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Constrained maintenance optimization under non-constant probabilities of imperfect inspections

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Abstract

In this research, we study a single-component system that is characterized by three distinct states, cf. the Delay Time Model: normal, defective, and failed. The system is maintained by an inspection policy with fixed time intervals, and preventive system replacement after a number of inspections. The inspections are imperfect, and the probability of an inspection error changes over the system's operation time. Our objective is to minimize the average cost over an infinite time horizon, under a reliability constraint. We present exact cost and reliability evaluations under a given maintenance policy, for a model considering non-constant probabilities of inspection errors. We call this model the *true model*. Furthermore, we compare the true model to a model that considers constant probabilities of inspection errors. We refer to this latter model as the *approximate model*. By means of a computational study, we find that the approximate model yields, on average, 26% higher costs than the true model. Furthermore, we find that the reliability constraint can be violated severely, by at most 25%, when using the approximate model.

Keywords: Maintenance Optimization, Delay-Time Model, False Positives, False Negatives, Human Factors

1. Introduction

Unexpected failures cause costly downtime for many advanced technical systems, such as airplanes, trains, baggage handling systems, and medical systems. Maintenance is done during system operation to avoid such unexpected failures. The costs of these maintenance activities comprise 15-60% of the total production costs in a manufacturer's facility (Mobley, 2002). Reducing the operating costs of such advanced technical systems can, therefore, be achieved by lowering the maintenance costs. To facilitate this, mathematical maintenance models and techniques are used to derive optimal maintenance policies.

The literature on maintenance optimization is rich and covers various areas such as system replacement, inspections, repair, and maintenance scheduling (Jardine and Tsang, 2013). These areas of maintenance optimization use modeling techniques that describe system degradation. A commonly used technique for modeling system degradation is the Delay Time Model (DTM). This model distinguishes three system states: normal, defective, and failed. The system operates

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properly in the normal state; operates in the defective state as well, but its defect can be revealed by inspections; or the system has failed. The DTM is typically studied under inspection based maintenance policies; i.e., inspections are done to reveal the system's degradation. A literature overview of the DTM up to 1999 is provided by Christer (1999), and Wang (2012) reviews the research from 1999 to 2012. The most recent advancements, since 2012, include postponements of maintenance actions when the defects are detected (van Oosterom et al., 2014), and the application of the DTM to systems that have redundant components (Wang, 2013). Furthermore, multiple different forms of preventive maintenance activities, such as routine service, preventive system replacement, and manual inspection, aided by condition-monitoring, are combined in two models, based on the DTM (Wang, 2013).

In this research, we focus on a single-component DTM. Our DTM is combined with an inspection based maintenance policy. Scarf et al. (2009) studied a general class of inspection policies, consisting of a fixed inspection interval length T and a number of inspections before preventive system replacement M. This implies that the system is preventively replaced at time MT, whatever its state may be.

The DTM literature considering inspection policies generally assumes that the inspections are perfect. However, due to (human) errors, inspections are usually not perfect in practice (Wickens, 1992). Therefore, imperfect inspections have been included in single-component DTMs by Okumura et al. (1996) and Berrade et al. (2013). Both works consider two types of imperfect inspection behavior, i.e., false positives and false negatives. A false positive corresponds to the judgment that a system is in its defective state, when it is actually in its normal state. False negatives correspond to the judgment that a system is in its normal state when, in fact, the system is in its defective state. We denote the probabilities of false positives and false negatives by α and β, respectively. For an overview, Table 1 is included.

Table 1: Probabilities of inspection behavior

		System State	
		Normal	Defective
Inspection outcome	Normal	$1-\alpha$	β
	Defective	α	$1-\beta$

The probabilities of false positives and false negatives, α and β, are assumed to be constant by Okumura et al. (1996) and Berrade et al. (2013). However, such an approach might be inaccurate, and therefore Wang (2010) explored the effects of non-constant probabilities of false negatives in a multi-component setting. Yet, a non-constant probability of false positives has not been explored in the DTM literature, nor in any of the other maintenance models, such as Markovian based maintenance models. Therefore, our work extends the literature by proposing a non-constant probability of false positives under a single-component DTM. Furthermore, we extend the literature by relating the probability of false negatives to the system's duration in the defective state, relative to its delay time. By this modeling of false negatives we deviate from Wang (2010), who only considers the duration in the defective state. Table 2 presents a schematic literature overview of DTMs with imperfect inspections.

Table 2: Imperfect inspections in the DTM literature

		False Positives	
		Constant	Non-constant
False Negatives	Constant	Okumura et al. (1996); Berrade et al. (2013)	
	Non-constant	Wang (2010)	This research

We borrow findings from different literature streams to base our non-constant probabilities on. In the literature on nondestructive testing (NDT), the Probability of Detection (POD) is studied. This POD is defined as the probability of finding a degradation when it is present (Berens, 1989). It reflects the probability of false negatives by β=1-POD (Nichols et al., 2008). As a system degrades, the POD increases; i.e., the probability of false negatives decreases with system degradation (Forsyth and Fahr, 1998). Wang (2010) captures such a relationship by relating the duration that a system has been defective (an indicator of system degradation) to the probability of false negatives. In this research, we conceptualize system degradation as the duration that a system has been defective relative to its delay time, which is a richer concept. For instance, weaker materials will have a lower delay time. Hence, under the same duration in the defective state, the weaker material (shorter delay time) will have a higher level of degradation than the strong one.

We consider the field of psychology for the non-constant probability of false positives. As the industry, nowadays, pays more attention to system reliability aspects, the pressure on maintenance staffs increases. Such an increased pressure tends to motivate employees to violate inspection guidelines (Latorella and Prabhu, 2000), and rely on self-developed decision making instead of following inspection guidelines. Psychological literature labels this type of decision making as heuristic decision making. To avoid any confusion with the terminology of 'heuristic' in operations research, we will refer to it as self-developed decision making. This self-developed decision making is a way of decision making in which not all information is taken into account. Yet, it may provide a convenient way to tackle a problem, compared to that of complete information (Kahneman and Frederick, 2002). Multiple types of self-developed decision making are defined throughout the literature. In this paper, we consider attribute substitution (Kahneman, 2003). For this type of self-developed decision making, an alternative attribute, e.g., a system's duration of normal operation, may be used for the decision making, rather than the actual condition obtained through inspections. To the authors' best knowledge, such an attribute substitution, with respect to false positives, in inspections has not been studied in maintenance modeling up to date.

