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Beta Working Paper series 499

BETA publicatie	WP 499 (working paper)
ISBN	
ISSN	
NUR	804
Eindhoven	March 2016

A Condition-Based Maintenance Model for a Single Component in a System with Scheduled and Unscheduled Downs

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March 28, 2016

Abstract

We introduce a maintenance model for a single component that is part of a complex engineering system and has a monotic, stochastic degradation process. For this component, a hard failure level is given and we use a condition-based maintenance (CBM) policy. The other components in the system are assumed to follow failure-based and periodic maintenance policies and their maintenance actions lead to unscheduled and scheduled downs of the system. These downs constitute opportunities for the single CBM component. If the degradation is at or above a control limit at an unscheduled or scheduled down, the component is replaced preventively and one saves on downtime and setup costs. We derive an efficient and accurate approximate evaluation procedure. Further, we show that this approximate evaluation leads to a close-to-optimal control limit policy when used within the optimization procedure and we show the potential savings when both unscheduled and scheduled downs are used as opportunities for the CBM component. Finally, we demonstrate that our model can be well used as a building block to solve multi-component CBM problems with many components.

***Key words*:** Condition-based maintenance, complex systems, opportunistic maintenance, scheduled downs, unscheduled downs

1. Introduction

Nowadays, the development of advanced sensor and ICT technology makes the remote acquisition of condition monitoring data (e.g., temperature of an engine, wearing of a brake) less costly. Based on the condition of a component/system, one can improve the diagnostics and prognostics of failures in order to reduce the maintenance related costs (e.g., downtime costs, set-up costs), which is the main idea behind condition-based maintenance (CBM); see Jardine et al. [13] and Peng et al. [24]. Considerable attention from researchers has been attracted to study CBM for complex engineering systems. These occur in many industries (aviation, oil-gas refinery, energy, automotive, semiconductor industry, and so on). It is usually not feasible to implement

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CBM for all components in a complex engineering system. Instead, there may be only a limited number of critical components that is under continuous condition monitoring. For the rest of the components, corrective maintenance and periodic preventive maintenance policies may be in place (see also [1], Section 2.2.1). Hence, it is a challenging problem to coordinate the maintenance actions of the different policies for the various components in a complex system.

The existing studies about CBM for multi-component systems often focus on proposing models to coordinate the maintenance activities among the components under a CBM policy. Relatively speaking, little research about CBM has been done to coordinate the maintenance activities of a mixture of different maintenance policies.

In this paper, we introduce a CBM model for a single component that is part of a complex system. The component follows a monotonic, stochastic degradation process, which is monitored continuously and has a pre-set, hard failure level (in calculations, we assume a random coefficient model and a Gamma process for the degradation process). Maintenance for this component can be initiated at any moment, but if it is executed at a moment that one has already another maintenance action for another component, then one saves *downtime costs* of the complex system and *maintenance setup costs*. The setup costs occur because an engineer has to go to the system or because the system has to go to a maintenance location. The downtime costs occur because complex engineering systems are generally used in primary processes of their users and those processes are stopped/disturbed when the complex system is down. These downtime costs are very high in many cases. We assume that the other components are subject to failure-based policies or to periodic maintenance policies (inspections, cleaning, lubrication). This leads to *unscheduled downs* and *scheduled downs*, respectively, for the complex system. The unscheduled downs follow a Poisson process and the scheduled downs occur according to a fixed maintenance interval. We analyse a *control limit policy*: If the degradation process is at or above the control limit at an unscheduled or a scheduled down, then a preventive maintenance action is executed. We derive an approximate evaluation procedure and optimize the control limit. We also show how our single-component model can be used for a multi-component CBM model.

The related literature consists of single-component and multi-component CBM models. Within the stream of single-component CBM models, there is literature in which the control limit and/or inspection interval is optimized, based on the stochastic degradation processes estimated from the condition monitoring data; see the reviews of Wang [33], Jardine et al. [13], and Peng et al. [24]. Wang [33] proposed a CBM model based on the general random coefficient model (cf. Lu and Meeker [19]) to determine the optimal control limit and the monitored interval in terms of cost, downtime and reliability. Gebraeel et al. [9, 10] extended the general degradation model to estimate the RUL distribution from sensor signals by a Wiener process and Bayesian updating. Using this technique, a single-unit replacement problem is formulated as a Markov decision process to develop a structured replacement policy in Elwany et al. [8]. There are also multiple models where a Gamma process is used for the degradation process. Dieulle et al. [7] and Park [22, 23] developed such models with a single-level control limit and Grall et al. [11] developed such a model with a multi-level control limit. Here, we distinguish scenarios with periodic inspection ([22]), aperiodic inspection ([7, 11]) and continuous monitoring ([17, 23]). The degradation process is also often modelled as a Markov process with discrete states. Optimal replacement policies were derived for observable Markov processes by Makis and Jiang [20] and Kharoufeh et al. [14] and from the evolution of the hidden states by Bunks et al. [4] and Lin and Makis [18]. Further, proportional hazards models are often used to relate the system's condition variables to the hazard function of a system, so that the maintenance policies can be optimized with respect to the optimal risk value of the hazard function; see Jardine et al. [12] and Vlok et al. [32]. Another type of model is the filtering model; see e.g. Wang et al. [34], who apply such a model to marine diesel engines. Motivated by the maintenance services practice at an OEM in the compressed air and generator industry, a recent contribution has been made by Poppe et al. [25]. They consider the same model as our model and look at the effect of using a second threshold level just below the hard failure level that initiates a maintenance action with a separate setup. This maintenance action is executed after a short preparation period. They show that the average cost can be reduced significantly if this extra type of maintenance action is much cheaper than the normal corrective maintenance action. Their calculations are based on a different approximate evaluation procedure and a different degradation process.

The stream of multi-component CBM models consists of only a limited number of papers. Bouvard et al. [3] converted a condition-based maintenance problem into a similar age-based maintenance clustering problem (cf. [36]), which yielded an optimal schedule with a dynamic maintenance interval. Wijnmalen and Hontelez [35] used a heuristic algorithm for computing control limits for components in systems under different discounted scenarios, which is formulated within a Markov decision framework. Castanier *et al.* [5] introduced a model to coordinate inspection/replacement of a two-component system via a Markov renewal process and minimize the long-run maintenance cost. However, this model becomes intractable when it would be extended to systems with many components. For systems with many components subject to *soft failures*, Zhu et al. [38] proposed a model with a control limit policy per component and a the joint maintenance interval of the system (in this system all maintenance actions are executed at the scheduled downs). Moreover, for larger scale problems, there is research based on Monte Carlo simulation and genetic algorithms; see Marseguerra et al. [21] and Barata et al. [2]. Alternatively, Tian et al. [28] proposed two maintenance policies for multi-component systems using the proportional hazard model, and Tian and Liao [29] propose the use of an artificial neural network. To compare age/time-based and condition-based maintenance policies, Koochaki et al. [15] evaluated the cost effectiveness of a three-component series system in the context of opportunistic maintenance via simulation. In their model, only unscheduled opportunities are considered, while our model includes both scheduled and unscheduled opportunities. De Jonge et al. [6] consider policies for a system with identical components for which the degradation is modelled by a so-called P-F curve.

The contribution of this paper is as follows. First, we introduce a new single-component CBM model with both scheduled and unscheduled opportunities for executing a maintenance action without separate downtime and setup costs. Second, we derive an efficient and accurate approximate evaluation procedure for a given control policy. Third, we show that the approximate evaluation can be well used to optimize the control limit, and we show the savings when both unscheduled and scheduled downs are used as opportunities by the CBM component. Fourth, we demonstrate that our model can be well used as a building block for multi-component CBM problems with many components. Hence, we also contribute to the literature in multi-component CBM models. For the use of our model as a building block for a multi-component system with a mix of condition-based, age-based, and failure-based components, we refer to [37].

The outline of this paper is as follows. The description of the system and the assumptions are given in Section 2. The approximate evaluation procedure is described in Section 3. In Section 4, we describe a case for lithography machines used within the semiconductor industry, and we apply our evaluation and optimization procedures. In Section 5, numerical experiments are performed to investigate the accuracy of our approximate evaluation and the optimization based on the approximate evaluation. Further, we show the potential savings that can be obtained by using both unscheduled and scheduled downs as opportunities for relatively cheap maintenance of the CBM component. Next, in Section 6, we demonstrate how our model can be used as a building block for large-scale, multi-component CBM problems. Finally, the conclusions are given in Section 7.

2. System Description

Consider a complex engineering system consisting of multiple components. One critical component is monitored continuously and maintained according to a condition-based maintenance policy. We call such a component a "CBM component". The degradation state of the CBM component $X(t)$ can be monitored continuously over time t, $t \in [0, \infty)$. We assume that the degradation process $X(t)$ is monotonic. When the degradation state $X(t)$ exceeds a predetermined warning limit H, the system operates under an unsatisfied condition. Hence, a maintenance action will be triggered immediately to restore the degradation level of the CBM component to its initial level. Such a system down due to the maintenance of the CBM component is called "CBMD" (a down due to the CBM component, see Figure 1). In this model, the warning limit H is a given parameter from technical experts, who have the knowledge on the physics of failures.

