

Another stream to which our work is related is that of spare parts management for complex systems. For a general review on this topic, see [14]. The scope of our work is the entire system lifecycle, including the after-sales service logistics, which is typically mentioned as an application where AM can have a large impact. There is, however, only a limited amount of literature that explores the effect of AM on after-sales service logistics. One article that explores the effect on optimal inventory levels of shorter production leadtime via AM is [15], which models a single stock point that follows a continuous review base-stock policy with lost sales. Their approach in numerically studying the effect of AM is to vary the ratio of regular and AM leadtime. They subsequently observe that lowering AM leadtimes leads to lower optimal base-stock levels compared to producing components via technology with a longer leadtime. Their analysis, however, assumes equal reliability for both part types, ignoring its effect on system downtime costs, and it does not take into account investment cost differences related to using different technologies. Another paper on spare parts and AM is[16], which investigates the positioning of AM capacity in a multi-echelon supply chain by conducting scenario simulation, in order to gain insight into when AM will outperform traditional production methods. However, their model does not take into account differences in component reliability, and requires assumptions related to the future state of technology. We avoid such assumptions, and allow for different reliability of both components, by focusing on identifying break-even characteristics of AM components.

Finally, in terms of the insights that we provide, our approach is related to that of [17], which compares the production costs for a traditional, high-pressure die-casted landing gear structure, to the production costs for a redesigned version that is produced via AM. With their detailed production cost model, they establish the break-even point in terms of volume produced, beyond which the traditionally produced part is cheaper. Their analysis, however, does not take into account design costs, or costs accumulated during the exploitation phase related to inventory holding, downtime and repair. We also provide analytical results of the break-even point, which [17] does not provide.

3 Model

In this section we introduce our modeling assumptions and define cost expressions related to the development, production and exploitation of regular and AM parts. These expressions are used to define break-even values in Section 4. A complete overview of all model variables and all model input parameters can be found in Table 1 and Table 2, respectively.

An OEM designs a critical component for one of its next generation capital goods, to which we refer as the system. The OEM estimates that it will sell N units of the system and the time until the systems are phased out is T months. We assume that the N systems are sold at time $t = 0$, at which point also the design costs and the production costs are incurred. During the exploitation phase, we assume the size of the installed base to remain constant. For the system, the OEM can design a component that is produced via traditional production technology, or he can design a component that is produced via AM. We call these components the regular part and the AM part, respectively. Variables for which the AM and regular characteristics can differ, receive a superscript R or A to denote characteristics of a regular part and an AM part, respectively. Superscript x is used to denote a characteristic that holds for both a regular and an AM component.

We assume that the characteristics of the regular component are known early in the design phase. This can realistically be expected, since such components are often upgraded from designs that were incorporated in previous system versions. The regular component would also be manufactured with a technology that is more mature, so that accurate estimations of its characteristics can be given.

Designing either a regular part or an AM part requires an investment in terms of design and testing costs. The investment costs for regular and AM components is denoted by I^R and I^A, respectively. We denote the expected difference between these two investment costs by I:

$$I = I^A - I^R.$$

We generally expect the investment costs for the AM part to exceed the investment costs for the regular part, due to unfamiliarity with AM technology and the design principles involved. However, I can be negative when large investment costs are associated with using traditional production technology. For instance in the case of expensive tooling, such as casting molds, that is not required when producing with AM.

From the investment costs, we subtract a potential performance benefit, $B(\cdot)$, to take into account potentially beneficial effects of using an AM part. One example of such benefits is in aviation, where AM components with a honeycombed interior can greatly reduce weight compared to regular components. This decreases the plane's fuel consumption, which leads to considerable savings. The value of the performance benefits over the product lifetime is:

$$B(\cdot) = b_p NT,$$

with b_p being defined as the value of the performance benefit per AM part per unit time. While b_p is typically positive, we do not require this in our analysis. Throughout our paper, for functions like $B(\cdot)$, we only explicitly write down arguments when they are required to denote dependencies, and we stick to the use of $(\cdot)$ otherwise. For instance, $B(N)$ denotes the performance benefits for an installed base of size N.

We define the net value of the investment costs and performance benefits as $K(\cdot) = I - B(\cdot)$. For the same reasons that I can be negative, it is possible for K to be negative. Additionaly, a negative value of $K(\cdot)$ can occur when the total performance benefits exceed the net difference between investment costs for the AM part and the regular part.

Besides the investment costs and performance benefits, there are also costs related to the production and exploitation of the systems. These costs depend on the respective parameters of the part that is installed. The production costs depend directly on the component production costs, $c_p^x \geq 0$, which can differ for the regular part and the AM part. This leads to the following expression for the unit production costs:

$$P^x(\cdot) = c_p^x N.$$

During the exploitation phase, costs related to repair and spare part inventory depend on the mean time between failures, τ^x, and the production leadtime, L^x. We assume that the production leadtimes are independent and identically distributed, with a constant mean over time. Furthermore, we assume that using AM always results in a shorter production leadtime, i.e., $L^A < L^R$.

The systems are supplied with spare parts from a single stockpoint that follows a continuous review $(S^x - 1, S^x)$ base stock policy. Inventory holding costs are incurred at a rate of h €/€/unit time, also for parts that are on order. Hence, the inventory holding costs are:

$$H^x(\cdot) = hc_p^x TS^x.$$

When a part fails, the defective part is replaced by a spare part from inventory, if one is available. In that case, a new part is ordered immediately. Otherwise an emergency shipment is conducted and the demand is lost to the stock point. In the former case, repair and downtime costs c_d are incurred, which includes the costs for order handling, failure diagnostics and transportation, but excluding component production costs. In the latter case, emergency system repair and downtime costs c_e are incurred, with $c_e > c_d$. The reason why downtime costs c_d are equal for the regular and the AM part, is that the time and resources required for failure diagnosis and component installation do not depend on the type of the component that is installed. Further, we assume that c_e is equal for the regular part and the AM part. The reason for this is that we estimate that the time it takes to perform the emergency transportation of the regular component is equivalent to the emergency production and transportation of an AM component at a nearby location. We also expect the additional costs for long-distance transportation by airplane to be comparable to the premium price paid for expediting an AM part at a local supplier.

We assume that the mean lifetime of components is generally distributed and that the number of systems that is served from the single stockpoint is sufficiently large. In this case, we may assume that the total demand process for spare parts follows a Poisson process with rate N/τ^x (see [18], p.14). Our assumptions imply that the on-hand stock process is identical to the process of the number of free servers of an $M/G/c/c$ queue with S^x parallel servers, arrival rate N/τ^x and service time L^x, i.e., an Erlang loss system. The emergency shipment probability, i.e., the probability of being out of stock, is then identical to the Erlang loss probability, $g^x(\cdot)$, for a system with S^x servers and a system load $a^x = NL^x/\tau^x$:

$$g^x(\cdot) = \frac{\frac{(a^x)^{S^x}}{S^x!}}{\sum_{i=0}^{S^x} \frac{(a^x)^i}{i!}}.$$

Using the Erlang loss rate, we can calculate the expected downtime and repair costs:

$$D^x(\cdot) = g^x(\cdot)\frac{NT\,(c_e - c_d)}{\tau^x} + \frac{NT(c_d + c_p^x)}{\tau^x}.$$

We now provide the cost function for the sum of the production costs, inventory holding costs and downtime and repair costs. This function, $C^x(\cdot)$, holds for both the regular and the AM parts. Note that $C^x(\cdot)$ is not the function for the total lifecycle costs, as it does not include the investment costs and potential performance benefits that AM can bring during the exploitation phase. It holds that:

$$C^x(\cdot) = P^x(\cdot) + H^x(\cdot) + D^x(\cdot).$$

To find the optimal base-stock level that minimizes total lifecycle costs, we can limit ourselves to minimizing $C^x(\cdot)$, since $K(\cdot)$ is independent of the base stock level S^x. Hence, we solve the optimization problem (Y^x):

$$(Y^x) \qquad \min_{S^x \in \mathbb{N}_0} \{C^x(S^x)\}$$

There is no closed-form solution for the optimal base-stock level $S^{x*}(\cdot)$. However, because the Erlang loss function is convex in S^x for a fixed τ^x (see [6]), we can easily find the optimum via a numerical search procedure. We define the following optimized cost function:

$$\hat{C}^x(\cdot) = C^x(S^{x*}(\cdot)). \tag{1}$$

4 Analysis

To identify when AM is preferable over traditional production, we require the break-even point where the total lifecycle costs of the regular component are equal to the total lifecycle costs for the AM components,

Table 1: Model variables

$B(\cdot)$	Total performance benefits from using AM parts
$\hat{C}^x(\cdot)$	Total costs for production, inventory holding and downtime/repair
$D^x(\cdot)$	Total downtime and repair costs
$g^x(\cdot)$	Erlang loss probability for an inventory system
$H^x(\cdot)$	Total inventory holding costs
I	Net difference in investment costs between AM and regular part
$K(\cdot)$	Net difference in investment costs and performance benefits
$P^x(\cdot)$	Initial part production costs

Table 2: Model input parameters

a^x	Load on the inventory system
b_p	Performance benefit per AM component per unit time
c_d	Downtime and repair costs incurred per failure when a part is available
c_e	Emergency downtime and repair costs incurred in an out-of-stock situation
c_p^x	Component production costs
h	Holding cost rate in Euro per Euro per year
I^x	Investment costs related to developing a part
L^x	Mean component production leadtime
N	Installed base size
T	Time horizon length
τ^x	Component mean time between failure

including performance benefits and investment costs. Knowing when both components perform equally well provides us with insight into the characteristics required for the AM part to outperform the regular part over the total system lifecycle. In Section 3 we introduced $K(\cdot)$ as a measure that includes performance benefits and the difference in development costs for AM and regular parts. Given that these play a large role in deciding whether or not to opt for an AM component, we evaluate how $K(\cdot)$ influences break-even characteristics in terms of component reliability in Section 4.1, and in terms of component production costs in Section 4.2.