Besides the imperfectness of inspections in maintenance, reliability aspects are becoming increasingly important according to the industry's and European Union's agenda (European Union, 2009). Hence, companies explicitly include reliability measures in their maintenance analyses. As Aven and Castro (2009) noted, reliability aspects can be included in the cost expression, but such an approach may be rather controversial, e.g., transforming human injuries into monetary terms. Therefore, they propose to include the reliability aspect in the form of a constraint.

In this research, we also take an explicit reliability constraint into account, and propose a constrained single-component model, considering non-constant probabilities of false positives and false negatives. We will refer to this model as the *true model*. Besides the true model, we also study the *approximate model*, which is a constrained single-component model, that considers constant probabilities of false positives and false negatives. Our analysis is directly transferable to a setting without a reliability constraint. Our objective is to minimize the average cost per time unit over an infinite time horizon, by optimizing the maintenance policy (M, T). The research's contributions are twofold: (1) we present exact cost and reliability evaluations of our true model; and (2) we compare our true model to the approximate model. We show that the approximate model may result in policies that violate the reliability constraint by 25%, and it yields, on average, 26% higher cost than the true model.

The remainder of this paper is organized as follows. In Section 2, we present the model. In Section 3, we give exact cost and reliability evaluations for a (M, T) policy, and we discuss the optimization procedure. In Section 4, we present a method for comparing the true and approximate model, and we present the computational results that compare both models in Section 5. In Section 6, we conclude and give potential directions for future research.

2. Model description

In this section, we describe our true model. However, the description and the reasoning also applies to the approximate model. The sole difference, in the latter case, is that the probabilities of false positives and false negatives are assumed to be constants.

Let us consider a single-component system operating over an infinite time horizon. The system has three states: normal, defective and failed. In the normal operating state, the system is working properly, without any detectable defects. In the defective state, inspections may reveal the system's defect. Yet, the system is still able to operate. The failed state of the system is self-announcing, and the system stops delivering its function immediately. To prevent the system from reaching its failed state, it is inspected periodically each $T > 0$ time units, it is preventively maintained upon detection of the defective state, or it is replaced preventively after $M - 1 \in \mathbb{N}$ inspections, at time MT. This implies that at time MT we do not perform another inspection, since the system is preventively replaced anyway. If the system fails, it is replaced correctively.

We assume perfect maintenance, implying that preventive maintenance is equivalent to preventive replacement. Hence, we will solely use the term preventive replacement in the remainder. Note that the (M, T) maintenance policy degenerates to an age-based maintenance policy when $M = 1$, degenerates to a pure inspection policy when $M = \infty$, and becomes a hybrid policy for any finite $M > 1$. We assume that inspections are the only means to detect the normal and defective state.

We denote the duration of the normal state, referred to as the time to defect, by the continuous random variable $X > 0$. This time to defect corresponds to the time between replacement (preventive or corrective), and the arrival time of the defect. The random time the system takes from defect arrival to failure, without doing any replacements, is referred to as the delay time and denoted by the continuous random variable $H > 0$. The sum of both random variables is the system's time to failure. The cumulative distribution function (cdf) and the probability density function (pdf), corresponding to both state durations, are defined by $F_X(\cdot)$ and $f_X(\cdot)$ for the time to defect, and by $F_H(\cdot)$ and $f_H(\cdot)$ for the delay time, respectively. Both cdfs, $F_X(\cdot)$ and $F_H(\cdot)$, are continuous functions.

The cost for doing an inspection is denoted by c_0, and the cost c_p is incurred for preventive replacement. When the system unexpectedly fails in between inspections, corrective replacement is immediately done with cost c_c, and the inspection schedule restarts; i.e., the first inspection is performed T time units after system failure. We assume that the failure cost, containing for example collateral damage cost, is included in the corrective replacement cost c_c. Furthermore, we assume that $0 < c_0 < c_p < c_0 + c_p < c_c$, and the time for inspections and maintenance is negligible.

In our model, the inspections can be imperfect; i.e., an inspection error can occur, and the inspection outcome differs from the system's true state. We take two classes of imperfect inspections into account: (1) false positives; and (2) false negatives. For an overview, see Table 1. The probabilities of imperfect inspections are assumed to be non-constant. Hence, we use two functions $\alpha(.)$ and $\beta(.)$ to describe the non-constant probabilities of false positives and false negatives, respectively.

For the non-constant probability of false positives, we assume that (some) engineers do not use the measurement outcome in their judgment, but replace this with a substitute attribute (Kahneman, 2003). We consider the time t that the system has been operating in its normal state as the substitute attribute. This time starts from the latest replacement moment (preventive or corrective). We assume that, if the time t of an inspection approaches a threshold value a, the engineers (using the time t as a substitute attribute) will become more tempted to engage in a false positive. The variable a might relate to a temporal parameter at which the maintenance engineers believe the system typically becomes defective, e.g. the mean time to defect. If the time t of an inspection exceeds the threshold value a, we assume that the engineers using the attribute substitution, will send the system to preventive maintenance. The probability of $\alpha(t)$ is then increasing for $t < a$, and remains constant for $t \geq a$. For an illustration of $\alpha(t)$, see Figure 1.

When the defect appears at the realization x of the time to defect, we assume that the system starts its degradation until failure. We conceptualize this system degradation by the failure progress. If the realized delay time is h; i.e., the failure occurs at h time units after the defect arrival, the failure progress is defined by the relative duration $(t - x)/h$ in the defective state, at time $x \leq t \leq \min\{x + h, MT\}$. Based on the POD literature, we assume that the inspection engineers have difficulty in determining whether degradation exists at the early stage of the failure progress. The more the system degrades, the easier the detection of the degradation becomes; i.e., the probability of false negatives $\beta((t - x)/h)$ is nonincreasing in $(t - x)/h$. See Berens (1989) for a more detailed discussion. An illustration for $\beta((t-x)/h)$ is presented in Figure 1.