Apart from this CBM component, all other components in the system are subject to either a failure-based or a periodic maintenance policy:

- *Failure-based maintenance policy*: For the components that are under a failure-based maintenance policy, the maintenance or replacement will be conducted immediately after the failure of the component. This will lead to *unscheduled downs* (USDs) of the system (see Figure 1). We assume that the interarrival times of the failures follow an exponential distribution with rate λ, so that the corrective maintenance actions cause USDs according to a homogeneous Poisson process. According to the Palm-Khintchine theorem (see e.g. [26]), even if the failure times of some components do not follow exponential distributions, the combination of a large amount of non-Poisson renewal processes will still have Poisson properties. Hence, this assumption about corrective maintenance is realistic if a failure-based maintenance policy is used for a sufficiently large amount of components in the system.

- *Periodic maintenance policy*: In many industries (aviation, oil-gas refinery, energy, automotive, semiconductor industry, and so on), periodic maintenance actions (inspection, cleaning, lubrication) for the system are taken every fixed interval (see e.g. [30]. This is a common practice because it facilitates the planning and coordination of maintenance resources (service engineers, maintenance equipments, spare parts) and users of complex engineering systems can take them into account in their production schedules. Let τ be the length of this fixed interval. When determining the fixed interval length, one can also take other factors than periodic maintenance into account, such as industrial regulations (e.g., a requirement that each system has to have an annual inspections like for cars in many countries) or commercial aspects (maintenance may be outsourced and the service company may want to show its presence sufficiently often). The periodic maintenance is planned and hence leads to *scheduled downs* (SDs) of the system (see Figure 1).

When a system down occurs (USD, SD, or CBMD), the system operation will be interrupted and it will cause high downtime costs for the system. Also, setup costs for maintenance will be incurred, either for sending a maintenance crew to the field or for bringing the system to a maintenance location. Downtime and setup costs for the multi-component system can be reduced by combining maintenance actions. In our model, we use the system downs caused by failure-based maintenance (at USDs) and periodic maintenance

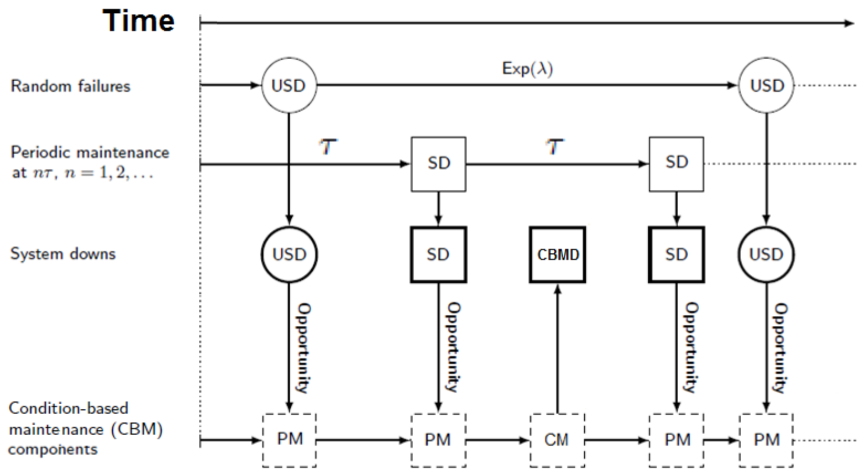


Figure 1: The maintenance policy of one CBM component

(at SDs) for other components as opportunities for the CBM component that we consider. We can do that by executing preventive maintenance before $X(t)$ reaches the warning limit H (see Figure 2). Obviously, we then loose a fraction of the useful lifetime of the CBM component.

In this paper, we distinguish three types of maintenance actions for the CBM component:

1. Corrective Maintenance at a CBMD (CM): When the system stops due to a CBMD, namely, at a time point where $X(t)$ crosses H (see Figure 2), a corrective maintenance (CM) action is taken with a cost c^{CM}, which includes the downtime and setup costs and the costs for replacing the failed CBM component by a spare part (which may be a new or a ready-for-use component).

2. Preventive Maintenance at an USD (PM-USD): When the system has an USD at a time instant t, it provides an opportunity for the CBM component to be maintained together with the component that failed and caused the USD. If the degradation $X(t)$ exceeds a control limit C ($X(t) \geq C$, see Figure 2), a preventive maintenance (PM) action will be taken with a cost c^{PM-USD}. This cost factor consists of the costs for replacing the CBM component by a spare part. In this case, no downtime and setup costs have to included. Hence, we assume that $c^{PM-USD} < c^{CM}$ (and generally c^{PM-USD} will be much smaller than c^{CM}).

3. Preventive Maintenance at a SD (PM-SD): When the system has an SD at time $n\tau$, $n \in \mathbb{N}$, it provides an opportunity for the CBM component to be maintained together with the components for which periodic maintenance takes place at this SD. If the degradation $X(t)$ exceeds the control limit C ($X(t) \geq C$, see Figure 2), a preventive maintenance (PM) action will be taken with a cost c^{PM-SD}. As for the previous action, this cost factor consists of the costs for replacing the CBM component by a spare part, and no downtime and setup costs have to be included. Hence, we assume that $c^{PM-SD} < c^{CM}$. Further, we assume that the cost factors c^{PM-SD} and c^{PM-USD} are almost equal. Under that assumption, it is reasonable to use the same control limit C. (In practice, it is possible that c^{PM-SD} is somewhat smaller than c^{PM-USD} because a spare part for a replacement at a SD may be provided from a central warehouse instead of a local warehouse and thus one avoid the charged costs for keeping spare parts available in local warehouses at close distance of installed systems.)

The interval τ for periodic maintenance is in terms of weeks or months and is small in comparison to the

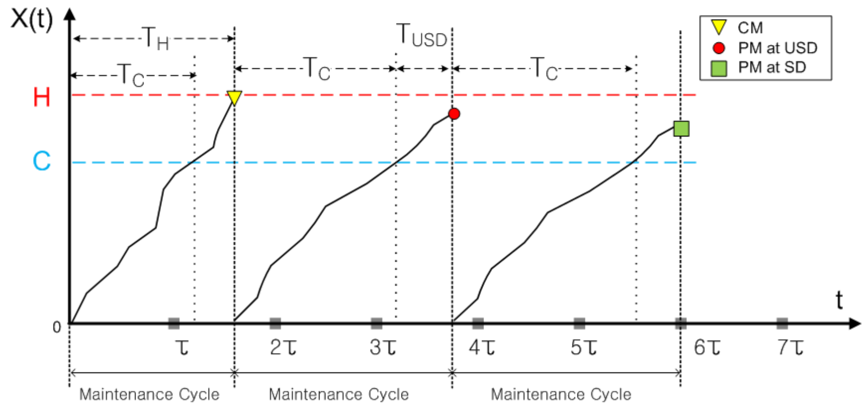


Figure 2: The degradation of the CBM component with three types of maintenance actions

long life cycles (10-40 years) of complex engineering systems. Hence, an infinite time horizon is assumed. Moreover, we assume that the CBM component is restored as good as new by any maintenance action (CM, PM-USD, or PM-SD), as shown in Figure 2.

The average costs per time unit under a given control limit are denoted by $Z(C)$. The objective is to minimize $Z(C)$ over all possible control limits $0 < C \leq H$.

An exact evaluation of the average costs $Z(C)$ can be done via simulation, and the optimization can be based on this evaluation by simulation. Because we have no proof that $Z(C)$ has a unique local minimum, we use enumeration for the optimization. However, simulation requires relatively large computation times. Hence, we develop an approximate evaluation procedure and that procedure can also be used within the optimization procedure.

3. Approximate Evaluation

The approximate evaluation starts with the definition of so-called *maintenance cycles*. We define a maintenance cycle as the time interval between two consecutive maintenance actions for the CBM component. We may distinguish two types of maintenance cycles. When the previous maintenance cycle ended with a PM-SD action, the next maintenance cycle starts at a SD. When the previous maintenance cycle ended with a CM or PM-USD action, the next maintenance cycle does *not* start at a SD. Hence, the time points at the beginning of the maintenance cycles do *not* constitute renewal points. Nevertheless, we pretend that these points are renewal points and we pretend that all cycles start at a SD. That are the only two approximate steps that we make. We denote the expected costs and the expected length per maintenance cycle by $K(C)$ and $L(C)$, respectively. By the renewal reward theorem, the average costs per time unit $Z(C)$ are equal to

$$Z(C) = \frac{K(C)}{L(C)}. \tag{1}$$

Below, we derive formulas for $K(C)$ and $L(C)$. They both follow from the analysis of a single maintenance cycle.

For each maintenance cycle, we denote the time since the start of the cycle by $\hat{t}$, and we pretend that the start of the cycle coincides with a SD; see also Figure 3.