Another reason for exogenizing $K(\cdot)$ in this manner, is that the components of $K(\cdot)$ can be estimated quite accurately early in the design stage. It is common for an OEM to estimate the costs associated with design activities. Estimating attainable performance benefits can be done in cooperation with an AM service provider who is knowledgeable on the design freedom that AM offers. For these reasons, it is possible to accurately estimate the value of $K(\cdot)$ for a particular part. The remainder of total lifecycle costs, especially those related to the reliability of AM parts and their production costs, are much more difficult to estimate, which is why providing break-even characteristics for these two component characteristics is the focus of our analysis.

Before we proceed, we provide several properties of the optimal cost function $\hat{C}^x(\cdot)$ in Lemma 1.

Lemma 1. *The optimal cost function $\hat{C}^x(\cdot)$ has the following properties:*

(i) $\hat{C}^x(\tau^x)$ *is strictly decreasing in* τ^x.

(ii) $\hat{C}^x(N)$ *is strictly increasing in* N.

(iii) $\hat{C}^x(c_p^x)$ *is strictly increasing in* c_p^x.

The proof of Lemma 1 and all further proofs can be found in the appendix.

4.1 Properties of the break-even reliability levels under equal production costs

In this section, we investigate the behavior of the break-even reliability characteristics of AM components in relation to several key parameters, most notably the value of $K(\cdot)$. For this part of the analysis, we evaluate the scenario where the component production costs are equal for regular and AM parts, i.e. $c_p^R = c_p^A = c_p$. In practice, this may occur, for example, when many production or assembly steps are required to produce a part. In that case, more expensive hours of the AM machine are offset by the production speed with which complex geometry is achieved or assembly steps are skipped.

We formally introduce the break-even reliability, $\tau^{A*}(\cdot)$, in Definition 1. In Lemma 2, we next show that $\tau^{A*}(K(\cdot))$ exists only upto a certain value of $K(\cdot)$ and that if it exists, it is unique for that value of $K(\cdot)$. We come back to the interpretation of these properties below.

Definition 1. *The break-even reliability of an AM component is $\tau^{A*}(\cdot)$ such that $\hat{C}^R(\tau^R) = \hat{C}^A(\tau^{A*}(\cdot)) + K(\cdot)$.*

Lemma 2. $\tau^{A*}(K(\cdot))$ *has the following properties:*

(i) $\tau^{A^*}(K(\cdot))$ *does not exists if* $K(\cdot) \geq K_{lim} = hc_pTS^{R^*} + g^R(\cdot)\frac{NT(c_e-c_d)}{\tau^R} + \frac{NTc_d}{\tau^R}$.

(ii) *For* $K(\cdot) \in (-\infty, K_{lim})$, $\tau^{A^*}(K(\cdot))$ *is uniquely defined.*

Unfortunately, there is no closed form solution to $\tau^{A*}(\cdot)$, mainly because it is integrated into the Erlang loss probability. For this reason, we can only evaluate it numerically up to an arbitrary accuracy, ε, which we define as:

$$\varepsilon = \left| \frac{\hat{C}^R(\cdot) - \hat{C}^A(\cdot) - K(\cdot)}{\hat{C}^R(\cdot)} \right| = 0.000001.$$

Determining $\tau^{A^*}(\cdot)$ is done iteratively by conducting a binary search, making use of the property that $\hat{C}^A(\tau^A)$ is decreasing in τ^A (Lemma 1(i)). Doing this for a range of values for $K(\cdot)$ yields a curve such as the one shown in Figure 1. The particular values used to generate Figure 1 can be found in Table 3.

Table 3: Numerical example parameters

h[€/€/month]	c_p[€]	c_d[€]	c_e[€]	L^A[months]	L^R[months]	τ^R[months]	N[-]	T[months]
0.02	40	200	800	0.5	3	10	100	180

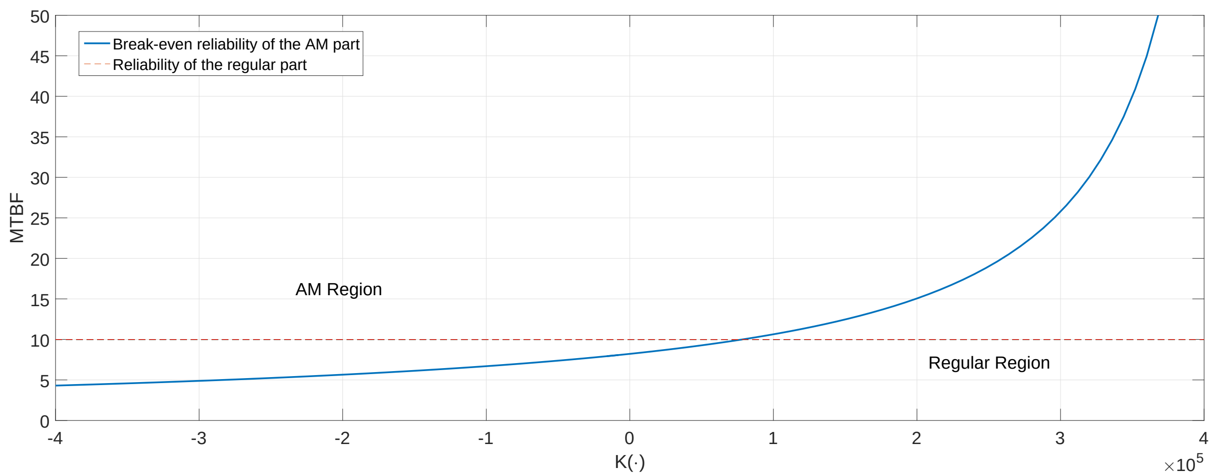

Figure 1: Break-even reliability as a function of $K(\cdot)$ for the numerical example of Table 3

In Figure 1, we notice some interesting behavior. First, we see that $\tau^{A^*}(\cdot)$ is increasing in $K(\cdot)$. Furthermore, $\tau^{A^*}(\cdot)$ goes to infinity when $K(\cdot)$ approaches K_{lim} as defined in Lemma 2(i), beyond which no break-even values can be found. We also see that when $K(\cdot) = 0$, the break-even reliability curve is below the regular part's reliability, and that the intersection point of the break-even reliability curve with the regular reliability occurs at a positive value of $K(\cdot)$. This implies that an OEM can invest more in developing an AM part, compared to developing the regular part, even if the AM part is not superior in terms of reliability. We show in Theorem 1 that these are structural properties of $\tau^{A*}(\cdot)$ as a function of $K(\cdot)$, and that they provide us with general insights into the required reliability of an AM component.

Theorem 1. $\tau^{A*}(K(\cdot))$ *has the following properties:*

(i) $\tau^{A^*}(K(\cdot))$ *is strictly increasing in* $K(\cdot)$ *for* $K(\cdot) \in (-\infty, K_{lim})$.

(ii) $\lim_{K(\cdot)\uparrow K_{lim}} \tau^{A^*}(K(\cdot)) = \infty$.

(iii) *If* $K \leq 0$ *then* $\tau^{A^*}(\cdot) < \tau^R$.

The intuitive explanation of the value of K_{lim} in Lemma 2(i) is the following: Opting for an AM component with a shorter production leadtime, leads to savings in inventory holding costs and downtime/repair costs.

However, these cost components for the regular part are finite, so that the maximum savings are also finite. Therefore, once $K(\cdot)$ exceeds the regular system's inventory holding and downtime/repair costs, $\tau^{A*}(\cdot)$ does not exist anymore.

For $K(\cdot) < K_{lim}$, Lemma 2(ii) states that $\tau^{A*}(K(\cdot))$ is unique for a given value of $K(\cdot)$. Its implication, in combination with Lemma 1(i), is that if the OEM can design an AM component at cost $K(\cdot)$, that is expected to attain $\tau^A > \tau^{A*}(K(\cdot))$, that AM part is preferable over the regular part. From now on, we refer to the area above the break-even curve of a graph, such as the one shown in Figure 1, as the AM region, and to the area below as the regular region.

Theorem 1(i) states that as the difference in required investments between the AM part and the regular part increases, i.e., if $K(\cdot)$ increases, the AM part break-even reliability also increases. Theorem 1(ii) states that when $K(\cdot)$ approaches the regular component's costs related to inventory holding and downtime/repair, $\tau^{A^*}(\cdot)$ tends to infinity.

When an AM component creates large performance benefits, $K(\cdot)$ can become negative. For such cases, Theorem 1(iii) describes that when differences in investment costs between the regular and AM part are at least canceled out by the benefits (i.e., $K(\cdot) = I^A - I^R - b_pNT \leq 0$), the break-even reliability is strictly below the reliability of the regular component. This also implies that when benefits are expected to equal the difference in required investment, i.e. $K(\cdot) = 0$, any AM part with reliability at least equal to that of the regular part is preferred over the regular part. This is especially useful for future applications of AM, since it is expected that investment costs for AM components will decrease as engineers become more familiar with its design principles and trial production costs decrease due to a better understanding of AM process parameters. This implies that in many future cases, I^A will be similar to I^R.

So far, we have considered the behavior of $\tau^{A^*}(\cdot)$ as a function of $K(\cdot)$. Other interesting behavior of $\tau^{A^*}(\cdot)$ relates to the size of the installed base and the length of the remaining time horizon. We would expect that an increase in the installed base size, or a longer remaining time horizon, has a positive effect on the size of the AM region, as it allows us to spread the investment costs over more parts or a longer time period. While this does seem to be the case when $K(\cdot)$ is large, we find that when performance benefits outweigh investment costs (i.e., $K(\cdot) < 0$), an increase in the remaining time horizon actually leads to a higher required reliability, thus decreasing the AM region. Figure 2 provides an example of this behavior, where on the right side of the graph, the break-even curve goes down when T increases from 60 months to 240 months, while on the left side, the break-even curve goes up.