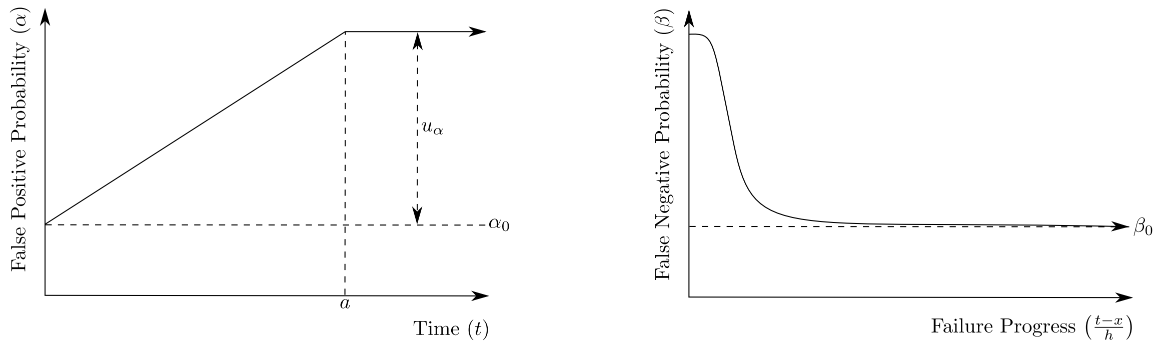

Figure 1: Illustration of non-constant probabilities α and β

We explicitly include a reliability constraint in our model, following the approach proposed by Aven and Castro (2009). The reliability constraint is commonly defined in industry by the maximum average number of failures per time unit over an infinite time horizon, which we denote by R_{max}. $R(M,T)$ corresponds to the average number of failures per time unit under a policy (M,T), over an infinite time horizon. We will refer to this $R(M,T)$ as the reliability in the remainder.

Our aim is to minimize the average cost per time unit $g(M,T)$ over an infinite time horizon, under a reliability constraint, by optimizing the maintenance policy decision variables M and T. This yields the following formulation of our optimization model:

$$\begin{aligned} \min_{M,T} \quad & g(M,T) \\ \text{s.t.} \quad & \\ & R(M,T) \leq R_{max} \\ & T > 0,\ M \in \mathbb{N}. \end{aligned} \tag{1}$$

2.1 Notation

3. Model analysis

This section presents the analysis of the optimization model from Equation (1). Again, we focus here on the true model, since the approximate model's analysis can directly be derived from the analysis below, when the probabilities of inspection errors are constant. First, we derive exact expressions for the average cost per time unit $g(M,T)$ and the reliability $R(M,T)$ in Section 3.1. The solution procedure for solving the optimization model is discussed in Section 3.2.

3.1 Evaluation of the cost and reliability function

Corrective and preventive replacement moments constitute renewal points. We define a renewal cycle as the time between two such successive renewal points. From renewal theory (Ross, 1983)

X	:	Continuous non-negative random variable representing the system's time to defect
H	:	Continuous non-negative random variable representing the system's delay time
$F_X(\cdot)$	:	Cumulative distribution function for the random variable X
$F_H(\cdot)$	:	Cumulative distribution function for the random variable H
$f_X(\cdot)$	:	Probability density function for the random variable X
$f_H(\cdot)$	:	Probability density function for the random variable H
$M-1$	:	Maximum number of inspections before preventive system replacement
T	:	Fixed inspection interval length
c_0	:	Cost per inspection
c_p	:	Cost for preventive replacement
c_c	:	Cost for corrective replacement
$\alpha(\cdot)$	:	Non-constant probability of a false positive inspection
$\beta(\cdot)$	:	Non-constant probability of a false negative inspection

we know that the average cost over one renewal cycle equals the average cost over an infinite time horizon $g(M,T)$. By the same reasoning, we know that the average number of failures per time unit over an infinite time horizon $R(M,T)$, equals the average number of failures per time unit in a renewal cycle. We use these properties to obtain expressions for $g(M,T)$ and $R(M,T)$. Then,

$$g(M,T) = \frac{C(M,T)}{L(M,T)}, \tag{2}$$

where $C(M,T)$ represents the expected renewal cycle cost, and $L(M,T)$ represents the expected renewal cycle length under (M,T). The reliability $R(M,T)$ is derived similarly:

$$R(M,T) = \frac{F(M,T)}{L(M,T)}, \tag{3}$$

where $F(M,T)$ denotes the expected number of failures in a renewal cycle under policy (M,T). The derivation for the terms $C(M,T)$, $F(M,T)$ and $L(M,T)$ is based on various event paths that may occur in a cycle. We study these event paths, and derive the corresponding probability expressions, which yield the expressions for $g(M,T)$ and $R(M,T)$.

Event Path Type 1 (E1). The system survives without any defect occurrence until time MT, at which the cycle ends. This implies that its time to defect has to exceed MT, and that no false positives occur at any of the inspections before M. We refer to this event path as $EventPath(1)$. Because the inspections are done every T time units, false positives do not occur at inspections $1, \ldots, M-1$, corresponding to the times $T, \ldots, (M-1)T$. Hence, the probability of false positives is evaluated at these time epochs. Since the probability of false positives is non-constant, a product series is included from inspections 1 to $M-1$. The probability expression for the Event Path Type 1 equals

$$\pi_1 = \int_{MT}^{\infty} \prod_{n=1}^{M-1} \left(1 - \alpha\left(nT\right)\right) f_X(x) dx = \left(1 - F_X(MT)\right) \prod_{n=1}^{M-1} \left(1 - \alpha\left(nT\right)\right).$$

Event Path Type 2 (E2). A false positive occurs at inspection $j \in \{1, \ldots, M-1\}$, thereby ending the cycle. This implies that the time to defect exceeds time jT, and before inspection j no false positives have occurred. This event path is referred to as $EventPath(2, j)$. The corresponding probability is:

$$\pi_{2,j} = \int_{jT}^{\infty} \prod_{n=1}^{j-1} (1 - \alpha(nT)) \alpha(jT) f_X(x) dx = (1 - F_X(jT)) \alpha(jT) \prod_{n=1}^{j-1} (1 - \alpha(nT)), \quad j \in \{1, \ldots, M-1\}.$$