Let $X(\hat{t})$ denote the degradation of the CBM component at time $\hat{t} \in [0, \infty)$ in one given maintenance

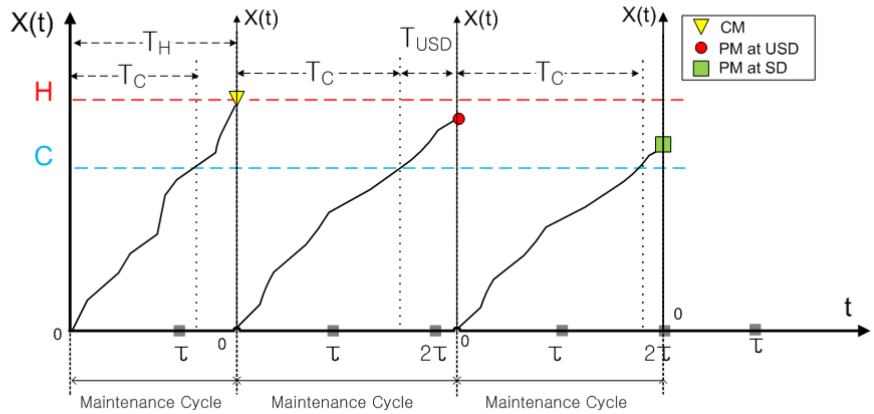


Figure 3: The degradation of the CBM component with three maintenance actions in renewal theory

cycle. Notice that the degradation process can be described by many different kinds of stochastic processes, e.g., a random coefficient model, a Gamma process, or a monotonic Markov Process. Because the degradation process is assumed to be monotonic, the probability that the degradation at time $\hat{t}$ exceeds a threshold χ is equal to the probability that the passage time T_χ of the threshold χ is less than time $\hat{t}$:

$$Pr\{T_\chi \leq \hat{t}\} = Pr\{X(\hat{t}) \geq \chi\}. \tag{2}$$

Let $F_{T_\chi}(\hat{t})$ and $f_{T_\chi}(\hat{t})$ be the cumulative density function (c.d.f.) and the probability density function (p.d.f.) of the passage time T_χ. Hence, the c.d.f. and p.d.f. of the passage time T_C and T_H can also be derived based on the degradation process $X(\hat{t})$. Because $X(\hat{t})$ will first cross the control limit C before reaching H, it holds that $T_C < T_H$ in each maintenance cycle. The CBM component will be replaced preventively if there are opportunities between T_C and T_H, and the first opportunity in the interval $[T_C, T_H)$ will be used. If no opportunity appears between T_C and T_H, the CBM component will fail at time T_H and then a CM action is executed.

To describe the possible events in a maintenance cycle, we distinguish two scenarios:

- ***Scenario 1:*** $(n-1)\tau \leq T_C < n\tau$ ***for some*** $n \in \mathbb{N}$ ***and*** $T_H < n\tau$
  In this case, no SD occurs between T_C and T_H. The first USD after T_C will appear at $T_C + T_{USD}$, where T_{USD} has an exponential distribution with rate λ. If $T_C + T_{USD} < T_H$, then the maintenance cycle will end with a PM-USD action at time $T_C + T_{USD}$. This happens with probability

$$\int_{(n-1)\tau}^{n\tau} \int_u^{n\tau} (1 - e^{-\lambda(v-u)}) f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du,$$

  where $f_{T_H|T_C}(v|u)$ is the p.d.f. of T_H given that $T_C = u$. If $T_C + T_{USD} \geq T_H$, then the maintenance cycle will end with a CM action at time T_H. This happens with probability

$$\int_{(n-1)\tau}^{n\tau} \int_u^{n\tau} e^{-\lambda(v-u)} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du.$$

- ***Scenario 2:*** $(n-1)\tau \leq T_C < n\tau$ ***for some*** $n \in \mathbb{N}$ ***and*** $T_H \geq n\tau$
  In this case, a SD occurs before T_H, and thus the maintenance cycle ends either with a PM-USD action or a PM-SD action. As in Scenario 1, The first USD after T_C appears at $T_C + T_{USD}$, where T_{USD} has

an exponential distribution with rate λ. If $T_C + T_{USD} < n\tau$, then the maintenance cycle will end with a PM-USD action at time $T_C + T_{USD}$. This happens with probability

$$\int_{(n-1)\tau}^{n\tau} \int_{n\tau}^{\infty} (1 - e^{-\lambda(n\tau - u)}) f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du.$$

If $T_C + T_{USD} \geq n\tau$, a PM-SD action will be taken at time $n\tau$. This happens with a probability

$$\int_{(n-1)\tau}^{n\tau} \int_{n\tau}^{\infty} e^{-\lambda(n\tau - u)} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du.$$

Define P_1, P_2, and P_3 as the probabilities that a maintenance cycle ends with a PM-USD action, a PM-SD action, and a CM action, respectively. From the descriptions of the two scenarions, it follows that:

$$\begin{aligned} P_1 &= \sum_{n=1}^{\infty} \Big\{ \int_{(n-1)\tau}^{n\tau} \int_{u}^{n\tau} (1 - e^{-\lambda(v-u)}) f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \\ &\qquad + \int_{(n-1)\tau}^{n\tau} \int_{n\tau}^{\infty} (1 - e^{-\lambda(n\tau-u)}) f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \Big\}, \qquad (3) \\ P_2 &= \sum_{n=1}^{\infty} \int_{(n-1)\tau}^{n\tau} \int_{n\tau}^{\infty} e^{-\lambda(n\tau-u)} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du, \qquad (4) \\ P_3 &= \sum_{n=1}^{\infty} \int_{(n-1)\tau}^{n\tau} \int_{u}^{n\tau} e^{-\lambda(v-u)} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du. \qquad (5) \end{aligned}$$

This leads to the following formula for the expected costs $K(C)$ per maintenance cycle:

$$K(C) = P_1\ c^{PM-USD} + P_2\ c^{PM-SD} + P_3\ c^{CM}. \qquad (6)$$

From the above scenarios, we find the following formula for the expected length $L(C)$ of a maintenance cycle:

$$\begin{aligned} L(C) &= \sum_{n=1}^{\infty} \Big\{ \int_{(n-1)\tau}^{n\tau} \int_{u}^{n\tau} \int_{0}^{v-u} (u+s)\lambda e^{-\lambda s}\ ds\ f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \\ &\qquad + \int_{(n-1)\tau}^{n\tau} \int_{u}^{n\tau} v e^{-\lambda(v-u)} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \\ &\qquad + \int_{(n-1)\tau}^{n\tau} \int_{n\tau}^{\infty} \int_{0}^{n\tau-u} (u+s)\lambda e^{-\lambda s}\ ds\ f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \\ &\qquad + \int_{(n-1)\tau}^{n\tau} \int_{n\tau}^{\infty} n\tau e^{-\lambda(n\tau-u)} f_{T_H|T_C}(v|u)\ dv\ f_{T_C}(u)\ du \Big\} \\ &= \sum_{n=1}^{\infty} \int_{(n-1)\tau}^{n\tau} \Big\{ \int_{u}^{n\tau} \left(\int_{0}^{v-u} (u+s)\lambda e^{-\lambda s}\, ds + v e^{-\lambda(v-u)} \right) f_{T_H|T_C}(v|u)\, dv \\ &\qquad + \int_{n\tau}^{\infty} \left(\int_{0}^{n\tau-u} (u+s)\lambda e^{-\lambda s}\, ds + n\tau e^{-\lambda(n\tau-u)} \right) f_{T_H|T_C}(v|u)\, dv \Big\} f_{T_C}(u)\, du. \end{aligned}$$

Using integration by parts, we obtain

$$\begin{aligned} \int_{0}^{v-u} (u+s)\lambda e^{-\lambda s}\, ds + v e^{-\lambda(v-u)} &= u + \frac{1}{\lambda}\left(1 - e^{-\lambda(v-u)}\right), \\ \int_{0}^{n\tau-u} (u+s)\lambda e^{-\lambda s}\, ds + n\tau e^{-\lambda(n\tau-u)} &= u + \frac{1}{\lambda}\left(1 - e^{-\lambda(n\tau-u)}\right). \end{aligned}$$

Hence, the formula $L(C)$ can be rewritten as

$$\begin{aligned} L(C) &= \sum_{n=1}^{\infty} \int_{(n-1)\tau}^{n\tau} \Big\{ \int_{u}^{n\tau} \left(u + \frac{1}{\lambda}\left(1 - e^{-\lambda(v-u)}\right)\right) f_{T_H|T_C}(v|u)\, dv \\ &\qquad + \int_{n\tau}^{\infty} \left(u + \frac{1}{\lambda}\left(1 - e^{-\lambda(n\tau-u)}\right)\right) f_{T_H|T_C}(v|u)\, dv \Big\} f_{T_C}(u)\, du \\ &= \mathbb{E}\{T_C\} + \sum_{n=1}^{\infty} \int_{(n-1)\tau}^{n\tau} \Big\{ \int_{u}^{n\tau} \frac{1}{\lambda}\left(1 - e^{-\lambda(v-u)}\right) f_{T_H|T_C}(v|u)\, dv \\ &\qquad + \frac{1}{\lambda}\left(1 - e^{-\lambda(n\tau-u)}\right) \int_{n\tau}^{\infty} f_{T_H|T_C}(v|u)\, dv \Big\} f_{T_C}(u)\, du. \end{aligned} \tag{7}$$

The formulas (3)-(5) for P_1, P_2, P_3 and formula (7) for $L(C)$ can be evaluated numerically once explicit expressions are given for f_{T_C} and $f_{T_H|T_C}(v|u)$; and $K(C)$ and $Z(C)$ are obtained by (6) and (1). Such explicit expressions are obtained when a specific degradation process is assumed; see Section 4. The integrals in formulas (3)-(5) and (7) can be evaluated by numerical integration, and the infinite sums in these formulas require an appropriate truncation (notice that the probability $Pr\{(n-1)\tau \le T_C < n\tau\} \to 0$ for $n \to \infty$ and similarly for the n-th term in each of these infinite sums; this can be used for the truncation of these infinite sums).