We now provide several properties of $\tau^{A*}(\cdot)$ related to changes in N and T, providing more insight into the applicability of AM for different installed base sizes and remaining time horizons. Since there is no closed form solution to $\tau^{A^*}(\cdot)$, we cannot explicitly evaluate its sensitivity to N or T in general. Therefore, we focus on two special cases that we can evaluate. These points are $K_1(\cdot)$ and $K_2(\cdot)$ and they are depicted in Figure 3.

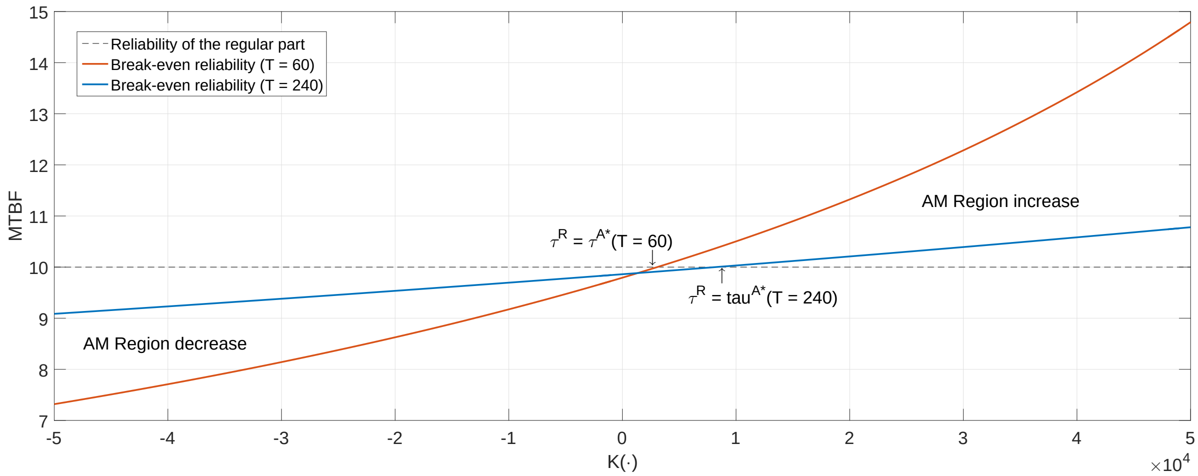

Figure 2: Sensitivity of Break-even reliability to T

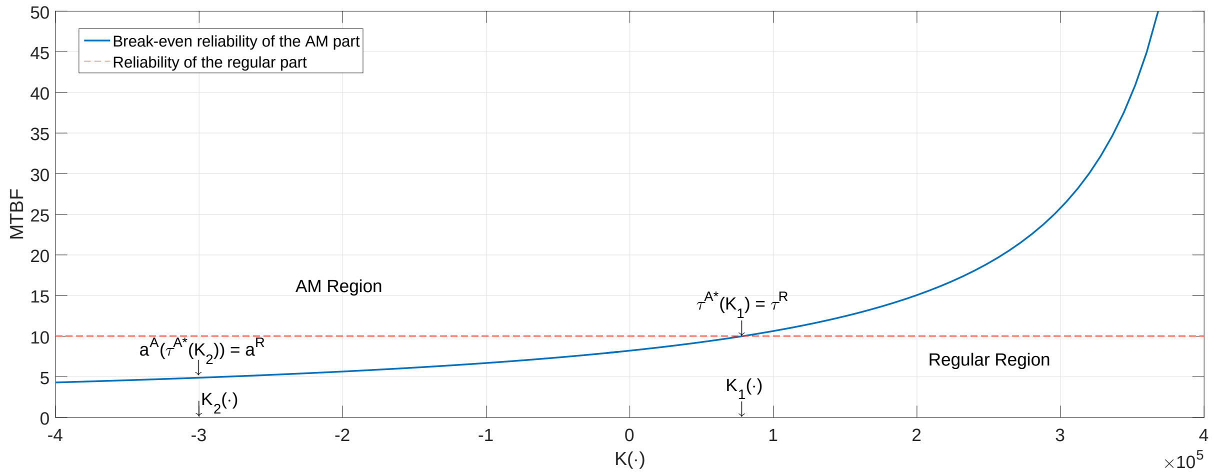

Figure 3: Example indication of the position of $K_1(\cdot)$ and $K_2(\cdot)$

The parameter values used to generate Figure 3 are the same as the parameter values for Figure 1, i.e., they are given in 3, except that we have set $L^A = L^R/2 = 1.5$ months. We first analyze the behavior of $K_1(\cdot)$ and after that proceed with properties on the behavior of $K_2(\cdot)$.

Definition 2. *$K_1(\cdot)$ is the value of $K(\cdot)$ such that $\tau^{A*}(K_1(\cdot)) = \tau^R$.*

$K_1(\cdot)$ is defined by the intersection point of τ^{A^*} with τ^R. This intersection point of the break-even reliability curve with the regular part's reliability is of particular practical significance, since it gives us insight into the amount by which the AM part's development costs, minus its performance benefits, may exceed those of the regular part, before we require it to be technologically superior in terms of reliability. We formalize several properties of $K_1(\cdot)$ in Theorem 2.

Theorem 2. *$K_1(\cdot)$ has the following properties:*

(i) $K_1(\cdot) > 0$.

(ii) $K_1(T)$ *is increasing in* T.

(iii) $\tau^{A^*}(K_1(T); T+\varepsilon) \leq \tau^{A^*}(K_1(T); T)$, *with* $\varepsilon > 0$.

The implication of Theorem 2(ii) and Theorem 2(iii) is that when the remaining time horizon increases, the break-even curve shifts below and to the right, increasing the AM region. This means that the OEM can then spend more on developing an AM component and that the AM part requires a lower reliability in order to break even with the regular part as T increases. Since $K_1(\cdot)$ is always positive, these findings imply that switching to AM components becomes more attractive for longer remaining time horizons, when there are no, or relatively small, performance benefits involved.

Remark: In our numerical experiment (Section 5), we see that for installed base sizes that are normally encountered in practice, an increase in N has the same implications as an increase in T. Counter examples to this behavior exist, however, for very small values of N. For example, using the parameters from Table 3, but setting $L^A = 2.9$, increasing N from 3 units to 4 units results in a decrease of $K_1(\cdot)$ from 8.45 to 8.14 Euro. In our numerical experiment (see Section 5), we examine the behavior of $K_1(N)$ in more detail for more realistic values of N.

Theorem 2 describes a positive influence of an increase in T on the AM region, i.e., a decrease in break-even reliability as T increases. We have also observed earlier, that there are cases where an increase in T has the opposite effect, see Figure 2, where on the left side the break-even reliability increases as T increases. We next examine this behavior in more detail. To do this, we first define a specific value $K(\cdot) = K_2(\cdot)$ for which we prove that an increase in T corresponds to an increase in required reliability.

Definition 3. $K_2(\cdot)$ *is the value of* $K(\cdot)$ *such that* $\frac{NL^A}{\tau^{A*}(K_2(\cdot))} = \frac{NL^R}{\tau^R}$.

Recall that $\tau^{A*}(\cdot)$ is decreasing as $K(\cdot)$ decreases (Theorem 1(i)). When we decrease $K(\cdot)$ far enough, we encounter a value $\tau^{A*}(K_2(\cdot))$ where the break-even reliability exactly off-sets the reduced production leadtime: At this point, it holds that the load on the Erlang loss inventory system of the regular part is equal to the load on the Erlang loss inventory system of the AM part (i.e., $a^R = a^A$). In the example of Figure 3, $L^R = 2L^A$, in which case we know that $a^R = a^A$ when $\tau^{A*}(K(\cdot)) = \tau^R/2 = 5$ months. Since we are investigating the scenario where both parts' production costs and emergency shipment costs are equal, we know that when $K(\cdot) = K_2(\cdot)$, the optimal base-stock levels for both parts will also be equal. This allows us to evaluate the behavior of $\tau^{A*}(K_2(\cdot))$, which we formally define in Theorem 3.

Theorem 3. *For* $\tau^{A*}(K_2(\cdot))$ *it holds that* $\tau^{A^*}(K_2(T); T) < \tau^{A^*}(K_2(T); T+\varepsilon)$.

Although we only prove Theorem 3 for a specific point $K_2(\cdot)$, experiments indicate that in general an increase in T results in greater $\tau^{A*}(\cdot)$ when $K(\cdot) \leq 0$. This implies that when AM performance benefits are expected to outweigh the required investment, we must also meet a larger reliability to break even as T increases, although $\tau^{A*}(K(\cdot) \leq 0)$ will never exceed τ^R (see Theorem 1(iii)).

Remark: Numerical experiments indicate that for practical examples, an increase in N has the same implications for $\tau^{A*}(K_2(\cdot))$ as an increase in T. However, counter examples to this behavior exist. For example, using the parameters from Table 3, but setting $L^R = 1$, $L^A = 0.9$, $c_d = 1$ and $h = 0.5$, increasing N from 1 unit to 2 units results in a decrease of $\tau^{A*}(K_2(\cdot))$ from 9 months to 8.95 months.

4.2 Properties of the break-even component costs under equal reliability

In this section, we investigate the behavior of the break-even component production costs. We consider a scenario where the reliability of an AM component is equal to that of the regular component, i.e. $\tau^R = \tau^A = \tau$. This is a scenario that can often occur in practice, especially for components that are less technically challenging to print, or that are not subjected to direct mechanical loads.