Event Path Type 3 (E3). The system becomes defective at time x, in an interval $[(i-1)T, iT)$, characterized by inspection $i \in \{1, \ldots, M\}$, and fails in this very interval, i.e., a false negative cannot occur. This implies that its delay time lies in the interval $[0, iT - x)$. Before defect arrival, no false positives occur. We denote this event path by $EventPath(3, i)$. The probability of $EventPath(3, i)$ is given by

$$\pi_{3,i} = \int_{(i-1)T}^{iT} \prod_{n=1}^{i-1} (1 - \alpha(nT)) \int_0^{iT-x} f_H(h) dh f_X(x) dx, \quad i \in \{1, \ldots, M\}.$$

Event Path Type 4 (E4). The system becomes defective in an interval $[(i-1)T, iT)$ characterized by inspection $i \in \{1, \ldots, M-1\}$. In contrast to the third type of event paths, the system does not fail in this interval, but fails in an interval $[jT, (j+1)T), j \in \{i, \ldots, M-1\}$. We refer to this event path as $EventPath(4, i, j)$. False negatives may occur at inspections i up to j, and we include a product series due to the non-constant false negative probability. Since the false negatives can only occur at inspection instances, we consider kT in the product series, where $k = i, \ldots, j$. No false positives occur before inspection i. Note that the event paths of type 4 are characterized by the inspections i and j, and therefore we obtain the probability expression:

$$\pi_{4,i,j} = \int_{(i-1)T}^{iT} \prod_{n=1}^{i-1} (1 - \alpha(nT)) \int_{jT-x}^{(j+1)T-x} \prod_{k=i}^{j} \beta\left(\frac{kT - x}{h}\right) f_H(h) dh f_X(x) dx,$$
$$i \in \{1, \ldots, M-1\}, j \in \{i, \ldots, M-1\}.$$

Event Path Type 5 (E5). The system becomes defective in an interval $[(i-1)T, iT)$, where $i \in \{1, \ldots, M-1\}$, and no false positives occur before defect arrival. The system's defect is revealed at inspection $j \in \{i, \ldots, M-1\}$, denoting that the system's delay time has to exceed $jT - x$. This event path is referred to as $EventPath(5, i, j)$. Note that the detection of the defect occurs at time jT. This means that for inspections i up to $j-1$, false negatives occur with a non-constant probability. The corresponding probability of $EventPath(5, i, j)$ corresponds to

$$\pi_{5,i,j} = \int_{(i-1)T}^{iT} \prod_{n=1}^{i-1} (1 - \alpha(nT)) \int_{jT-x}^{\infty} \prod_{k=i}^{j-1} \beta\left(\frac{kT - x}{h}\right) \left(1 - \beta\left(\frac{jT - x}{h}\right)\right) f_H(h) dh f_X(x) dx,$$
$$i \in \{1, \ldots, M-1\}, j \in \{i, \ldots, M-1\}$$

Event Path Type 6 (E6). The system becomes defective at time x in $[(i-1)T, iT)$, where $i \in \{1, \ldots, M\}$, and remains defective until the system is renewed at time MT. Before the defect arrives, no false positives occur. From the defect arrival at time x to MT, the system remains defective. This implies that inspections i up to $M-1$ undergo false negatives. We denote this event path by $EventPath(6,i)$, with corresponding probability

$$\pi_{6,i} = \int_{(i-1)T}^{iT} \prod_{n=1}^{i-1} (1-\alpha(nT)) \int_{MT-x}^{\infty} \prod_{k=i}^{M-1} \beta\left(\frac{kT-x}{h}\right) f_H(h)dhf_X(x)dx, \quad i \in \{1,\ldots,M\}.$$

From the six different types of event paths, we derive the expression for the expected renewal cycle costs $C(M,T)$. For E3 and E4 corrective replacement costs c_c are incurred, since these event path types end in system failure. Furthermore, for E3 and E4, $i-1$ and j inspections are done, respectively. Consequently the cost of E3 corresponds to $(i-1)c_0 + c_c$, and for E4 the cost is given by $jc_0 + c_c$. Since all other types of event paths, E1, E2, E5, and E6 end in preventive replacement, cost c_p are incurred. For these event types, the number of inspections and the total inspection cost can be determined. By the above expressions for the probabilities of the event paths, the expected cycle cost $C(M,T)$ are obtained by summing the event paths over all possible i and j:

$$\begin{aligned} C(M,T) = {} & ((M-1)c_0 + c_p)\pi_1 + \sum_{j=1}^{M-1}(jc_0+c_p)\pi_{2,j} + \sum_{i=1}^{M}((i-1)c_0+c_c)\pi_{3,i} + \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(jc_0+c_c)\pi_{4,i,j} \\ & + \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(jc_0+c_p)\pi_{5,i,j} + \sum_{i=1}^{M}((M-1)c_0+c_p)\pi_{6,i}. \end{aligned} \tag{4}$$

We derive the expression for the expected renewal cycle length $L(M,T)$ in a similar way to $C(M,T)$. For E3 and E4, the cycle ends in system failure. Therefore, the cycle length depends on the realizations of x and h, and involves the integration over x and h. The other types of event paths renew at the time epoch of a preventive replacement (at an inspection or at MT). When summing over all possible values of i and j for all event paths, we obtain

$$\begin{aligned} L(M,T) = {} & MT\pi_1 + \sum_{j=1}^{M-1} jT\pi_{2,j} + \sum_{i=1}^{M}\int_{(i-1)T}^{iT}\prod_{n=1}^{i-1}(1-\alpha(nT))\int_0^{iT-x}(x+h)f_H(h)dhf_X(x)dx \\ & + \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}\int_{(i-1)T}^{iT}\prod_{n=1}^{i-1}(1-\alpha(nT))\int_{jT-x}^{(j+1)T-x}\prod_{k=i}^{j}\beta\left(\frac{kT-x}{h}\right)(x+h)f_H(h)dhf_X(x)dx \\ & + \sum_{i=1}^{M-1}\sum_{j=i}^{M-1} jT\pi_{5,i,j} + \sum_{i=1}^{M} MT\pi_{6,i}. \end{aligned} \tag{5}$$

The expected number of failures in a renewal cycle $F(M,T)$ corresponds to the probability of corrective replacement in a renewal cycle. By summing over all possible values of i and j for event paths of types 3 and 4, we find

$$F(M,T) = \sum_{i=1}^{M}\pi_{3,i} + \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}\pi_{4,i,j}.$$