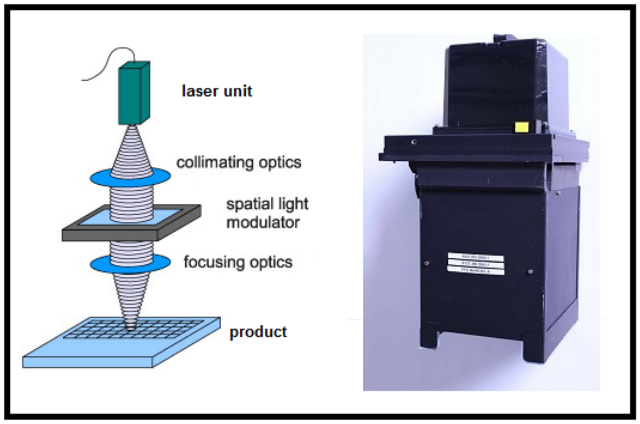
4. Case Study

As a demonstration of our model, we provide a case of lithography machines used in the semiconductor industry. What we describe below is based on a study that we did at a manufacturer of lithography machines; see [30]. Because of their confidentiality, we are not allowed to give the real-life data for cost factors and parameters of degradation processes. Instead, we give modified data, which are still somewhat representative. More importantly, the description below shows how our model could be used in practice.

The lithography machines are complex engineering systems processing the pure-silicon-made wafers to semiconductor integrated circuits, also known as micro-chips. The laser unit in the machine is considered as one of the most important components, whose degradation is continuously monitored. The measurement of its physical condition is the output power in Watts. When the degradation of output power exceeds a certain limit, bad chips are produced and a maintenance action is needed. Considering the laser unit as the CBM component, the degradation of output power over time is obtained from the historical data of multiple laser units.

As mentioned in the literature review in Section 1, there are several approaches to model the stochastic degradation paths of a component. In this case, we model the degradation process $X(\hat{t})$ per maintenance cycle by the following two approaches: (i) the *Random coefficient model* (cf. [19]), because it is relatively flexible and convenient for describing the degradation paths derived from the physics of failures; (ii) the *Gamma process* (cf. [31]), which is a process that is often used in the literature.

Fitting Option 1 - Random coefficient model: $X(\hat{t}; \Phi, \Theta)$ is a random variable with a given set of constant parameters $\Phi = \{\phi_1, ..., \phi_Q\}, Q \in \mathbb{N}_0$, and a set of random parameters, $\Theta = \{\theta_1, ..., \theta_V\}, V \in \mathbb{N}_0$, with given probability distributions. For example, based on the physics of failures, it may be known that the degradation behavior is described by a simple polynomial function: $X(\hat{t}; \Phi, \Theta) = \phi_1 + \theta_1 \hat{t}^{\phi_2}$ (so, $\Phi = \{\phi_1, \phi_2\}$ and $\Theta = \{\theta_1\}$). For the laser unit, the behavior is well described by the following linear function: $X(\hat{t}) = \theta_1 \hat{t}$,



Laser unit

Lithography machine

Figure 4: A laser unit in a lithography machine

where θ_1 is a random variable. Under this modelling, the speed of degradation is constant over time for each laser unit, but this speed varies for the various laser units in installed systems. Then equation (2) can be written in terms of F_{θ_1} (the c.d.f. of $\theta_1 \geq 0$):

$$Pr\{T_\chi \leq \hat{t}\} = Pr\{\theta_1 \hat{t} \geq \chi\} = Pr\{\theta_1 \geq \frac{\chi}{\hat{t}}\} = 1 - F_{\theta_1}\left(\frac{\chi}{\hat{t}}\right). \tag{8}$$

The p.d.f. of the passage time T_χ is equal to $(\chi/\hat{t}^2) f_{\theta_1}(\chi/\hat{t})$, where f_{θ_1} is the p.d.f. of θ_1. This expression can be further rewritten when a distribution is given for θ_1. For the laser unit, it is reasonable to assume that θ_1 follows a Weibull distribution with shape parameter β and scale parameter α. Then the p.d.f. of the passage time T_χ can be written as

$$\begin{aligned} f_{T_\chi}(\hat{t}) &= \frac{\chi}{\hat{t}^2} f_{\theta_1}(\chi/\hat{t}) = \frac{\chi}{\hat{t}^2}\frac{\beta}{\alpha}\left(\frac{\chi}{\alpha\hat{t}}\right)^{\beta-1} \exp\{-\left(\frac{\chi}{\alpha\hat{t}}\right)^{\beta}\} \\ &= \frac{\beta\alpha}{\chi}\left(\frac{\chi}{\alpha\hat{t}}\right)^{\beta+1} \exp\{-\left(\frac{\chi}{\alpha\hat{t}}\right)^{\beta}\}, \quad \hat{t} > 0. \end{aligned} \tag{9}$$

Taking $\chi = C$ in equation (9) gives the formula for the p.d.f. of T_C, f_{T_C}, which is needed in the formulas

Table 1: Input parameters (because of confidentiality, these input parameters are modified data and not the real-life data)

Parameter	Explanation
$c^{PM-SD} = 26.5$	Costs of preventive maintenance at a scheduled down [thousand Euro]
$c^{PM-USD} = 28.8$	Costs of preventive maintenance at an unscheduled downs [thousand Euro]
$c^{CM} = 44.5$	Costs of corrective maintenance [thousand Euro]
$\tau = 91$	Fixed time interval for the scheduled downs [day]
$\lambda = 8.86 * 10^{-3}$	Poisson arrival rate of unscheduled downs [per day]
$H = 88$	Failure threshold [Watt]
$\beta = 3.73$	Shape parameter of the Weibull distribution for θ_1 under Fitting Option 1
$\alpha = 0.159$	Scale parameter of the Weibull distribution for θ_1 under Fitting Option 1
$\gamma = 0.221$	Shape parameter of the Gamma process under Fitting Option 2
$\eta = 1.85$	Scale parameter of the Gamma process under Fitting Option 2

(3)-(5) and (7) to compute P_1, P_2, P_3 and $L(C)$. Because of the RCM that we assumed, the value of T_H is directly coupled to the value of T_C. If $T_C = u$, then $T_H = (H/C) \cdot u$, i.e., $(T_H|T_C = u)$ is deterministic and this property can be used to first simplify the formulas for P_1, P_2, P_3 and $L(C)$ (see also [38], where this is shown for a different CBM model). $\diamondsuit$

Fitting Option 2 - Gamma process: $X(\hat{t})$ is a Gamma process with shape parameter γ, scale parameter η, and initial degradation level 0 at $\hat{t} = 0$. The random increments throughout the process are independently and identically distributed. This implies that the degradation increment $X(\hat{t}_2) - X(\hat{t}_1)$ between two time points $\hat{t}_1$ and $\hat{t}_2$, $0 \leq \hat{t}_1 < \hat{t}_2$, is Gamma distributed with shape parameter $\gamma(t_2 - t_1)$ and scale parameter η (the corresponding p.d.f. is $\eta^{\gamma(t_2-t_1)} x^{\gamma(t_2-t_1)-1} e^{-\eta x}/\Gamma(\gamma(t_2 - t_1))$), and this increment is independent of the degradation process until time point $\hat{t}_1$. Then, for each $\chi > 0$, equation (2) can be rewritten as (use that $X(\hat{t})$ is Gamma distributed with shape parameter $\gamma\hat{t}$ and scale parameter η):

$$Pr\{T_\chi \leq \hat{t}\} = Pr\{X(\hat{t}) \geq \chi\} = 1 - F_{X(\hat{t})}(\chi) = \frac{\Gamma(\gamma\hat{t}, \eta\chi)}{\Gamma(\gamma\hat{t})}, \quad \hat{t} > 0, \tag{10}$$

where

$$\begin{aligned} \Gamma(\gamma\hat{t}) &= \int_0^\infty y^{\gamma\hat{t}-1} e^{-y} dy, \\ \Gamma(\gamma\hat{t}, \eta\chi) &= \int_{\eta\chi}^\infty y^{\gamma\hat{t}-1} e^{-y} dy \end{aligned}$$

(see also [27]). $\diamondsuit$

The degradation parameters (i.e., α, β, γ and η) can be estimated from real-life data. For this estimation, one can follow the standard methods in the literature (see [27]). In Table 1, we give modified input data for the laser units (as stated above, because of their confidentiality, we are not allowed to give the real-life data). We apply both the random coefficient model (Fitting Option 1) and the Gamma process (Fitting Option 2) for the degradation process.

The optimal control limit C^* in terms of a percentage of H can be found by minimizing the average costs $Z(C)$ via enumeration, where the evaluations are done via the approximate evaluation procedure of Section 3. As a comparison, we also simulate the average costs $\hat{Z}(C^*)$ under control limit C^* (see Appendix A for a description of the simulation procedure). Figure 5 shows the behavior of the average costs $Z(C)$ as a function of the control limit C (as percentage of H) for both degradation processes. In this figure, we also