We formally introduce the break-even component production cost, $c_p^{A*}(\cdot)$, in Definition 4. In Lemma 4, we next show that $c_p^{A*}(\cdot)$ exists only upto a certain value of $K(\cdot)$, and that if it exists, it is unique for that value of $K(\cdot)$.

Definition 4. *The break-even condition for the AM component's production costs is defined as* $c_p^{A*(\cdot)}$ *such that* $\hat{C}^R(c_p^R) = \hat{C}^A(c_p^{A*(\cdot)}) + K(\cdot)$.

Theorem 4. $c_p^{A*}(K(\cdot))$ *has the following properties:*

(i) $c_p^{A*}(K(\cdot))$ *does not exist if* $K(\cdot) > \hat{C}^R(c_p^R)$.

(ii) *For* $K(\cdot) \in (-\infty, \hat{C}^R(c_p^R)]$, $c_p^{A*}(K(\cdot))$ *is uniquely defined.*

(i) $c_p^{A*}(K(\cdot))$ *is strictly decreasing in* $K(\cdot)$.

We are able to separate $c_p^{A*}(\cdot)$ from the break-even equation of Definition 4 by reordering:

$$c_p^{A*}(\cdot) = \frac{c_p^R\left(N + hTS^{R*}(\cdot) + \frac{NT}{\tau}\right) + (g^R(\cdot) - g^A(\cdot))\frac{NT(c_e - c_d)}{\tau} - K(\cdot)}{N + hTS^{A*}(\cdot) + \frac{NT}{\tau}}. \tag{2}$$

Unfortunately, we can only evaluate $c_p^{A^*}(\cdot)$ numerically, since $S^{A*}(\cdot)$ depends on $c_p^{A*}(\cdot)$. There is also no closed form expression for the optimal base stock level $S^{x*}(\cdot)$. We can still, however, obtain some insight into the behavior of the break-even component costs.

Similar to what we did in the previous section, we manipulate $K(\cdot)$ to obtain insight into how $c_p^{A*}(\cdot)$ behaves under the influence of required design investments and expected performance benefits. Figure 4 shows an example of typical behavior of $c_p^{A*}(K(\cdot))$. The parameters used are in Table 3.

Because $\hat{C}^x(c_p^x)$ is increasing in c_p^x (Lemma 1), we require c_p^A to be below the break-even point in order for AM to be preferable. From Figure 4 we observe that $c_p^{A*}(\cdot)$ is decreasing in $K(\cdot)$ and that at some point, see

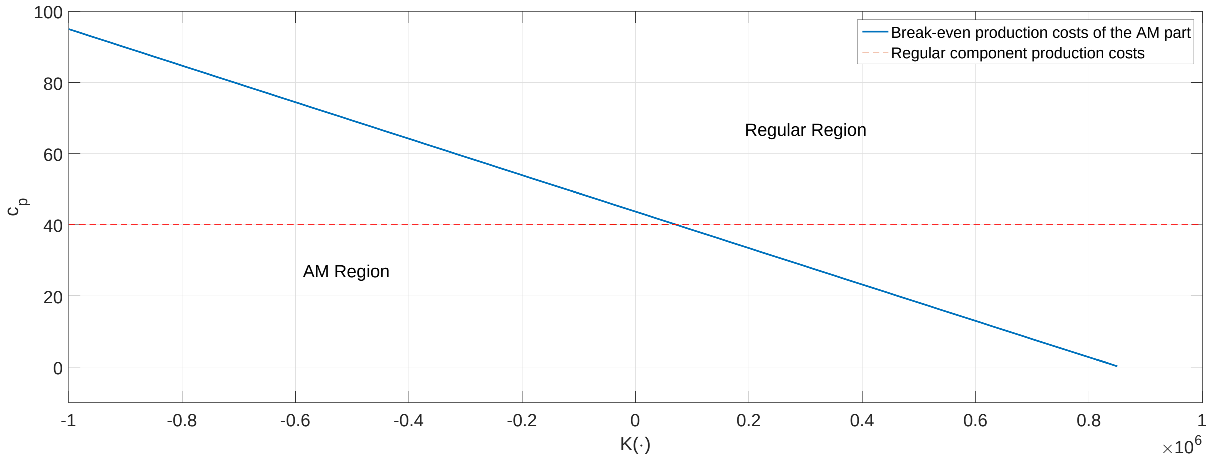

Figure 4: Break-even production costs as a function of $K(\cdot)$

Lemma 4(i), the break-even production costs cease to exist, which is due to our requirement that $c_p^x \geq 0$. The value of $K(\cdot)$ where $c_p^{A*}(\cdot)$ ceases to exist can be found via eq. (2):

$$c_p^{A*} = 0 \Rightarrow c_p^R \left(N + hTS^{R*}(\cdot)\right) + g^R(\cdot) \left(\frac{NT(c_e - c_d)}{\tau}\right) = K(\cdot).$$

That the break-even production costs cease to exist exactly at $\hat{C}^R(c_p^R)$ can intuitively be explained by observing that even when the AM component production costs are zero, we cannot save more than the lifecycle costs of the regular component related to production, inventory holding and emergency shipments. Hence, if $K(\cdot)$ increases beyond $\hat{C}^R(c_p^R)$, no break-even values can be found. This bound on the maximum $K(\cdot)$ can be used as an early go or no-go for the evaluation of AM as a production option, because the value of $K(\cdot)$ can be estimated early in the design process. We also see that when $K(\cdot) = 0$, the break-even production costs of the AM component are greater than the production costs of the regular part. These observations are similar to the properties of $\tau^{A*}(K(\cdot))$.

Finally, we note that the intersection point of $c_p^{A*}(\cdot)$ with C_p^R occurs at the exact same value of $K(\cdot)$ as $K_1(\cdot)$ from Section 4.1, since we have assumed equal reliability and because the component production costs are also equal at this intersection point. Hence, both models are equivalent at $K(\cdot) = K_1(\cdot)$, and the insights from Theorem 2 also hold for the intersection point where $c_p^{A*}(\cdot) = C_p^R$.

5 Numerical experiment

To generate insight into the applicability of AM in practice we conduct a numerical experiment on a range of input parameters. Our goal is to provide managerial insights into situations where AM is most suitable to replace traditional technology, and to provide insights into AM characteristics that require the most attention for the technology to become more widely applicable. We report the following outcome variables:

(i) $K_1(\cdot)/c_p$: AM is not a mature manufacturing technology yet, and high investment costs are often required for its application. Investigating $K_1(\cdot)/c_p$ gives insight into how much higher I^A may be relative to I^R in the absence of large performance benefits. A high ratio of $K_1(\cdot)/c_p$ indicates that a significant investment can be made towards the development of an AM component, if that component's reliability is comparable to that of its regular counterpart.

(ii) $\tau^{A*}(K(\cdot)=0)/\tau^R$: In the long term, as AM matures and design engineers gain more experience with its application, the AM development costs will decrease. In that case, we expect AM and regular investment costs to be balanced, i.e., $I^R \approx I^A$, which makes the investigation of $\tau^{A*}(K(\cdot)=0)/\tau^R$ relevant. From Lemma 2 we know that this ratio is strictly less than one. The closer this ratio is to one, the closer the reliability of the AM component must be to the reliability of the regular component in the absence of performance benefits.

To investigate the behavior of the output variables mentioned above, we set up a full factorial experiment over the parameter values defined in Table 4. Each parameter has three possible values, the middle values being commonly encountered in practice. The other values are more extreme values that may apply in specific cases. For instance, an installed base size of 25 units can apply to radar installations for a specific class of naval vessels.

Table 4: Parameter values for numerical experiment

c_p[**€**]	N	T[**months**]	L^A[**months**]	c_d/c_p	c_e/c_d	τ^R[**months**]
250,1000,4000	25,100,400	60,120,240	0.25,0.5,1	2,4,8	4,8,16	12,24,48

The remaining parameters are fixed at the following levels: $h = 0.02$€/€/month and $L^R = 3$ months. All variations provide a total of 2187 combinations. Table 5 contains the most interesting results from the experiment. The remaining results can be found in Appendix 8.1.

From Table 5 we identify interesting behavior for the application of AM in the short to mid-term future, as described by the ratio $K_1(\cdot)/c_p$. Firstly, c_d/c_p has almost no effect on the value of $K_1(\cdot)/c_p$. This indicates that whether the costs due to asset downtime are relative small or large compared to the component production costs does not affect the application of AM much. On the other hand, we see that $K_1(\cdot)/c_p$ is very sensitive to an increase in τ^R or L^A. For increases in τ^R, this is due to the fact that a reliable regular component has few failures and, therefore, requires low base-stock levels and few emergency shipments. This limits the amount of costs that can be saved by using a quickly producible AM component. An increase in L^A similarly decreases the ratio of $K_1(\cdot)/c_p$. As the AM leadtime becomes closer to the regular leadtime, the costs that can be saved in the after-sales service supply chain decrease. This diminishes the amount that can be invested in developing an AM part. Finally, an increase in N allows for spending more on an AM design, as the additional investment is spread over a larger installed base.

Table 5: Results from the numerical experiment

		$K_1(\cdot)/c_p$			$\tau^{A*}(K(\cdot)=0)/\tau^R$		
Parameter	**Parameter value**	**Average**	Min	Max	**Average**	Min	Max
c_d/c_p	2	107	5	676	0.943	0.872	0.973
	4	109	5	685	0.969	0.928	0.986
	8	111	6	693	0.984	0.961	0.993
τ^R	12	176	15	693	0.970	0.909	0.993
	24	97	9	379	0.966	0.894	0.992
	48	54	5	213	0.961	0.872	0.992
L^A	0.25	128	8	693	0.959	0.872	0.989
	0.5	113	7	616	0.965	0.896	0.991
	1	87	5	480	0.973	0.924	0.992
N	25	24	5	72	0.956	0.872	0.989
	100	71	15	213	0.967	0.910	0.991
	400	232	49	693	0.973	0.930	0.992

We observe that AM is currently already suitable for application to component design if the systems have a sizeable installed base and a long system lifetime. This fits well with the capital goods setting and matches with the findings described in Theorem 2. We observe that the additional costs for developing an AM component, compared to the costs for developing the regular component, may often be several hundred times larger than the component production costs. However, we also find that the reliability of the regular component that is considered should not be too large, as this diminishes the potential cost savings in terms of after-sales service logistics costs. If we cannot reduce inventory costs and sufficiently decrease the number of emergency shipments that are performed, the AM investment costs quickly become a deterrent to apply AM.