3.2 Optimization procedure

For optimizing our problem, cf. Equation (1), we enumerate (M, T) combinations. We restrict ourselves to considering $M \in \{1, \ldots, M_{max}\}$, with M_{max} corresponding to the enumeration upper bound. For our computational study, we assume $M_{max} = 40$. For each $M \in \{1, \ldots, M_{max}\}$, we enumerate the inspection interval lengths $T_M \in \{\delta, 2\delta, \ldots, k_{max}(E[X] + E[H])\}$ with a stepsize of $\delta = (E[X] + E[H])/50$. Here, $k_{max}(E[X] + E[H])$ denotes the upper bound of the considered inspection interval length T_M, with $k_{max} = 2$; and $E[\cdot]$ corresponds to the expected value of a random variable. We denote the cost minimizer that satisfies the reliability constraint by $\tilde{T}_M$. Next, we improve our result by doing a local search around $\tilde{T}_M$. That is, we define a new search space, bounded below by $\underline{T}_M = \tilde{T}_M - \delta$ and bounded above by $\overline{T}_M = \tilde{T}_M + \delta$. Furthermore, we refine our stepsize to $\Delta = (\overline{T}_M - \underline{T}_M)/50 = \delta/25$. We enumerate all possible (M, T) combinations in $(\underline{T}_M, \overline{T}_M)$, for $M \in \{1, \ldots, M_{max}\}$ with stepsize Δ. From this local search, we select the cost minimizer that satisfies the reliability constraint, and denote this by ${T_M}'$. It may occur that the reliability constraint is not binding at ${T_M}'$, while it is violated at ${T_M}' + \Delta$. In this case, we do a bisection search from ${T_M}'$ to ${T_M}' + \Delta$ to find a policy that is binding. Subsequently, we compare the costs of the policy with a non-binding reliability constraint, to the policy with a binding reliability constraint, and we denote the policy with minimum costs by (M, T^*_M) for the true model, and $(M, \widehat{T}_M)$ for the approximate model, $M \in \{1, \ldots, M_{max}\}$. As a final step, we select the maintenance policy from all (M, T^*_M) and $(M, \widehat{T}_M)$, that minimizes the costs and satisfies the reliability. We denote these optimal policies by (M^*, T^*) and $(\widehat{M}, \widehat{T})$, respectively.

4. Comparing the true and approximate model

In this section we present a method for comparing the true and approximate model. Both models are compared, under their optimal policies, such that the average probabilities of false positives and false negatives are equal.

The average probabilities are determined from the true model, since these averages depend on the policy chosen. Of all inspections, under policy (M, T), that are executed when the component is in the normal state, $\mu_\alpha(M, T)$ denotes the fraction at which a false positive occurs. That is, $\mu_\alpha(M, T)$ is equal to the average number of false positives $z_\alpha(M, T)$ per time unit, divided by the total number of executed inspections per time unit $t_\alpha(M, T)$ when the system is in its normal state. Note that $\mu_\alpha(M, T)$ is defined over an infinite time horizon. $\mu_\beta(M, T)$ corresponds to the fraction at which false negatives are generated, under policy (M, T), based on all inspections that are done when the system is in its defective state. That is, $\mu_\beta(M, T)$ equals the average number of false negatives per time unit $z_\beta(M, T)$, divided by the total number of inspections done per time unit $t_\beta(M, T)$ when the system is defective.

The optimization model for the true model will result in an optimal policy (M^*, T^*), and average probabilities equal $\mu_\alpha(M^*, T^*)$, $\mu_\beta(M^*, T^*)$. Under the approximate model, the probabilities of inspection errors are constant and remain unaffected by the maintenance policy. By setting the functions $\alpha(t) = \tilde{\alpha}$ and $\beta((t-x)/h) = \tilde{\beta}$, the true model (described and analyzed in Sections 2 and 3) reduces to the approximate model. The optimal policy for this approximate model is denoted by $(\widehat{M}, \widehat{T})$, and the average probabilities $\mu_\alpha(\widehat{M}, \widehat{T}) = \tilde{\alpha}$, $\mu_\beta(\widehat{M}, \widehat{T}) = \tilde{\beta}$. We set $\tilde{\alpha} = \mu_\alpha(M^*, T^*)$ and $\tilde{\beta} = \mu_\beta(M^*, T^*)$, because we want to compare the two models, under their optimal policies, such that the average probabilities of false positives and false negatives are equal. Our procedure for comparing the true and approximate model is illustrated in Figure 2.

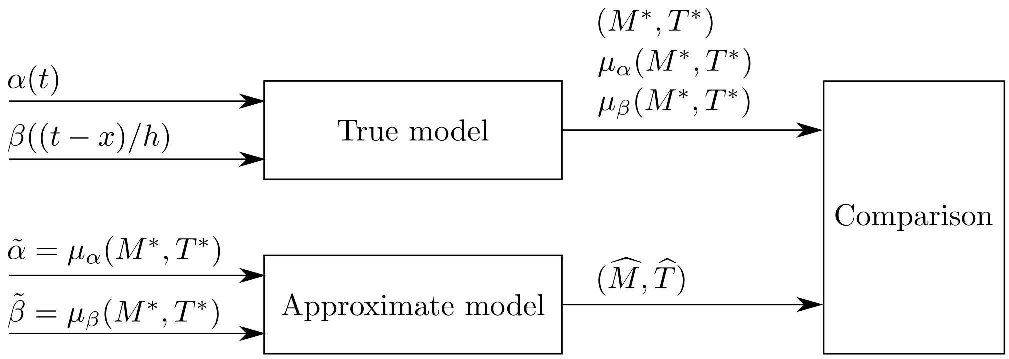

Figure 2: Comparing a non-constant to a constant probabilities model

We remark that no inspections are done when $M = 1$, since this corresponds to an age-based maintenance policy. Consequently, $\mu_\alpha(M, T)$ and $\mu_\beta(M, T)$ are not defined for $M = 1$. Hence, we assume that if $M^* = 1$, then $\widehat{M} = 1$ as well, and consequently $T^* = \widehat{T}$.

In Section 4.1, we define the expressions used to calculate the average probabilities of false positives and false negatives under an arbitrary (M, T) policy. These will be used to make the comparison between the true and approximate model. Section 4.2 presents the measures of comparison.