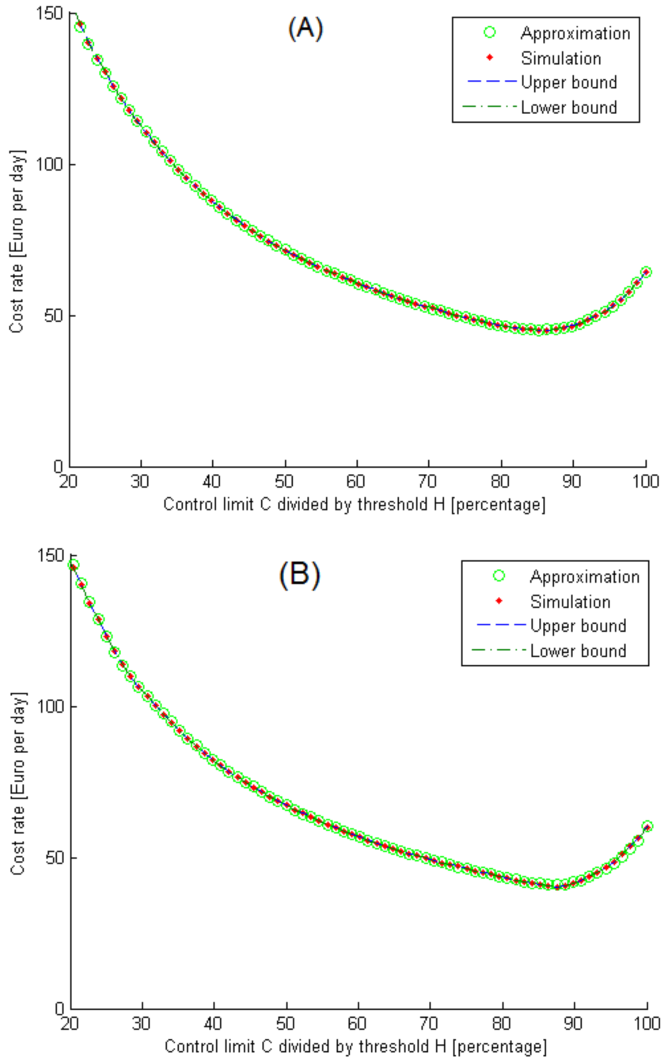


Figure 5: Average cost rate [Euro per day] as a function of the control limit C (denoted as percentage of H) for two fitting options: (A) the random coefficient model; (B) the Gamma process

give the simulated costs, together with the lower bound and upper bound of the 95% confidence interval. We can observe that the approximate costs are within the confidence intervals for all values of the control limit C.

The most important numerical results are also given in Table 2. Under the use of the random coefficient model, the optimal maintenance policy obtained via the approximate evaluation has a control limit that is 85.71% of the threshold ($C^*/H = 85.71\%$) and a minimum cost rate of 45.09 Euro per day (see also Figure 5-A). In the case of the Gamma process, the optimal maintenance policy obtained via the approximate evaluation has a control limit that is 87.18% of H with a minimum cost rate around 40.99 Euro per day (see also Figure 5-B).

To further investigate the differences between our approximate results and the simulation results, Table 2 shows: (i) the results obtained via the approximate evaluation, being C^*, its minimum cost rate $Z(C^*)$, its probabilities P_1, P_2, P_3, and its expected cycle length $L(C^*)$; (ii) the simulation results under the optimal control limit C^* obtained via approximate evaluation, where $\hat{Z}(C^*)$ denotes the simulated average costs,

Table 2: The optimal maintenance policies under the parameter setting given in Table 1

Fitting Option 1: the random coefficient model (RCM)

Approximation Result	Simulation Result 1
$Z(C^*) = 45.09$ [Euro per day] $C^*/H = 85.71\%$ $\left\{P_1, P_2, P_3\right\} = \{0.3075, 0.6350, 0.0576\}$ $L(C^*) = 627.4$ [day]	$\hat{Z}(C^*) = 45.16 \pm 0.024$ [Euro per day] $C^*/H = 85.71\%$ $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.3062, 0.6333, 0.0605\}$ $\hat{L}(C^*) = 627.6$ [day]
Simulation Result 2	
$\hat{Z}(\hat{C}^*) = 45.14 \pm 0.021$ [Euro per day] $\hat{C}^*/H = 85.23\%$ $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.3086, 0.6412, 0.0502\}$ $\hat{L}(\hat{C}^*) = 623.8$ [day]	Gap 1: $\lvert\frac{\left(\hat{Z}(C^*) - Z(C^*)\right)}{\hat{Z}(C^*)}\rvert = 0.16\%$ Gap 2: $\lvert\frac{\left(\hat{Z}(\hat{C}^*) - \hat{Z}(C^*)\right)}{\hat{Z}(\hat{C}^*)}\rvert = 0.04\%$

Fitting Option 2: the Gamma process (GP)

Approximation Result	Simulation Result 1
$Z(C^*) = 40.99$ [Euro per day] $C^*/H = 87.18\%$ $\left\{P_1, P_2, P_3\right\} = \{0.3102, 0.6563, 0.0335\}$ $L(C^*) = 679.76$ [day]	$\hat{Z}(C^*) = 41.01 \pm 0.051$ [Euro per day] $C^*/H = 87.18\%$ $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.3096, 0.6512, 0.0392\}$ $\hat{L}(C^*) = 681.98$ [day]
Simulation Result 2	
$\hat{Z}(\hat{C}^*) = 40.57 \pm 0.038$ [Euro per day] $\hat{C}^*/H = 85.75\%$ $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.3122, 0.6635, 0.0243\}$ $\hat{L}(\hat{C}^*) = 682.79$ [day]	Gap 1: $\lvert\frac{\left(\hat{Z}(C^*) - Z(C^*)\right)}{\hat{Z}(C^*)}\rvert = 0.05\%$ Gap 2: $\lvert\frac{\left(\hat{Z}(\hat{C}^*) - \hat{Z}(C^*)\right)}{\hat{Z}(\hat{C}^*)}\rvert = 1.06\%$

$\hat{P}_1, \hat{P}_2, \hat{P}_3$ denote the simulated probabilities for the three maintenance actions at the end of a maintenance cycle, and $\hat{L}(C^*)$ denotes the simulated mean cycle length; (iii) the optimal control limit $\hat{C}^*$ obtained via simulation-based optimization with its minimum cost rate $\hat{Z}(\hat{C}^*)$, its probabilities $\hat{P}_1, \hat{P}_2, \hat{P}_3$ for the three maintenance actions, and its mean cycle length $\hat{L}(\hat{C}^*)$.

Based on the results in Table 2, we observe that the absolute value $|\big(\hat{Z}(C^*) - Z(C^*)\big)/\hat{Z}(C^*)|$, denoted as Gap 1, is only 0.16% in the RCM case and 0.05% in the GP case, which shows that our approximate evaluation is very close to the true costs under the same control limit C^*. The absolute value $|\big(\hat{Z}(\hat{C}^*) - \hat{Z}(C^*)\big)/\hat{Z}(\hat{C}^*)|$, denoted as Gap 2, is only 0.04% in the RCM case and 1.06% in the GP case. This implies that the deviation of C^* from $\hat{C}^*$ does not lead to a large deviation for the simulated optimal costs, which is due to the fact that the average costs function $\hat{Z}(C)$ is flat in the neighborhood of its minimum. Hence, in practice, the optimal maintenance policy obtained via our approximate evaluation results in average costs that are very close to the true minimum average costs. Also notice that the approximated values for P_1, P_2, P_3 and $L(C)$ are close to the simulated values $\hat{P}_1, \hat{P}_2, \hat{P}_3$ and $\hat{L}(C)$. Therefore, we can conclude that the gaps are small and our approximate evaluation is accurate in this case study.

5. Numerical Experiments

In this section, we conduct the following numerical experiments based on full factorial test beds. In Section 5.1, we investigate the accuracy of the approximate evaluation. In Section 5.2, we optimize the control limit based on approximate evaluations and we investigate the gap with the true optimal control limit. In

Table 3: Parameter setting for Test bed 1

Parameter	Explanation
$E[T_H] = 1$	Mean lifetime
$H = 100\%$	Hard failure threshold
$C = \{30\%, 50\%, 70\%\}$	Control limit
$\sigma = 1/2 * \{50\%, 100\%, 150\%\}$	Standard deviation of component lifetime
$\lambda = 2 * \{50\%, 100\%, 150\%\}$	Poisson arrival rate for unscheduled downs
$\tau = 0.2 * \{50\%, 100\%, 150\%\}$	Fixed time interval for scheduled downs

Section 5.3, we evaluate the cost reduction potential of our proposed policy under various parameter settings. Throughout this section, we follow the same notation as in Section 4 to distinguish results obtained via the approximate evaluation and simulated results.

5.1 Accuracy of the approximate evaluation

The accuracy of our approximate evaluation can be assessed based on the gap between the approximation result $Z(C)$ and the simulation result $\hat{Z}(C)$. W.l.o.g., we take the mean life time $E[T_H]$ of the CBM component equal to 1 time unit. This normalizes the time unit. Similarly, we can take the hard failure level H equal to 1 which normalizes the degradation level. By fitting the two moments of the component life time, the shape and scale parameters of the Weibull distribution for the RCM case (see Fitting Option 1 in Section 4) and the Gamma distribution for the GP case (see Fitting Option 2 in Section 4) can be determined. The standard deviation σ is varied in our experiment. The larger σ, the larger the stochasticity of the degradation process. Next, τ and λ are varied, because they determine the frequency of the opportunities for PM-SD and PM-USD maintenance actions (see Section 2). Finally, we vary the control limit C, which we express as percentage of H. We use the following three values for the control limit C: 30%, 50%, 70%. Each of the other three parameters has a basic value multiplied by a set of factors, $\{50\%, 100\%, 150\%\}$. The basic values are found back in Table 3. For each parameter, the chosen values are ordered in increasing order and denoted with subindices 1, 2, 3. Hence, we have a full factorial test bed Λ with instances $(C_j, \sigma_l, \lambda_k, \tau_m)$, where $j, l, k, m = \{1, 2, 3\}$. This test bed, denoted as Test Bed 1, consists of $|\Lambda| = 81$ instances.