In the long term, we expect $K_1(\cdot)$ to be closer to zero in the absence of performance benefits, as investment costs for AM decrease. Investigating $K_1(\cdot) = 0$, we observe several effects on the required reliability that an AM component must have. We find that when component production costs are small compared to costs related to asset downtime, i.e., the ratio of c_d/c_p is high, then the AM component must be almost as reliable as the regular component. This effect is due to the downtime costs becoming a more influential component of the total lifecycle costs, so that any increase in failure rate also has a high impact. We also see that changes in τ^R, L^A or N have a small effect on the break-even reliability at $K(\cdot) = 0$.

Furthermore, even in the most extreme cases, the AM component may only be 13% less reliable than the regular component. This indicates that the reliability that an AM part can achieve is crucial for its application. In a capital goods setting, the costs related to machine downtime are simply too big to allow for substantial reductions in reliability, even with the logistical benefits that AM offers. This also implies that even in the absence of performance benefits, it is very beneficial to apply AM if there is an opportunity to use AM to increase a component's reliability. For example by integrating multiple components into one part, and thus removing potential failure modes. In such cases, a small increase in reliability compared to the regular component can have a large effect on total lifecycle costs.

6 Case studies

We perform case studies to illustrate the practical applicability of our model, as well as the current performance of AM compared with traditional technology.

The first case study is performed at a company that manufactures access equipment and its spare parts. The component that we evaluate is a stainless steel hydraulic valve block that is used to control the bucket movement of a 60m boom lift (see Figure 5). Such a component has to be able to withstand large hydraulic pressure, but the amount of pressure is also predictable and the AM component is expected to cope with this type of load as well as the regular version does, i.e., the reliability of the AM version is expected to be equal to that of the regular version. Therefore, we focus on determining the required production costs for the AM component to outperform the regular version. For this, we use the method of analysis described in Section 4.2. The data required for the analysis is shown in Table 6. Downtime costs c_d have been estimated based on one day of downtime for a large rental company at a cost of €475 in lost revenue per failure. Emergency downtime costs $c_e = 4c_d$. This ratio of c_e/c_d is fairly low, but reasonable for such types of equipment, which are typically not crucial to entire business processes. The AM leadtime of two weeks is typical for the service that third party AM service providers guarantee. The other parameters have been determined from company records.

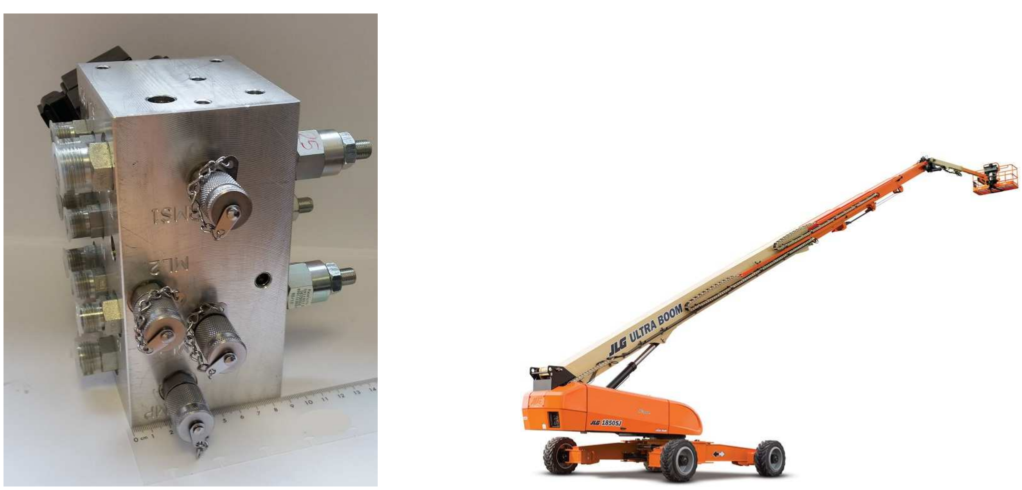

Figure 5: The regular valve block and boom lift of case study 1

Table 6: Data for evaluating case study 1

c_p^R	416.91	€
c_d	475	€
c_e	1900	€
h	0.015	€/€/month
L^A	0.5	months
L^R	4.78	months
N	400	units
τ^x	120	months
T	360	months

The data in Table 6 is used to generate the break-even curve that is shown in Figure 6. To estimate whether or not this component is suitable for AM we must also determine its position on the graph. To do this, we require estimates for the values of c_p^A and $K(\cdot)$.

The AM production costs are largely determined by the component weight. The regular part weighs 8kg and to serve as an indication of the weight of the AM component, we refer to a similar valve block that has previously been designed for AM in a cooperation between Layerwise, an AM service provider, and the VTT research center of Finland. Using a topologically optimized AM design resulted in a 76% weight reduction over the regular version. A similar weight reduction for our component would result in a weight of 1.92kg. The density of stainless steel 316 is 7860kg/cm^3, which results in a component volume of 244cm^3. The AM cost of stainless steel is estimated to be €3.14 per cm^3 (see [19]), bringing our estimate of c_p^A to €767.

To estimate $K(\cdot)$, we require an estimate of the difference between the regular and AM design costs. The regular part is a typical valve block that will be easy to design, while the design engineers can use of-the-shelf software to optimize the topology of the AM part. We estimate that the difference between these design costs is negligible. However, the AM part requires extensive testing to fine-tune production parameters. Typically, at least ten trial production runs are required to find the correct combination of production parameters and subsequent testing of the products is required. Based on the estimate for c_p^A, which largely determines the costs of a trial production run, we find that a value of $I =$ €10,000 is realistic.

We next look at possible performance benefits. The company indicates that reducing the weight of a component that is situated in the bucket of the boom lift also creates a certain performance benefit, as the weight reduction directly increases the weight carrying capacity of the equipment. This results in a competitive advantage for the company. The exact value of this advantage, however, is difficult to determine. Therefore, we ignore this and assume that $K(\cdot) = I$.

Note that these are rough estimates for c_p^A and $K(\cdot)$, which is common business practice when evaluating new products or components early in the development process. We will see that these rough estimates, in combination with the model's break-even curves are sufficient to draw conclusions about this case.

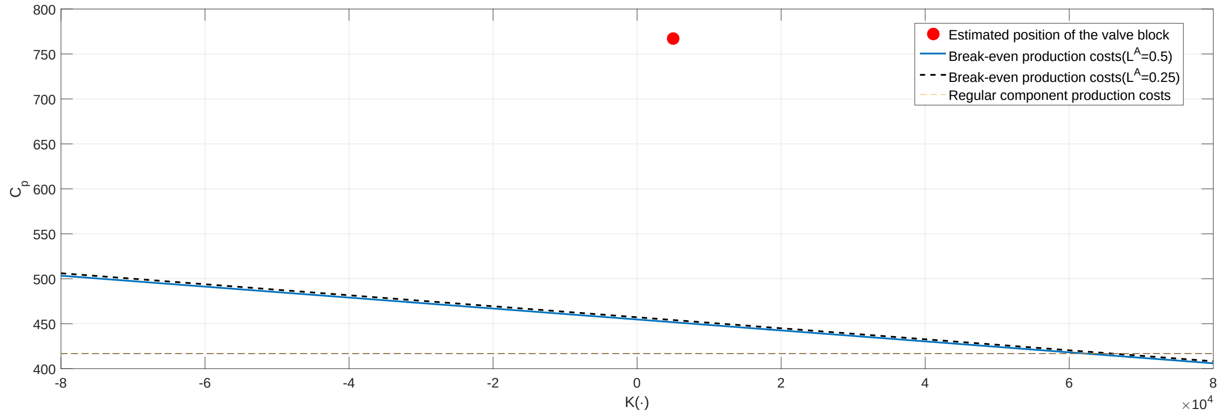

Figure 6: Outcome of case study 1

In Figure 6 we can clearly see that the estimated AM component production costs are much higher than the break-even production costs, indicating that the traditional technology is preferred for this component. The clear difference also illustrates why rough estimates for c_p^A and $K(\cdot)$ will generally be sufficient to draw conclusions from our model output. One reason why traditional technology is still prefered for this component is that the component's reliability relative to the system lifetime is large, which has been identified as a detriment to the application of AM (Section 5). We also observe that much of the potential benefit AM can bring in terms of after-sales costs has already been attained, as a further reduction of the AM leadtime from two weeks to one week has little effect. Further improvement can only come from an increase in reliability, which is unlikely for this component type, or a decrease in AM production costs. From Figure 6 we observe that AM production costs must decrease by approximately 40% for the AM part to become preferred over the regular version, which may well occur as AM technology continues to develop rapidly.

Figure 7: The regular aluminium bracket of case study 2

The second case we examine is one from the aviation industry. Figure 7 shows an aileron bracket used to control the roll of an aircraft, in this case a business jet. Each jet has two of these parts, situated at the end of each wing. The characteristics for the regular part come from company records and are shown in Table 7 below.