4.1 Average false positive and false negative probabilities

In this section, we derive expressions for $\mu_\alpha(M, T)$ and $\mu_\beta(M, T)$. We know that $\mu_\alpha(M, T) = z_\alpha(M, T)/t_\alpha(M, T)$. From renewal theory (Ross, 1983), it follows that $z_\alpha(M, T)$ and $t_\alpha(M, T)$ equal the average number of false positives and the expected total number of inspections done when the system is in its normal state, over one renewal cycle, respectively. That is, $z_\alpha(M, T) = Z_\alpha(M, T)/L(M, T)$, and $t_\alpha(M, T) = T_\alpha(M, T)/L(M, T)$, where $Z_\alpha(M, T)$ denotes the expected number of false positives in a renewal cycle, $T_\alpha(M, T)$ the expected total number of inspections in a renewal cycle when the system is in its normal state, and $L(M, T)$ represents the expected renewal cycle length. Rewriting the expression for $\mu_\alpha(M, T)$ yields

$$\mu_\alpha(M, T) = \frac{Z_\alpha(M, T)}{T_\alpha(M, T)}, \ \forall M \geq 2.$$

To derive expressions for $Z_\alpha(M, T)$ and $T_\alpha(M, T)$ we use the event path types from Section 3.1. The average number of false positives that occur in a renewal cycle $Z_\alpha(M, T)$ equals the sum of the probabilities of each possible *EventPath*$(2, j)$. Hence, $Z_\alpha(M, T) = \sum_{j=1}^{M-1} \pi_{2,j}$. The total expected number of inspections $T_\alpha(M,T)$ is derived by considering all event paths from Section 3.1. Multiplying the probability of an event path by the corresponding number of inspections (when the system is in its normal state), and summing over all possible i and j, we obtain

$$T_\alpha(M,T) = (M-1)\pi_1 + \sum_{j=1}^{M-1} j\pi_{2,j} + \sum_{i=1}^{M}(i-1)(\pi_{3,i}+\pi_{6,i}) + \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(i-1)(\pi_{4,i,j}+\pi_{5,i,j}).$$

Similar to $\mu_\alpha(M,T)$, we derive

$$\mu_\beta(M,T) = \frac{Z_\beta(M,T)}{T_\beta(M,T)},\ \forall M \geq 2,$$

with $Z_\beta(M,T)$ the average number of false negatives per renewal cycle, and $T_\beta(M,T)$ the expected total number of inspections when the system is defective over one renewal cycle.

To derive the expected number of false negatives in a renewal cycle, $Z_\beta(M,T)$, we use the event path types E4-E6 from Section 3.1. Each of these event paths occurs with probability $\pi_{4,i,j}$, $\pi_{5,i,j}$ and $\pi_{6,i}$ respectively. The expected number of false negatives in a renewal cycle is given by

$$Z_\beta(M,T) = \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(j-i+1)\pi_{4,i,j} + \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(j-i)\pi_{5,i,j} + \sum_{i=1}^{M-1}(M-i)\pi_{6,i}.$$

Similarly, we derive the expression for the expected total number of inspections when the system is defective:

$$T_\beta(M,T) = \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}(j-i+1)(\pi_{4,i,j}+\pi_{5,i,j}) + \sum_{i=1}^{M-1}(M-i)\pi_{6,i} = Z_\beta(M,T) + \sum_{i=1}^{M-1}\sum_{j=i}^{M-1}\pi_{5,i,j}.$$

4.2 Comparison

We compare the true and approximate model by focusing on the cost and reliability difference between both models. That is, we evaluate the cost and reliability of $(\widehat{M},\widehat{T})$, the optimal policy of the approximate model, and compare this to the cost and reliability of (M^*,T^*), the optimal policy of the true model. The cost and reliability functions under non-constant probabilities are presented in Section 3.1, and denoted by $g(M,T)$ and $R(M,T)$, respectively.

To compare the reliability between both models, we study the relative reliability difference

$$\Delta R = \frac{R(\widehat{M},\widehat{T}) - R(M^*,T^*)}{R(M^*,T^*)}.$$

Besides the relative reliability difference ΔR, we also study the number of infeasible solutions generated by the approximate model. That is, the number of instances in which $R(\widehat{M},\widehat{T}) > R_{max}$.

Finally, we are also interested in the relative cost difference between both optimal policies, (M^*,T^*) and $(\widehat{M},\widehat{T})$. We denote this relative cost difference by

$$\Delta g = \frac{g(\widehat{M},\widehat{T}) - g(M^*,T^*)}{g(M^*,T^*)}.$$

5. Computational study

In this section we present our computational study, in which we focus on five main factors. We define a base instance, and then vary each factor to a high and low level. This implies that we create a testbed consisting of $2 \times 5 + 1 = 11$ instances. We compute the optimal policies for the true and approximate model, the relative cost differences, and the relative reliability differences, (M^*, T^*), $(\widehat{M}, \widehat{T})$, Δg, and ΔR respectively.

5.1 Testbed

We relate the false positive probability to the time t that the system has been operating in its normal state, counted from the last renewal point, see Section 2. We assume that the probability of a false positive is related to t in a piecewise manner, for simplicity. That is, we assume that $\alpha(t)$ increases to a threshold a, and from this point stays constant; see Figure 1. Then,

$$\alpha(t) = \begin{cases} \alpha_0 + (u_\alpha t)/a & \text{if } t \leq a \\ \alpha_0 + u_\alpha & \text{otherwise} \end{cases},$$

where α_0 represents the false positive probability that does not depend on the time t. For the probability of false negatives, we follow the literature on the Probability of Detection (POD). Since β=1-POD, and the relationship between the POD and system degradation is best modeled by a log odds distribution (Berens, 1989; Georgiou, 2006), we relate the failure progress to the probability of false negatives in log odds terms as well. Hence,

$$\beta\left(\frac{t-x}{h}\right) = \beta_0 + \frac{1-\beta_0}{1+e^{\gamma+\eta \ln\left(\frac{t-x}{h}\right)}}, \qquad \text{if } x \leq t \leq x+h,$$

where $\eta \geq 0$ and γ are shape parameters, and β_0 corresponds to the false negative probability that does not depend on the failure progress. For further details on the log-odds distribution, see Georgiou (2006). Figure 3 illustrates the proposed functions for the probability of false positives related to t, and for the probability of false negatives related to the failure progress $(t-x)/h$, under various parameter settings.