Notice that no cost parameters are chosen as factors in this test bed, since the average costs $Z(C)$ are fully determined by the probabilities of the three maintenance actions and the expected cycle length. This also helps to reduce the size of the test bed. To compare the approximate and simulation results, we compare the obtained values for the probabilities P_1, P_2, P_3 and the mean cycle length $L(C)$. To see how much the approximate and simulation results differ, we define a deviation vector $\left[\delta_1, \delta_2, \delta_3, \delta_4\right] = \left[\hat{P}_1 - P_1, \hat{P}_2 - P_2, \hat{P}_3 - P_3, (\hat{L}(C) - L(C))/\hat{L}(C)\right]$. The deviation vectors of 81 instances are shown in Tables 9-12 of Appendix B. There are three levels for each parameter C, σ, λ, τ. We categorize the instances containing a specific level of a certain parameter into a subset. For example, the subset of instances containing level C_1 for C is defined as $\Lambda_{C_1} = \left\{(C_1, \sigma_l, \lambda_k, \tau_m) | l, k, m \in \{1, 2, 3\}\right\}$. For each of these subsets, the average of the absolute deviation vectors (denoted by AAD) and the maximum of the absolute deviation vectors (denoted by MAD) are summarized in Table 4. The computation time of the approximate evaluation was in the order of a few seconds per instance.

The first insight from Table 4 is that the AAD and MAD of δ_1, δ_2, δ_3 and δ_4 are small for the RCM case. For this case, the approximate evaluation is accurate under all parameter settings. For the GP case, the AAD and MAD values for δ_1 and δ_4 are equally small as in the RCM case. For δ_2 and δ_3, the AAD and

Table 4: The average absolute difference (AAD) and maximum absolute difference (MAD) between the approximate results and simulation results under two fitting options for the degradation process

Fitting Option 1: the random coefficient model (RCM)

	$\lvert\delta_1\rvert$ $\{AAD, MAD\}$	$\lvert\delta_2\rvert$ $\{AAD, MAD\}$	$\lvert\delta_3\rvert$ $\{AAD, MAD\}$	$\lvert\delta_4\rvert$ $\{AAD, MAD\}$
Λ_{C_1}	$\{0.0047, 0.0156\}$	$\{0.0048, 0.0160\}$	$\{0.0003, 0.0004\}$	$\{0.52\%, 1.48\%\}$
Λ_{C_2}	$\{0.0030, 0.0143\}$	$\{0.0031, 0.0149\}$	$\{0.0005, 0.0007\}$	$\{0.21\%, 1.01\%\}$
Λ_{C_3}	$\{0.0020, 0.0168\}$	$\{0.0039, 0.0295\}$	$\{0.0023, 0.0127\}$	$\{0.12\%, 0.62\%\}$
Λ_{σ_1}	$\{0.0043, 0.0168\}$	$\{0.0057, 0.0295\}$	$\{0.0017, 0.0127\}$	$\{0.33\%, 1.40\%\}$
Λ_{σ_2}	$\{0.0028, 0.0148\}$	$\{0.0034, 0.0151\}$	$\{0.0009, 0.0053\}$	$\{0.26\%, 1.30\%\}$
Λ_{σ_3}	$\{0.0026, 0.0150\}$	$\{0.0028, 0.0154\}$	$\{0.0005, 0.0014\}$	$\{0.26\%, 1.48\%\}$
Λ_{λ_1}	$\{0.0012, 0.0048\}$	$\{0.0019, 0.0143\}$	$\{0.0010, 0.0095\}$	$\{0.17\%, 0.58\%\}$
Λ_{λ_2}	$\{0.0031, 0.0109\}$	$\{0.0038, 0.0209\}$	$\{0.0011, 0.0122\}$	$\{0.29\%, 1.06\%\}$
Λ_{λ_3}	$\{0.0054, 0.0168\}$	$\{0.0061, 0.0295\}$	$\{0.0011, 0.0127\}$	$\{0.39\%, 1.48\%\}$
Λ_{τ_1}	$\{0.0005, 0.0014\}$	$\{0.0007, 0.0019\}$	$\{0.0005, 0.0007\}$	$\{0.07\%, 0.19\%\}$
Λ_{τ_2}	$\{0.0031, 0.0140\}$	$\{0.0031, 0.0138\}$	$\{0.0004, 0.0007\}$	$\{0.25\%, 0.95\%\}$
Λ_{τ_3}	$\{0.0060, 0.0168\}$	$\{0.0081, 0.0295\}$	$\{0.0022, 0.0127\}$	$\{0.53\%, 1.48\%\}$
Λ	$\{0.0032, 0.0168\}$	$\{0.0039, 0.0295\}$	$\{0.0010, 0.0127\}$	$\{0.28\%, 1.48\%\}$

Fitting Option 2: the Gamma process (GP)

	$\lvert\delta_1\rvert$ $\{AAD, MAD\}$	$\lvert\delta_2\rvert$ $\{AAD, MAD\}$	$\lvert\delta_3\rvert$ $\{AAD, MAD\}$	$\lvert\delta_4\rvert$ $\{AAD, MAD\}$
Λ_{C_1}	$\{0.0017, 0.0055\}$	$\{0.0016, 0.0040\}$	$\{0.0008, 0.0040\}$	$\{0.16\%, 0.50\%\}$
Λ_{C_2}	$\{0.0017, 0.0078\}$	$\{0.0069, 0.0314\}$	$\{0.0066, 0.0294\}$	$\{0.20\%, 0.44\%\}$
Λ_{C_3}	$\{0.0052, 0.0186\}$	$\{0.0298, 0.0600\}$	$\{0.0339, 0.0635\}$	$\{0.37\%, 1.12\%\}$
Λ_{σ_1}	$\{0.0021, 0.0078\}$	$\{0.0080, 0.0504\}$	$\{0.0073, 0.0511\}$	$\{0.19\%, 0.43\%\}$
Λ_{σ_2}	$\{0.0034, 0.0186\}$	$\{0.0125, 0.0516\}$	$\{0.0141, 0.0598\}$	$\{0.26\%, 0.88\%\}$
Λ_{σ_3}	$\{0.0032, 0.0154\}$	$\{0.0177, 0.0600\}$	$\{0.0198, 0.0635\}$	$\{0.29\%, 1.12\%\}$
Λ_{λ_1}	$\{0.0019, 0.0077\}$	$\{0.0148, 0.0600\}$	$\{0.0150, 0.0635\}$	$\{0.24\%, 0.70\%\}$
Λ_{λ_2}	$\{0.0030, 0.0186\}$	$\{0.0119, 0.0509\}$	$\{0.0137, 0.0598\}$	$\{0.26\%, 1.12\%\}$
Λ_{λ_3}	$\{0.0038, 0.0184\}$	$\{0.0115, 0.0479\}$	$\{0.0125, 0.0575\}$	$\{0.24\%, 0.88\%\}$
Λ_{τ_1}	$\{0.0013, 0.0030\}$	$\{0.0041, 0.0207\}$	$\{0.0033, 0.0191\}$	$\{0.18\%, 0.50\%\}$
Λ_{τ_2}	$\{0.0026, 0.0097\}$	$\{0.0144, 0.0600\}$	$\{0.0148, 0.0635\}$	$\{0.22\%, 0.55\%\}$
Λ_{τ_3}	$\{0.0048, 0.0186\}$	$\{0.0197, 0.0535\}$	$\{0.0231, 0.0607\}$	$\{0.34\%, 1.12\%\}$
Λ	$\{0.0029, 0.0186\}$	$\{0.0127, 0.0600\}$	$\{0.0137, 0.0635\}$	$\{0.25\%, 1.12\%\}$

MAD values are larger than in the RCM case. The AAD values are still sufficiently small, but the MAD values show that the deviations for P_2 and P_3 go up to 6%.

5.2 Quality of the heuristic policy obtained via approximate evaluation

The results for the case of Section 4 show that the optimal policy obtained via our approximate evaluation was close to optimal. In this section, we verify the quality of the optimal control limit policy C^* which is obtained when the approximate evaluation is used in the optimization procedure. This policy may deviate from the true optimal policy $\hat{C}^*$ and thus is a heuristic. We consider the same gaps as in Section 4: (i) Gap 1, $\big(\hat{Z}(C^*) - Z(C^*)\big)/\hat{Z}(C^*)$, shows the accuracy in average costs of the approximate evaluation for control limit policy C^*; (ii) Gap 2, $\big(\hat{Z}(\hat{C}^*) - \hat{Z}(C^*)\big)/\hat{Z}(\hat{C}^*)$, shows how much the heuristic policy C^* deviates in true average costs from the true optimal policy $\hat{C}^*$.