Table 7: Data for evaluating case study 2

c_p^R	450	€
c_d	600	€
c_e	25000	€
h	0.018	€/€/month
L^A	0.5	months
L^R	2.25	months
N	340	units
τ	120	months
T	180	months

In addition, the engineering department of the company where the case study is conducted, has performed a redesign of the regular bracket. The resulting AM part is made out of titanium, instead of aluminium. Titanium is approximately 60% heavier compared to aluminium, but due to design improvements, the AM part is 25% lighter than the regular part, saving 80 gram per component. A recent report estimates that one kilogram of weight saved results in €183.60 in fuel costs saved per year per airplane (see [20]). Savings of 160 gram per airplane, over 170 airplanes with 2 components each, hence $N = 340$, that are used for a period of 15 years implies $B(\cdot) =$ €75K. Designing the AM component cost €5K more than the regular part. This brings the final value of $K(\cdot)$ to €-70K. The production costs for the AM part are €1000, compared to €450 for the regular part.

The result of the analysis is shown in Figure 8. We see that also in this second case, traditional technology is still preferred. As in case study 1, a key reason for this is the high production costs of AM components, although this is expected to decrease substantially in the coming years. Another reason is that the component is only featured twice per airplane for a fleet of 170 business jets. Had this component been featured on commercial airliners, with their higher utilization, longer lifecycle and with a much larger installed base, the fuel savings would have been enough to prefer the AM component over the regular component.

7 Conclusions

In this paper, we introduce and develop a model for evaluating two production methods that can be used to produce two differently designed, but functionally the same system components. The practical motivation for this model is the potential that additive manufacturing offers compared to traditional technology, which in our case is increased design freedom and reduced production leadtimes. The former can create performance

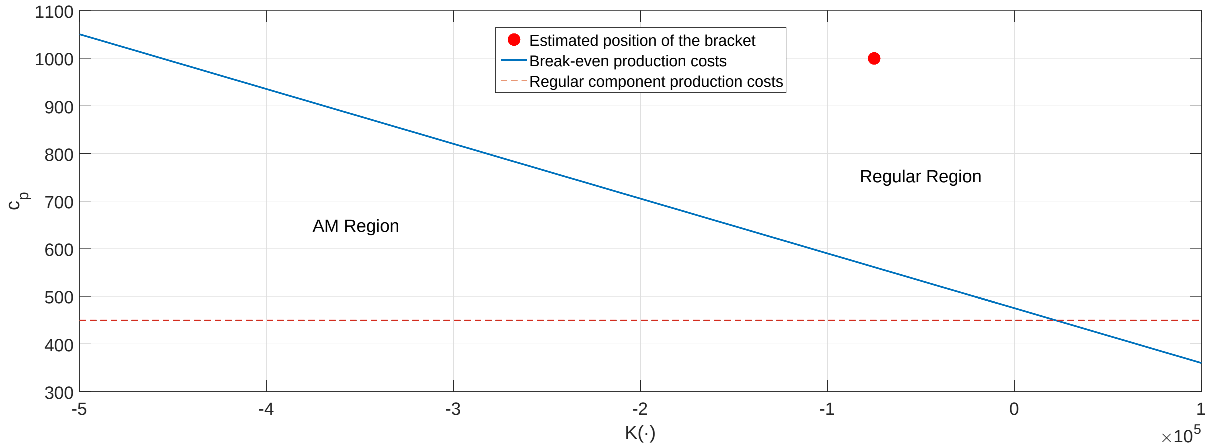

Figure 8: Outcome of case study 2

benefits, while the latter is very beneficial for the after-sales service logistics. We evaluate the OEM's design decision to opt for either the regular component or its AM counterpart by modeling total lifecycle costs, taking into account design costs, logistical costs, including maintenance and downtime costs, and performance benefits. The break-even characteristics that our model generates allow the OEM to decide on which design option to select early on in the design process.

Through our model analysis, a numerical experiment and two case studies, we gain managerial insights into the applicability of AM in a component design setting. In our numerical experiment we find that AM component development costs are often allowed to be high relative to the regular component development costs, especially when the system lifetime is long, or when installed base sizes are large, as this allows the additional development costs to be spread out. This confirms results obtained by our model analysis. Large installed base sizes, or small expected component lifetimes compared to the system lifetime, also increase the logistical costs for inventory holding and downtime when using traditional production technology. This creates larger potential savings that can be obtained by making use of AM's short production leadtimes. These savings allow for larger AM investment costs. The OEM must remain aware, however, of a bound on the maximum additional AM development costs.

For some components, AM can generate large performance benefits, as we have seen in our second case study. Not all components, however, can generate such performance benefits. In the absensce of performance benefits, we find through our numerical experiment and our first case study, that compared to their regular counterpart the reliability of AM components is much more important than their production costs. In both case studies, AM component production costs are allowed to exceed regular production costs by 25% and the AM component would still be preferred due to the logistical benefits involved, while the numerical experiment shows that, even in the most extreme case, the AM component may only be 13% less reliable than its regular counterpart. The importance of component reliability implies that one promising group of components are those for which an AM design can integrate multiple parts into a single assembly, thus reducing failure rates and component production costs.

Finally, our case studies and numerical experiments show that further leadtime reductions have only a limited influence on the break-even values we find in practice. This indicates that designing and producing reliable AM parts should be the primary focus of the AM industry and OEMs that are interested in this technology. We expect that as AM technology develops and its production costs decrease, an increasing number of components will meet one or more of the criteria we have identified for successful AM implementation.

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A Appendix

This appendix contains additional results from the numerical experiment in Section A.1. In Section A.2, we present an additional lemma that is used in some of the proofs. The proofs for the lemmas and theorems we present in our paper are in Sections A.3 through A.8.

A.1 Additional numerical experiment results

This appendix contains additional results from the numerical experiment of Section 5.

Table 8: Results from the numerical experiment

		$K_1(\cdot)/c_p$			$\tau^{A*}(K(\cdot)=0)/\tau^R$		
Parameter	**Parameter value**	**Average**	Min	Max	**Average**	Min	Max
	4	107	5	673	0.967	0.891	0.993
c_e/c_p	8	109	5	683	0.966	0.884	0.993
	16	111	6	692	0.964	0.872	0.993
	60	63	5	261	0.959	0.872	0.989
T	120	98	8	405	0.967	0.896	0.992
	240	167	13	693	0.971	0.909	0.993
	250	109	5	693	0.956	0.872	0.993
c_p	1000	109	5	693	0.956	0.872	0.993
	4000	109	5	693	0.956	0.872	0.993

A.2 Lemma 3

For some proofs, we require a property that relates to the optimal base-stock levels, which we formally define in the following lemma:

Lemma 3. *Under optimal base-stock level $S^{x*}(\cdot)$, it holds that $(g^x(S^{x*}(\cdot))-g^x(S^{x*}(\cdot)+\delta))\frac{N(c_e-c_d)}{\tau} \leq \delta hc_p^x$, for $\delta \in \mathbb{N}^+$.*

Proof. We know, due to the optimality of S^{x*} that $\Delta(C^x(S^{x*})) = C^x(S^{x*}+\delta) - C^x(S^{x*}) \geq 0$, for $\delta \in \mathbb{N}^+$:

$$\Delta C^x(S^{x*}) = \left[c_p^x N + hc_p^x T(S^{x*}(\cdot)+\delta) + g^x(S^{x*}+\delta)\frac{NT\,(c_e-c_d)}{\tau^x} + \frac{NT(c_d+c_p^x)}{\tau^x}\right]$$
$$- \left[c_p^x N + hc_p^x T S^{x*}(\cdot) + g^x(S^{x*})\frac{NT\,(c_e-c_d)}{\tau^x} + \frac{NT(c_d+c_p^x)}{\tau^x}\right]$$
$$= \delta hc_p^x T + (g^x(S^{x*}+\delta) - g^x(S^{x*}))\,\frac{NT\,(c_e-c_d)}{\tau^x}.$$

From this, it follows that:

$$(g^x(S^{x*}) - g^x(S^{x*}+\delta))\,\frac{N\,(c_e - c_d)}{\tau^x} \leq \delta h c_p^x.$$

Note that the non-strict inequality is the results of the possibility that it may occur that base stock level $S^{x*}(\cdot)$ and $S^{x*}(\cdot)+1$ are both optimal. In all cases where there is a unique optimal base stock level, or when $\delta > 1$, the inequality is strict.

A.3 Proof of Lemma 1

The proofs for Lemma 1(ii) and Lemma 1(iii) follow the same procedure as the proof of Lemma 1(i). Let $\varepsilon > 0$.

(i)

$$\begin{aligned}
\hat{C}(\tau+\varepsilon) - \hat{C}(\tau) &= C(S^*(\tau+\varepsilon);\tau+\varepsilon) - C(S^*(\tau);\tau) \\
&\leq C(S^*(\tau);\tau+\varepsilon) - C(S^*(\tau);\tau) \\
&= \left[g(S^*(\tau);\tau+\varepsilon)\frac{NT(c_e-c_d)}{\tau+\varepsilon} - g(S^*(\tau);\tau)\frac{NT(c_e-c_d)}{\tau} \right] \\
&\quad + \left[\frac{NTc_d}{\tau+\varepsilon} - \frac{NTc_d}{\tau} \right] \\
&< 0.
\end{aligned}$$

The weak inequality is obtained since $C(S^*(\tau);\tau+\varepsilon) \geq C(S^*(\tau+\varepsilon);\tau+\varepsilon)$. In the next equality the holding costs cancel out because both cost functions have the same base stock level. The strict inequality follows from the fact that $g(S^*_\tau;\tau)$ is strictly decreasing in τ for a fixed base-stock level (see [6]).