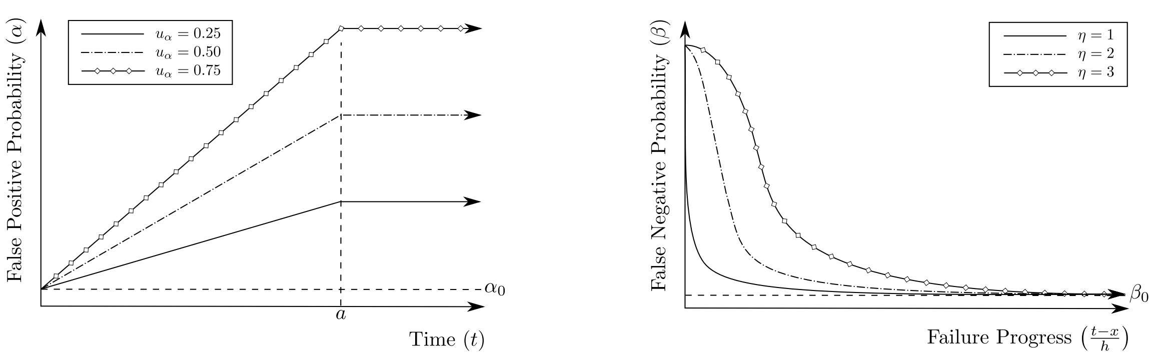

Figure 3: Functions $\alpha(t)$ and $\beta((t-x)/h)$ for various parameter settings

We explore the effects of five factors: the reliability objective R_{max}, the inspection cost parameter c_0, the bandwidth of the non-constant probability of false positives (and defining the slope) u_α, the shape parameter η for the probability of false negatives, and the coefficient of variation of the delay time CV_H. Note that a decrease in γ yields a similar result as an increase in η, as the $\ln(\cdot)$ term is negative. We use Weibull distributions in our computational study for the time to defect and the delay time. We find the shape and scale parameters of the Weibull distributions by first searching for the shape parameter that yields the corresponding coefficient of variation. We start by restricting our focus to the coefficient of variation, since this is solely determined by the shape parameter. Subsequently, we determine the scale parameter to obtain the given expected time to defect $E[X]$ and delay time $E[H]$. All factor choices are presented in Table 3, and the values of our base instance and all other model parameters is listed in Table 4.

R_{max}	c_0	u_α	η	CV_H
1.00×10^{-4}; 1.00×10^{-6}; 1.00×10^{-8}	50; 100; 200	0.25; 0.50; 0.75	1.0; 2.0; 3.0	0.25; 0.50; 0.75

Table 3: Factor choices

R_{max}	$E[X]$	$E[H]$	CV_H	CV_X	c_0	c_p	c_c	α_0	u_α	a	β_0	η	γ
1.00×10^{-6}	900	100	0.50	0.50	100	1,000	2,000	0.05	0.50	900	0.05	2.0	5

Table 4: Values of factors and parameters of the base instance

5.2 Results and managerial insights

Our numerical results are presented in Table 5. We will first study the results of the true model (columns 2-6), and secondly we will discuss the results of the approximate model (columns 9-13). Finally, we elaborate on the cost and reliability differences that we observe in columns 14 and 15 in Table 5.

All true model instances satisfy the reliability constraint. This constraint is binding for each of the true model instances, i.e., $R(M^*, T^*) = R_{max}$. When the reliability constraint becomes tighter, we observe that M^* increases; and T^*, M^*T^*, and the mean cycle length decrease. This implies that more inspections are planned in order to satisfy the constraint, and preventive replacement is done earlier. In case the inspections become more expensive or more prone to false positives (u_α increases), we see that M^* decreases, T^* increases, and M^*T^* and $L(M^*, T^*)$ decrease. This behavior indicates that inspections become less attractive when the inspection cost or the false positive probability (in terms of u_α) increases. If the false negative probability increases (in terms of η), we observe a decrease in M^*, T^*, M^*T^*, and in $L(M^*, T^*)$. This indicates that earlier preventive replacement and more frequent inspections are done in order to prevent the system from failing. This same behavior is observed when the variation in the delay time increases.

		(M^*,T^*)	$L(M^*,T^*)$	M^*T^*	$g(M^*,T^*)$	$R(M^*,T^*)$	μ_α	μ_β	$(\widehat{M},\widehat{T})$	$L(\widehat{M},\widehat{T})$	$\widehat{M}\widehat{T}$	$g(\widehat{M},\widehat{T})$	$R(\widehat{M},\widehat{T})$	$\Delta g(\%)$	$\Delta R(\%)$
R_{max}	1.00×10^{-4}	(3,132.93)	335.64	398.80	3.63	1.00×10^{-4}	0.16	0.21	(2,168.29)	290.95	336.57	3.88	1.06×10^{-4}	6.97	5.56
	1.00×10^{-6}	(9,16.60)	109.60	149.44	14.73	1.00×10^{-6}	0.09	0.43	(1,51.32)	51.32	51.32	19.49	1.00×10^{-6}	32.29	-0.01
	1.00×10^{-8}	(11,3.60)	29.95	39.60	59.39	1.00×10^{-8}	0.06	0.72	(1,12.06)	12.06	12.06	82.95	1.00×10^{-8}	39.67	0.00
c_0	50	(15,14.33)	121.72	214.93	11.63	1.00×10^{-6}	0.10	0.44	(4,17.68)	59.69	70.74	19.00	1.25×10^{-6}	63.43	25.17
	100	(9,16.60)	109.60	149.44	14.73	1.00×10^{-6}	0.09	0.43	(1,51.32)	51.32	51.32	19.49	1.00×10^{-6}	32.29	-0.01
	200	(3,26.13)	73.07	78.40	18.98	1.00×10^{-6}	0.07	0.45	(1,51.32)	51.32	51.32	19.49	1.00×10^{-6}	2.64	0.00
u_α	0.25	(10,15.86)	118.82	158.62	14.30	1.00×10^{-6}	0.07	0.44	(2,29.42)	56.54	58.84	19.46	1.12×10^{-6}	36.08	12.27
	0.50	(9,16.60)	109.60	149.44	14.73	1.00×10^{-6}	0.09	0.43	(1,51.32)	51.32	51.32	19.49	1.00×10^{-6}	32.29	-0.01
	0.75	(8,17.42)	101.87	139.38	15.12	1.00×10^{-6}	0.10	0.43	(1,51.32)	51.32	51.32	19.49	1.00×10^{-6}	28.86	0.00
η	1	(9,17.92)	117.13	161.26	13.74	1.00×10^{-6}	0.09	0.16	(8,16.48)	89.03	131.86	16.87	1.07×10^{-6}	22.77	6.67
	2	(9,16.60)	109.60	149.44	14.73	1.00×10^{-6}	0.09	0.43	(1,51.32)	51.32	51.32	19.49	1.00×10^{-6}	32.29	-0.01
	3	(8,15.84)	97.77	126.72	16.00	1.00×10^{-6}	0.08	0.63	(1,51.32)	51.32	51.32	19.49	1.00×10^{-6}	21.80	0.00
CV_H	0.25	(10,31.22)	194.52	312.22	8.22	1.00×10^{-6}	0.12	0.31	(2,50.84)	94.07	101.68	11.70	1.23×10^{-6}	42.33	22.60
	0.50	(9,16.60)	109.60	149.44	14.73	1.00×10^{-6}	0.09	0.43	(1,51.32)	51.32	51.32	19.49	1.00×10^{-6}	32.29	-0.01
	0.75	(6,10.70)	54.82	64.22	26.31	1.00×10^{-6}	0.07	0.54	(1,32.29)	32.29	32.29	30.97	1.00×10^{-6}	17.74	0.00