In this experiment, we take the same instances as in Test Bed 1 in Section 5.1. However, the control limit is no longer an input variable since the control limit is optimized in this experiment. The resulting test bed

Table 5: The average absolute difference (AAD) and the maximum absolute difference (MAD) for $(\hat{C}^* - C^*)/H$, Gap 1, and Gap 2 under two fitting options for the degradation process

Fitting Option 1: the random coefficient model (RCM)

	$(\hat{C}^* - C^*)/H$ $\{AAD, MAD\}$	Gap 1 $\{AAD, MAD\}$	Gap 2 $\{AAD, MAD\}$
Ω_{σ_1}	$\{0.97\%, 6.57\%\}$	$\{0.20\%, 0.30\%\}$	$\{0.12\%, 0.41\%\}$
Ω_{σ_2}	$\{0.55\%, 1.36\%\}$	$\{0.20\%, 0.64\%\}$	$\{0.10\%, 0.19\%\}$
Ω_{σ_3}	$\{0.90\%, 3.72\%\}$	$\{0.17\%, 0.35\%\}$	$\{0.14\%, 0.33\%\}$
Ω_{λ_1}	$\{0.83\%, 3.72\%\}$	$\{0.14\%, 0.30\%\}$	$\{0.12\%, 0.33\%\}$
Ω_{λ_2}	$\{1.13\%, 6.57\%\}$	$\{0.21\%, 0.63\%\}$	$\{0.15\%, 0.41\%\}$
Ω_{λ_3}	$\{0.45\%, 1.36\%\}$	$\{0.22\%, 0.64\%\}$	$\{0.10\%, 0.17\%\}$
Ω_{τ_1}	$\{0.31\%, 1.36\%\}$	$\{0.11\%, 0.35\%\}$	$\{0.10\%, 0.21\%\}$
Ω_{τ_2}	$\{0.73\%, 3.72\%\}$	$\{0.15\%, 0.26\%\}$	$\{0.15\%, 0.33\%\}$
Ω_{τ_3}	$\{1.37\%, 6.57\%\}$	$\{0.32\%, 0.64\%\}$	$\{0.11\%, 0.41\%\}$
Ω	$\{0.81\%, 6.57\%\}$	$\{0.19\%, 0.64\%\}$	$\{0.12\%, 0.41\%\}$

Fitting Option 2: the Gamma process (GP)

	$(\hat{C}^* - C^*)/H$ $\{AAD, MAD\}$	Gap 1 $\{AAD, MAD\}$	Gap 2 $\{AAD, MAD\}$
Ω_{σ_1}	$\{1.76\%, 3.82\%\}$	$\{3.61\%, 4.93\%\}$	$\{0.68\%, 1.69\%\}$
Ω_{σ_2}	$\{2.67\%, 5.45\%\}$	$\{3.98\%, 6.76\%\}$	$\{1.03\%, 1.74\%\}$
Ω_{σ_3}	$\{3.71\%, 7.33\%\}$	$\{3.18\%, 7.13\%\}$	$\{1.92\%, 3.93\%\}$
Ω_{λ_1}	$\{2.72\%, 6.65\%\}$	$\{3.98\%, 7.13\%\}$	$\{1.44\%, 3.93\%\}$
Ω_{λ_2}	$\{2.53\%, 5.14\%\}$	$\{3.56\%, 6.90\%\}$	$\{1.11\%, 2.79\%\}$
Ω_{λ_3}	$\{2.89\%, 7.33\%\}$	$\{3.23\%, 6.52\%\}$	$\{1.08\%, 2.58\%\}$
Ω_{τ_1}	$\{1.21\%, 1.79\%\}$	$\{1.11\%, 2.36\%\}$	$\{0.68\%, 1.69\%\}$
Ω_{τ_2}	$\{2.17\%, 4.30\%\}$	$\{3.59\%, 4.70\%\}$	$\{1.33\%, 2.73\%\}$
Ω_{τ_3}	$\{4.77\%, 7.33\%\}$	$\{6.06\%, 7.13\%\}$	$\{1.62\%, 3.93\%\}$
Ω	$\{2.72\%, 7.33\%\}$	$\{3.59\%, 7.13\%\}$	$\{1.21\%, 3.93\%\}$

is denoted by Ω and consists of instances $(\sigma_l, \lambda_k, \tau_m)$, $l, k, m = \{1, 2, 3\}$. This test bed is denoted as test bed 2 and consist of $|\Omega| = 27$ instances. For the cost factors, we use the same factors as in Section 4; see Table 1 for their values. The deviation $(\hat{C}^* - C^*)/H$, Gap 1, and Gap 2 for all 27 instances are listed in Table 13 of Appendix B. For each factor, we categorize the instances containing a specific level into a subset. For example, the subset of all instances containing σ_1 is defined as $\Omega_{\sigma_1} = \{(\sigma_1, \lambda_k, \tau_m) | k, m \in \{1, 2, 3\}\}$. For each of these subsets, the average and maximum absolute deviation, denoted by AAD and MAD are listed in Table 5 for both fitting options for the degradation process. The computation times of the results obtained via approximate evaluation were in the order of a few minutes per instance.

The first insight from Table 5 is that the AAD and MAD values are almost all small for the RCM case. The MAD value for $(\hat{C}^* - C^*)/H$ in the whole test bed is 6.57%, which is significant. This MAD value is obtained for the instance $(\sigma_1, \lambda_2, \tau_3)$ (see Table 13). The value for Gap 1 for this instance is only 0.09%, which is again low, and the value for Gap 2 is 0.41%. The latter value is the highest value for Gap 2 among all instances, but is still low. In general, even if the absolute value for $(\hat{C}^* - C^*)/H$ is somewhat larger, the values for Gap 1 and Gap 2 are very low. This is due to the accuracy of the approximate evaluation and the flatness of the average costs $\hat{Z}(C)$ in the neighbourhood of the optimum. We can conclude that the quality of our heuristic is excellent for the RCM case.

For the GP case, the AAD and MAD value for $(\hat{C}^* - C^*)/H$ in the whole test bed is 2.72% and 7.33%, respectively. This is reasonable, but less good than in the RCM case. The AAD and MAD value for Gap 1 is 3.59% and 7.13%, respectively. This is relatively high, and shows that the approximate evaluation for the control limit policy C^* is much less accurate than in the RCM case. Fortunately, the AAD and MAD value for Gap 2 is 1.21% and 3.93%, respectively. This is less good than in the RCM case, but it shows that the average costs of the heuristic policy C^* are close to the optimal costs. We may conclude that the quality of our heuristic is good for the GP case.

5.3 Cost reduction potential

As stated in the introduction, our model for a single CBM component with USDs and SDs as opportunities for preventive maintenance is new. Hence, in this section, we compare the optimal control limit policy within our model to three other policies. This shows the value of using both USDs and SDs and thus also the value of our new model. The three other policies that we consider are: 1) an *only-SD-opportunistic policy*, which means that only SDs are considered as opportunities; 2) an *only-USD-opportunistic policy*, which means that only USDs are considered as opportunities; 3) a *failure-based policy*, which means that neither USDs or SDs are considered as opportunities for preventive maintenance. For the only-SD-opportunistic policy and the only-USD-opportunistic policy, we use a control limit policy with an optimized control limit. This optimization can be done via special cases of our model with $\lambda = 0$ and $\tau = \infty$, respectively. For the failure-based policy, the average costs are equal to $c^{CM}/E[T_H] = 44.5/1 = 44.5$ thousand Euro per time unit. They are constant over all combinations of instances and the two fitting options for the degradation process, because they only depend on the factors c^{CM} and $E[T_H]$ and these factors are the same in all instances and under both fitting options.

For this experiment, we can use the same test bed as in Section 5.2, i.e., Test bed 2. The average costs of the optimal control limit policy that uses USDs and SDs are denoted by Z. The average costs of the optimal only-SD-opportunistic policy and the optimal only-USD-opportunistic policy are denoted by $\tilde{Z}_1$ and $\tilde{Z}_2$, respectively. The average costs under the failure-based policy are denoted by $\tilde{Z}_3$. Because $\tilde{Z}_3$ is the same in all instances and under both fitting options, we use $\tilde{Z}_3$ as the basis for three comparisons that we make: A) the cost savings percentage of including opportunities at both USDs and SDs, denoted by $\triangle_0 = (\tilde{Z}_3 - Z)/\tilde{Z}_3$; B) the cost savings percentage of using only opportunities at SDs, denoted by $\triangle_1 = (\tilde{Z}_3 - \tilde{Z}_1)/\tilde{Z}_3$; C) the cost savings percentage of using only opportunities at USDs, denoted by $\triangle_2 = (\tilde{Z}_3 - \tilde{Z}_2)/\tilde{Z}_3$. The mean cost savings, and their minimum and maximum over subsets of instances and the whole test bed are given in Table 6. The results per instance are listed in Table 14 in Appendix B. All results in this experiment are based on the approximate evaluation procedure.

Let us first consider the results for the RCM case in Table 6. We see that the cost savings are on average equal to 8.4% when only USDs are used as opportunities for preventive maintenance; see the mean value for $\triangle_2$ over the whole test bed. This percentage increases strongly as a function of the rate with which these USDs occur. For the subset Ω_{λ_3}, the mean time between two successive USDs is $1/\lambda_3 = 1/3 = 0.33$ time units (recall that the mean lifetime $E[T_H] = 1$ time unit) and the average value of $\triangle_2$ is 11.5%. The average costs savings when only SDs are used are equal to 22.6%, and these savings decrease strongly as a function of the maintenance interval length. For the subset Ω_{τ_3}, the maintenance interval length $\tau_3 = 0.3$ time units and the average value of $\triangle_1$ is 14.1%. This percentage may be compared to the 11.5% for $\triangle_2$ in subset Ω_{λ_3}, and shows that it is slightly better to have SDs with deterministic interarrival times than an (almost) equal amount of USDs with exponential interarrival times (notice that this includes the effect of slightly cheaper preventive maintenance costs at SDs than at USDs). The cost savings when both USDs and SDs are used are equal to 28.9%; see the mean value for $\triangle_2$ over the whole test bed. This is significantly more than when

Table 6: The cost savings percentages $\triangle_0$, $\triangle_1$, $\triangle_2$ under two fitting options for the degradation process

Fitting Option 1: the random coefficient model (RCM)