(ii) Let $N \in \mathbb{N}^+$, then:

$$\begin{aligned}
\hat{C}(N+1) - \hat{C}(N) &= C(S^*(N+1);N+1) - C(S^*(N);N) \\
&\geq C(S^*(N+1);N+1) - C(S^*(N+1);N) \\
&= \left[c_p(N+1) + hc_pTS^*(N) + g(S^*(N);N+1)\frac{(N+1)T(c_e-c_d)}{\tau} + \frac{(N+1)Tc_d}{\tau} \right] \\
&\quad - \left[c_p(N) + hc_pTS^*(N) + g(S^*(N);N)\frac{NT(c_e-c_d)}{\tau} + \frac{NTc_d}{\tau} \right] \\
&= c_p + \left[g(S^*(N);N+1)\frac{(N+1)T(c_e-c_d)}{\tau} - g(S^*(N);N)\frac{(N)T(c_e-c_d)}{\tau} \right] + \frac{Tc_d}{\tau} \\
&> 0.
\end{aligned}$$

The strict inequality follows from the fact that the Erlang loss probability, $g(S;N)$, is strictly decreasing in the service rate (see [21]) and hence strictly increasing in N.

(iii)

$$\begin{aligned}\hat{C}(S^*(c_p+\varepsilon);c_p+\varepsilon)-\hat{C}(S^*(c_p);c_p) &\geq \hat{C}(S^*(c_p+\varepsilon);c_p+\varepsilon)-C(S^*(c_p+\varepsilon);c_p)\\ &=\varepsilon\left(N+hTS^*(c_p+\varepsilon)\right)\\ &>0.\end{aligned}$$

A.4 Proof of Lemma 2

(i) We are interested in finding $\tau^{A*}(\cdot)$ such that $\hat{C}^R(\tau^R)=\hat{C}^A(\tau^{A*}(\cdot))+K(\cdot)$. If $\hat{C}^R(\tau^R)$ is given and $K(\cdot)$ increases, then $\hat{C}^A(\tau^{A*}(\cdot))$ must decrease. From Lemma 1(i), we know that $\hat{C}^A(\tau^A)$ is decreasing in τ^A, so in order to find the minimal possible life cycle costs when using an AM component, we are interested in:

$$\lim_{\tau^A\to\infty}\hat{C}^A(\tau^A)=\lim_{\tau^A\to\infty}\left[c_pN+hc_pTS^{A^*}(\tau^A)+g^A(\tau^A)\frac{NT(c_e-c_d)}{\tau^A}+\frac{NTc_d}{\tau^A}\right]=c_pN,$$

which holds since both $S^{A*}(\tau^A)$ and $g^A(S^{A*};\tau^A)$ go to zero. In other words, $\hat{C}^A(\tau^A)$ is strictly larger than c_pN for every possible value of τ^A. If we next assume that $K(\cdot)\geq K_{lim}=hc_pTS^{R^*}+g^R(\cdot)\frac{NT(c_e-c_d)}{\tau^R}+\frac{NTc_d}{\tau^R}$, we see that there exists no break-even reliability $\tau^{A*}(\cdot)$:

$$\begin{aligned}\hat{C}^A(\tau^{A*}(\cdot))&=\hat{C}^R(\tau^R)-K(\cdot)\\ &\leq\left[c_pN+hc_pTS^{R^*}(\cdot)+g^R(\cdot)\frac{NT(c_e-c_d)}{\tau^R}+\frac{NTc_d}{\tau^R}\right]-\left[hc_pTS^{R^*}(\cdot)+g^R(\cdot)\frac{NT(c_e-c_d)}{\tau^R}+\frac{NTc_d}{\tau^R}\right]\\ &=c_pN.\end{aligned}$$

For $\tau^{A*}(\cdot)$ to exist on the entire interval $K\in(-\infty,K_{lim})$, we require that $\hat{C}^A(\tau^{A*}(\cdot))$ can take on any value on the interval (C_pN,∞). We find:

$$\lim_{\tau^A\downarrow 0}\hat{C}^A(\tau^A)=\lim_{\tau^A\downarrow 0}\left[c_pN+hc_pTS^{A^*}(\tau^A)+g^A(\tau^A)\frac{NT(c_e-c_d)}{\tau^A}+\frac{NTc_d}{\tau^A}\right]>\lim_{\tau^A\downarrow 0}\frac{NTc_d}{\tau^A}=\infty.$$

(ii) This follows from the fact that $\hat{C}^x(\tau^x)$ is strictly decreasing in τ^x (see Lemma 1(i))

A.5 Proof of Theorem 1

(i) By Definition 1, it holds that $\hat{C}^A(\tau^{A*}(\cdot))=\hat{C}^R(\tau^R)-K(\cdot)$. This implies that if $K(\cdot)$ increases, $\hat{C}^A(\tau^{A*}(\cdot))$ decreases and from Lemma 1(i) we know that $\hat{C}^A(\tau^A)$ is strictly decreasing in τ^{A*}.

(ii) From the proof of Lemma 2(i), we know that $\lim_{\tau^A\to\infty}\hat{C}^A(\tau^A)=C_pN$ and that $\hat{C}^R(\tau^R)-K_{lim}=c_pN$, which, in combination with Theorem 1(i) implies the result.

(iii) We assume that $\tau^{A^*}(\cdot)\geq\tau^R$ and show that then $K(\cdot)>0$, which proves by contradiction that if

$K(\cdot) \leq 0$, then $\tau^{A^*}(\cdot) < \tau^R$.

$$
\begin{aligned}
K(\cdot) &= C^R(S^{R*}(\tau^R); \tau^R) - C^A(S^{A*}(\tau^{A^*}(\cdot)); \tau^{A^*}(\cdot)) \\
&\geq C^R(S^{R*}(\tau^R); \tau^R) - C^A(S^{R*}(\tau^R); \tau^{A^*}(\cdot)) \\
&= \left[c_p N + hc_p T S^{R^*}(\tau^R) + g^R(S^{R^*}(\tau^R); \tau^R)\frac{NT(c_e - c_d)}{\tau^R} + \frac{NTc_d}{\tau^R}\right] \\
&\quad - \left[c_p N + hc_p T S^{R^*}(\tau^R) + g^A(S^{R^*}(\tau^R); \tau^{A^*}(\cdot))\frac{NT(c_e - c_d)}{\tau^{A^*}(\cdot)} + \frac{NTc_d}{\tau^{A^*}(\cdot)}\right] \\
&= \left[g^R(S^{R^*}(\tau^R); \tau^R)\frac{NT(c_e - c_d)}{\tau^R} - g^A(S^{R^*}(\tau^R); \tau^{A^*}(\cdot))\frac{NT(c_e - c_d)}{\tau^{A^*}(\cdot)}\right] + \left[\frac{NTc_d}{\tau^R} - \frac{NTc_d}{\tau^{A^*}(\cdot)}\right] \\
&> 0.
\end{aligned}
$$

The weak inequality is obtained since $C^A(S^{A*}(\tau^{A^*}(\cdot)); \tau^{A^*}(\cdot)) \leq C^A(S^{R*}(\tau^R); \tau^{A^*}(\cdot))$. The strict inequality follows from the fact that the Erlang Loss probability is strictly increasing in the system load (see [21]), so that $g^A(S^{R^*}(\tau^R); \tau^{A^*}(\cdot)) < g^R(S^{R^*}(\tau^R); \tau^R)$, since $L^A < L^R$ and $\tau^{A^*}(\cdot) \geq \tau^R$.

A.6 Proof of Theorem 2

(i) This follows from Theorem 1(i) and Theorem 1(iii).

(ii) Let $\varepsilon > 0$ and $\Delta K_1(T) = K_1(T + \varepsilon) - K_1(T)$. We then need to prove that $\Delta K_1(T) > 0$.

From Definition 2, we know that $\tau^{A*}(K_1(T + \varepsilon)) = \tau^{A*}(K_1(T)) = \tau^R$ and we denote this value by τ. From Definition 1, we know that $\hat{C}^R(T) - \hat{C}^A(T) = K_1(T)$. This means that:

$$
\begin{aligned}
&\Delta K_1(T) \\
&= \left[\hat{C}^R(T + \varepsilon) - \hat{C}^A(T + \varepsilon)\right] - \left[\hat{C}^R(T) - \hat{C}^A(T)\right] \\
&= \left[\hat{C}^R(T + \varepsilon) - \hat{C}^R(T)\right] + \left[\hat{C}^A(T) - \hat{C}^A(T + \varepsilon)\right] \\
&= \left[hc_p(T + \varepsilon)S^{R*}(\cdot) + g^R(S^{R*}(\cdot))\frac{N(T + \varepsilon)(c_e - c_d)}{\tau} + \frac{N(T + \varepsilon)c_d}{\tau}\right] \\
&\quad - \left[hc_p T S^{R*}(\cdot) + g^R(S^{R*}(\cdot))\frac{NT(c_e - c_d)}{\tau} + \frac{NTc_d}{\tau}\right] + \left[hc_p T S^{A*}(\cdot) + g^A(S^{A*}(\cdot))\frac{NT(c_e - c_d)}{\tau} + \frac{NTc_d}{\tau}\right] \\
&\quad - \left[hc_p(T + \varepsilon)S^{A*}(\cdot) + g^A(S^{A*}(\cdot))\frac{N(T + \varepsilon)(c_e - c_d)}{\tau} + \frac{N(T + \varepsilon)c_d}{\tau}\right] \\
&= \left[hc_p \varepsilon S^{R*}(\cdot) + g^R(S^{R*}(\cdot))\frac{N\varepsilon(c_e - c_d)}{\tau} + \frac{N\varepsilon c_d}{\tau}\right] - \left[hc_p \varepsilon S^{A*}(\cdot) + g^A(S^{A*}(\cdot))\frac{N\varepsilon(c_e - c_d)}{\tau} + \frac{N\varepsilon c_d}{\tau}\right] \\
&= \varepsilon \left(hc_p(S^{R*}(\cdot) - S^{A*}(\cdot)) + (g^R(S^{R*}) - g^A(S^{A*}))\frac{N(c_e - c_d)}{\tau}\right).
\end{aligned}
$$

Under the condition that optimal base-stock levels are non-decreasing in the system load, i.e., $S^{A*}(\cdot) \leq S^{R*}(\cdot)$, we make a case distinction. The first case is that $S^{A*}(\cdot) = S^{R*}(\cdot)$ and we denote this value by S, while the second case is $S^{A*}(\cdot) < S^{R*}(\cdot)$. For the first case, we find:

$$\begin{aligned}\Delta K_1(T) &= \varepsilon\left(hc_p(S^{R*}(\cdot) - S^{A*}(\cdot)) + (g^R(S^{R*}(\cdot)) - g^A(S^{A*}(\cdot)))\frac{N(c_e - c_d)}{\tau}\right)\\ &= \varepsilon\left(hc_p(S - S) + (g^R(S) - g^A(S)\frac{N(c_e - c_d)}{\tau}\right)\\ &= \varepsilon(g^R(S) - g^A(S))\frac{NT(c_e - c_d)}{\tau}\\ &> 0,\end{aligned}$$

because the Erlang loss probability is strictly increasing in the load (see [21]), so that $g^A(S) < g^R(S)$ because $L^A < L^R$.