Table 5: Experiment results

Let us now consider the true model's cost effects of the factor perturbations. We observe that the costs $g(M^*,T^*)$ increase when the reliability constraint becomes tighter and when CV_H increases, since more frequent inspections are required. The costs also increase if the probabilities of false positives and false negatives increase.

From Table 5, we derive that the approximate model satisfies the reliability constraint in 6 out of the 11 different instances, but violates the constraint in the remaining 5. We see that constraint violation occurs when the approximate model plans inspections ($\widehat{M} > 1$). Four of the five instances, that violate the reliability constraint, have sufficiently low average probabilities of false positives μ_α and false negatives μ_β to make inspections attractive. The fifth instance ($c_0 = 50$) plans inspections because the inspection costs c_0 are low. Since inspections are planned, an approximation is used which induces a violation of the reliability constraint. For the optimal policies $(\widehat{M},\widehat{T})$, we observe that the approximate model prefers preventive replacement over inspections, when the reliability constraint becomes tighter and when CV_H increases. Furthermore, we see that when the probability of inspection errors increases, the approximate model also prefers preventive replacement. When we study the expected cycle lengths and costs of the approximate model, we find the same behavior as for the true model. That is, we observe similar results with respect to $L(\widehat{M},\widehat{T})$ and $g(\widehat{M},\widehat{T})$, compared to the results of the true model. Hence, we will not elaborate upon these implications.

Let us now compare the true model to the approximate model. We first focus on the reliability measure. We observe negligible reliability differences for instances with $\widehat{M} = 1$. When the approximate model plans no inspections, this model yields the same results as the true model. Hence, the approximate model satisfies the reliability constraint (in a binding way). When the approximate model plans inspections, $\widehat{M} > 1$, we observe reliability differences up to 25%. These differences are equivalent to a measure of constraint violation, as the true model's reliability $R(M^*,T^*)$ is binding. We now turn our attention to comparing both models regarding the relative cost difference. For a tightening R_{max}, we see that Δg increases, and that M^* increases, while $\widehat{M}$, T^*, and $\widehat{T}$ decrease. This means that, for a stricter reliability constraint, the true model plans more inspections, while the approximate model avoids inspections and does earlier preventive replacement. Hence, the relative cost difference Δg increases. When c_0 and u_α increase, we observe that M^* and $\widehat{M}$ decrease, T^* and $\widehat{T}$ increase, and Δg decreases. That is, when less inspections are scheduled, due to changes in the inspection costs and changes in u_α, the constant probability approximation is applied to fewer inspections per time unit, and thus Δg decreases. For false negatives, when η increases from 1 to 2, Table 5 shows that T^* and $\widehat{M}$ decrease. As a result, relatively many inspections are planned before preventive replacement for the true model compared to the approximate model, as the approximate model avoids inspections. Hence, Δg increases. However, we observe that, when η increases further from 2 to 3, $(\widehat{M}, \widehat{T})$ remains unaffected, and M^* decreases. Therefore, the true model relies more on preventive replacement, and thus becomes more similar to the outcome of the approximate model, which prefers preventive replacement. This reduces Δg. An increase in the coefficient of variation of the delay time CV_H induces lower T^*, M^*, and $\widehat{M}$. That is, a more variable delay time causes more frequent inspections for the true model, and preventive replacement is done after a lower number of inspections. Because preventive replacement occurs after fewer inspections, the true model's optimal policy relies less on inspections and more on preventive replacement, as does the approximate model. Consequently, Δg decreases.

Overall, the approximate model may result in infeasible maintenance policies that violate the reliability constraint by 25%. Additionally, the approximate model yields 26% higher costs, on average, compared to the results of the true model, which may increase to 63%, dependent on the factor choice. Such a relative cost difference Δg corresponds to large absolute cost differences, and might therefore substantially affect practical maintenance decision making.

6. Conclusion

In this paper, we considered a single-component system, which is periodically inspected and subject to imperfect inspections and a reliability constraint. We presented exact cost and reliability evaluations of a maintenance policy (M, T) for a single-component DTM including non-constant probabilities of false positives and false negatives. Furthermore, we proposed a method that allows us to compare our true model, with non-constant probabilities of imperfect inspections, to the approximate model, that considers constant probabilities of imperfect inspections. In our numerical study, we illustrate that applying a heuristic of constant probabilities of imperfect inspections can yield policies which are infeasible and violate the reliability constraint by 25%. Moreover, we find that the average cost of using the approximate model is 28% higher than the average cost obtained under the true model. The maximum cost difference that we found in our computational study was 63%.

The model described in this paper is computationally demanding for evaluation. The enumerative optimization procedure proposed in this research, can therefore become computationally prohibitive. Hence, for further research, we suggest to explore some structural properties in the

(M, T) solutions to cut off part of the search space. Additionally, it would be interesting to explore other functions for the probability of false positives and false negatives in further research.

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