For the second case, we recall the property defined in Lemma 3:

$$(g^A(S^{A*}(\cdot)) - g^A(S^{A*}(\cdot) + \delta))\frac{N(c_e - c_d)}{\tau} \leq \delta hc_p,$$

which means that:

$$(g^A(S^{A*}(\cdot)) - g^A(S^{R*}(\cdot)))\frac{N(c_e - c_d)}{\tau} \leq \left(S^{R*}(\cdot) - S^{A*}(\cdot)\right) hc_p.$$

This in turn implies that:

$$\begin{aligned}\Delta K_1(T) &= \varepsilon\left(hc_p(S^{R*}(\cdot) - S^{A*}(\cdot)) + (g^R(S^{R*}) - g^A(S^{A*}))\frac{N(c_e - c_d)}{\tau}\right)\\ &= \varepsilon\left(hc_p(S^{R*}(\cdot) - S^{A*}(\cdot)) - (g^A(S^{A*}) - g^R(S^{R*}))\frac{N(c_e - c_d)}{\tau}\right)\\ &\geq \varepsilon\left(hc_p(S^{R*}(\cdot) - S^{A*}(\cdot)) - (S^{R*}(\cdot) - S^{A*}(\cdot))hc_p\right)\\ &= 0.\end{aligned}$$

Note that equality can only occur when the optimal base stock level is not unique, i.e., costs are equal under base stock level $S^{A*}(\cdot)$ and base stock level $S^{A*}(\cdot) + 1$. Furthermore, it must be the case that $S^{A*}(\cdot) = S^{R*}(\cdot) - 1$, otherwise $\delta > 1$ and we regain a strict inequality as a result of Lemma 3. In all other cases, it will hold that $\Delta K_1(T) > 0$.

(iii) This proof follows directly from the property described in Theorem 2(ii) and Theorem 1(i).

A.7 Proof of Theorem 3

We define the term Δ_T as the difference between two break-even equations according to Definition 1, one for T and one for $T + \varepsilon$, and both for $K_2(T)$ as defined in Definition 3. Both equations should be equal to zero, so that also $\Delta_T = 0$. We prove for the case where $\tau^{A*}(K_2(T); T + \varepsilon) \leq \tau^{A*}(K_2(T); T)$, then $\Delta_T > 0$. Since we know that if $\tau^{A*}(K_2(T); T + \varepsilon) \leq \tau^{A*}(K_2(T); T)$, then $\Delta_T > 0$, we know that it must hold that

$\tau^{A*}(K_2(T); T+\varepsilon) > \tau^{A*}(K_2(T); T)$, which completes our proof.

$$
\begin{aligned}
\Delta_T &= \left[\hat{C}^R(\tau^R; T) - \hat{C}^A(\tau^{A*}(K_2(T); T); T) - K_2(T)\right] \\
&\quad - \left[\hat{C}^R(\tau^R; T+\varepsilon) - \hat{C}^A(\tau^{A*}(K_2(T); T+\varepsilon); T+\varepsilon) - K_2(T)\right] \\
&= \left[\hat{C}^R(\tau^R; T) - \hat{C}^A(\tau^{A*}(K_2(T); T); T)\right] - \left[\hat{C}^R(\tau^R; T+\varepsilon) - \hat{C}^A(\tau^{A*}(K_2(T); T+\varepsilon); T+\varepsilon)\right] \\
&\geq \left[\hat{C}^R(\tau^R; T) - \hat{C}^R(\tau^R; T+\varepsilon)\right] + \left[\hat{C}^A(\tau^{A*}(K_2(T); T); T+\varepsilon) - \hat{C}^A(\tau^{A*}(K_2(T); T); T)\right] \\
&= \left[hc_pT(S^{R*}(\cdot) - S^{R*}(\cdot)) - \varepsilon hc_p S^{R*}(\cdot) + (g^R(\cdot) - g^R(\cdot))\frac{NT(c_e - c_d)}{\tau^R} - g^R(\cdot)\frac{\varepsilon N(c_e - c_d)}{\tau^R} - \frac{\varepsilon N c_d}{\tau^R}\right] \\
&\quad + \left[hc_pT(S^{A*}(\cdot) - S^{A*}(\cdot)) + \varepsilon hc_p S^{A*}(\cdot) + (g^A(\cdot) - g^A(\cdot))\frac{NT(c_e - c_d)}{\tau^{A*}(K_2(T); T)}\right. \\
&\quad \left. + g^A(\cdot)\frac{\varepsilon N(c_e - c_d)}{\tau^{A*}(K_2(T); T)} + \frac{\varepsilon N c_d}{\tau^{A*}(K_2(T); T)}\right] \\
&= \varepsilon\left[\left(\frac{Nc_d}{\tau^{A*}(K_2(T); T)} - \frac{Nc_d}{\tau^R}\right) + N(c_e - c_d)\left(\frac{g^A(\cdot)}{\tau^{A*}(K_2(T); T)} - \frac{g^R(\cdot)}{\tau^R}\right)\right] \\
&> 0.
\end{aligned}
$$

First, we provide both break-even equations and we let $K_2(T)$ cancel out from both break-even equations. Then, we substitute $\tau^{A*}(K_2(T); T)$ for $\tau^{A*}(K_2(T); T+\varepsilon)$. This results in the first inequality, because $\hat{C}^A(\tau^A)$ is decreasing in τ (see Lemma 1(i)), which results in a cost decrease if we increase $\tau^{A*}(K_2(T); T+\varepsilon)$ up to $\tau^{A*}(K_2(T); T)$. The inequality is non-strict to also include the case where $\tau^{A*}(K_2(T); T+\varepsilon)$ is equal to $\tau^{A*}(K_2(T); T)$. Then we write the entire equation. For the final equality, note that we have equal loads on the inventory systems, i.e., $a^R = a^A$, and we have equal cost structures. This implies that $S^{R*}(\cdot) = S^{A*}(\cdot)$ and $g^R(\cdot) = g^A(\cdot)$. We also know that $S^{x*}(T) = S^{x*}(T+\varepsilon)$, since Lemma 3 shows that the optimal base stock level is independent of T. Hence, many terms cancel out. Finally, we make use of the fact that $\tau^{A*}(K_2(T); T) < \tau^R$, which results in the final inequality.

A.8 Proof of Theorem 4

(i) Suppose $K(\cdot) > \hat{C}^R(c_p^R)$, then the following holds via eq. (**??**):

$$
\begin{aligned}
c_p^{A^*} &= \frac{c_p^R(N + hTS^{R*}(c_p^R)) + (g^R(\cdot) - g^A(\cdot))\frac{NT(c_e - c_d)}{\tau} - K(\cdot)}{N + hTS^{A*}(c_p^{A^*}) + \frac{NT}{\tau}} \\
&\leq \frac{c_p^R(N + hTS^{R*}(c_p^R)) + (g^R(\cdot))\frac{NT(c_e - c_d)}{\tau} - K(\cdot)}{N + hTS^{A*}(c_p^{A^*}) + \frac{NT}{\tau}} \\
&= \frac{\hat{C}^R(c_p^R) - K(\cdot)}{N + hTS^{A*}(c_p^{A^*}) + \frac{NT}{\tau}} \\
&< 0,
\end{aligned}
$$

where the weak inequality holds due to the fact that the Erlang loss probability $g^A \in (0, 1]$. Hence, we require a negative production cost of the AM component in order to meet break-even, which is clearly not feasible.

(ii) This follows from the fact that $\hat{C}(\cdot)$ is increasing in c_p^x (Lemma 1).

(iii) Let $\varepsilon > 0$ and assume that $c_p^{A*}(K(\cdot)) = c_p^{A*}(K(\cdot) + \varepsilon)$. By Definition 4 we find the following expression:

$$\begin{aligned}\left[\hat{C}^R(c_p^R) - \hat{C}^A(c_p^{A^*}(K(\cdot)))\right] - \left[\hat{C}^R(c_p^R) - \hat{C}^A(c_p^{A^*}(K(\cdot) + \varepsilon))\right] &= \hat{C}^A(c_p^{A^*}(K(\cdot) + \varepsilon)) - \hat{C}^A(c_p^{A^*}(K(\cdot))) \\ &= \varepsilon,\end{aligned}$$

which implies that in order to compensate for the increase in $K(\cdot)$ and meet the break-even condition of Definition 4, $c_p^{A*}(K(\cdot) + \varepsilon)$ must be smaller than $c_p^{A*}(K(\cdot))$, since $C^A(c_p^A)$ is increasing in c_p^A (Lemma 1(iii)).