	$\triangle_0$			$\triangle_1$			$\triangle_2$		
	mean	min	max	mean	min	max	mean	min	max
Ω_{σ_1}	29.7%	23.0%	35.3%	23.4%	11.6%	31.6%	8.6%	5.1%	11.7%
Ω_{σ_2}	28.7%	22.7%	34.7%	22.5%	14.6%	29.7%	8.4%	5.0%	11.5%
Ω_{σ_3}	28.5%	23.0%	34.3%	21.9%	16.2%	29.5%	8.2%	4.8%	11.3%
Ω_{λ_1}	29.5%	23.7%	35.3%	22.6%	11.6%	31.6%	5.0%	4.8%	5.1%
Ω_{λ_2}	28.8%	22.7%	35.0%	22.6%	11.6%	31.6%	8.7%	8.5%	8.9%
Ω_{λ_3}	28.5%	22.7%	34.7%	22.6%	11.6%	31.6%	11.5%	11.3%	11.7%
Ω_{τ_1}	34.5%	33.7%	35.3%	30.3%	29.5%	31.6%	8.4%	4.8%	11.7%
Ω_{τ_2}	29.0%	27.8%	30.9%	23.4%	20.2%	27.0%	8.4%	4.8%	11.7%
Ω_{τ_3}	23.4%	22.7%	25.0%	14.1%	11.6%	16.2%	8.4%	4.8%	11.7%
Ω	28.9%	22.7%	35.3%	22.6%	11.6%	31.6%	8.4%	4.8%	11.7%

Fitting Option 2: the Gamma process (GP)

	$\triangle_0$			$\triangle_1$			$\triangle_2$		
	mean	min	max	mean	min	max	mean	min	max
Ω_{σ_1}	27.4%	23.6%	31.4%	21.0%	16.5%	25.9%	11.5%	8.8%	14.0%
Ω_{σ_2}	26.3%	23.0%	30.1%	20.8%	15.4%	26.4%	10.5%	7.5%	13.1%
Ω_{σ_3}	26.1%	22.9%	29.7%	20.7%	12.2%	27.9%	9.5%	6.4%	12.4%
Ω_{λ_1}	26.7%	22.9%	31.4%	20.8%	12.2%	27.9%	7.6%	6.4%	8.8%
Ω_{λ_2}	26.6%	22.9%	31.2%	20.8%	12.2%	27.9%	10.8%	9.8%	11.8%
Ω_{λ_3}	26.5%	23.0%	31.0%	20.8%	12.2%	27.9%	13.2%	12.4%	14.0%
Ω_{τ_1}	30.2%	29.5%	31.4%	26.7%	25.9%	27.9%	10.5%	6.4%	14.0%
Ω_{τ_2}	26.3%	25.6%	27.4%	21.1%	20.6%	21.9%	10.5%	6.4%	14.0%
Ω_{τ_3}	23.2%	22.9%	24.0%	14.7%	12.2%	16.5%	10.5%	6.4%	14.0%
Ω	26.6%	22.9%	31.4%	20.8%	12.2%	27.9%	10.5%	6.4%	14.0%

only SDs or only USDs are used. Hence, it is beneficial to use both SDs and USDs if possible. Finally, we observe that the average values of $\triangle_0$, $\triangle_1$, $\triangle_2$ decrease only slightly as a function of the standard deviation of the lifetime, or, equivalently, the stochasticity of the degradation process.

When we consider the results for the GP case, we see exactly the same effects as for the RCM case. In fact, the cost savings percentages are remarkably similar in both cases. On one hand, this is surprising because the degradation behavior under RCM is really different than under GP. On the other hand, we saw already that the stochasticity of the degradation process has a limited effect on the cost savings. This seems to be related to each other.

6. Demonstration for a multi-component systems

In this section, we demonstrate that our single component model can be used as a building block for the analysis of multi-component systems. Let us consider a system consisting of 20 CBM components with degradation processes that are modeled by the random coefficient model described in Section 4. The components are numbered from 1 to 20. In Table 7, the following input parameters are given: 1) α_i and β_i are the scale and shape parameters of the Weibull distribution for the factor θ_1 used in the description of the degradation process for component i; 2) C_i^{PM-USD}, C_i^{PM-SD}, and C_i^{CM} are the costs for a PM-USD, PM-SD, and CM action, respectively. The rest of the parameter setting is the same as in the case described in Section 4 (see Table 1); for all components, $H = 88$, $\tau = 91$ days and $\lambda = 8.86 * 10^{-3}$ USDs per time unit.

Table 7: Input parameters for the multi-component system with 20 CBM components

Component	α_i	β_i	C_i^{PM-USD} ($\times 10^3$ Euro)	C_i^{PM-SD} ($\times 10^3$ Euro)	C_{cpm} ($\times 10^3$ Euro)
1	0.159	3.37	28.8	26.5	44.5
2	0.167	3.55	29.3	27.0	45.6
3	0.176	3.72	29.9	27.6	46.7
4	0.184	3.90	30.4	28.1	47.8
5	0.192	4.08	30.9	28.6	48.9
6	0.201	4.26	31.4	29.1	50.0
7	0.209	4.43	32.0	29.7	51.1
8	0.218	4.61	32.5	30.2	52.2
9	0.226	4.79	33.0	30.7	53.3
10	0.234	4.97	33.5	31.2	54.4
11	0.243	5.14	34.1	31.8	55.6
12	0.251	5.32	34.6	32.3	56.7
13	0.259	5.50	35.1	32.8	57.8
14	0.268	5.68	35.6	33.3	58.9
15	0.276	5.85	36.2	33.9	60.0
16	0.285	6.03	36.7	34.4	61.1
17	0.293	6.21	37.2	34.9	62.2
18	0.301	6.39	37.7	35.4	63.3
19	0.310	6.56	38.3	36.0	64.4
20	0.318	6.74	38.8	36.5	65.5

The rate λ denotes the rate for USDs that occur because of failures of other components than the 20 CBM components.

For each CBM component, we follow a control limit policy with control limit C_i. We optimize them via an iterative procedure. In an initial iteration, for each component i, we apply our single-component model with rate λ for the USDs and interval length τ for the SDs. We use the approximate evaluation procedure and optimize the control limit C_i. This gives a first optimized level C_i^* for component i. What we also get is an estimate $\lambda_i = P_3/L(C_i^*)$ that denotes the number of corrective maintenance actions for component i per time unit. These are extra USDs that can be used by the other CBM components. This is used in the further iterations.

In each next iteration, for each CBM component i, we take the USDs caused by other CBM components into account, and we approximate their arrival process by a Poisson process. The total rate of the USDs then becomes equal to $\lambda + \sum_{j\in\{1,\ldots,20\}\setminus\{i\}} \lambda_j$. Subsequently, we recalculate the optimal control limit C_i^* for component i and we recalculate the rate λ_i with which USDs are caused. This iterative process is continued until we have convergence for the optimal control limits C_i^*. At the end of this procedure, we also obtain the average costs $Z_i(C_i^*)$ for each component i and the total costs $\sum_{i\in\{1,\ldots,20\}} Z_i(C_i^*)$. In addition, we apply simulation to obtain the true average costs $\hat{Z}_i(C_i^*)$ for each component i and the true total costs $\sum_{i\in\{1,\ldots,20\}} \hat{Z}_i(C_i^*)$.

We have no guarantee for the convergence of the above procedure, but we obtained convergence for all instances to which this procedure was applied. This holds also for the generalized procedure of [37] for systems with a mix of CBM, age-based, and failure-based components.

For our problem, the above procedure leads to the results listed in Table 8. In the last column, we list the relative gap $|(\hat{Z}_i(C_i^*) - Z_i(C_i^*))/\hat{Z}_i(C_i^*)|$, and similarly for the total costs. The computation time was 30.2 minutes on a PC with a 2.5 GHz processor and 4 G RAM. This shows that a system with 20

Table 8: Output for the multi-component system with 20 CBM components

Component	C_i^*	$Z_i(C_i^*)$ (Euro/day)	$\hat{Z}_i(C_i^*)$ (Euro/day)	Gap
1	86.41%	43.84	44.01	0.4%
2	85.71%	48.05	48.46	0.9%
3	85.60%	52.43	52.54	0.2%
4	84.96%	56.99	57.21	0.4%
5	83.85%	61.68	61.47	0.3%
6	83.45%	66.50	66.81	0.5%
7	83.33%	71.51	71.84	0.5%
8	82.81%	76.70	76.93	0.3%
9	81.54%	81.96	82.08	0.1%
10	80.60%	87.28	86.99	0.3%
11	80.00%	92.69	92.01	0.7%
12	79.89%	98.25	97.76	0.5%
13	79.89%	104.02	104.38	0.4%
14	79.77%	110.05	110.74	0.6%
15	79.66%	116.43	117.97	1.3%
16	79.93%	123.21	124.82	1.3%
17	79.35%	130.45	131.55	0.8%
18	78.41%	138.10	139.22	0.8%
19	76.58%	146.01	146.36	0.2%
20	75.00%	153.84	152.46	0.9%
Total		1859.99	1865.61	0.0%

components can be solved within a reasonable computation time. This calculation time seems to be linear in the number of components (with more components, the number of iterations is about the same, and the calculation time per component is linear as a function of the number of components). The results in Table 8 also show that components with the shortest lifetimes (the components with the highest indices) and highest costs for maintenance actions get the lowest control limits C_i^*. Further, we observe that the gaps $|(\hat{Z}_i(C_i^*) - Z_i(C_i^*))/\hat{Z}_i(C_i^*)|$ are small and the gap for the total costs is almost zero.

7. Conclusions

In this paper, we introduced a new model for a single CBM component that is part of a complex engineering system and has both USDs and SDs as opportunities for preventive maintenance. For the degradation process, we applied both a random coefficient model and a Gamma process. We developed an efficient approximate evaluation procedure for control limit policies and showed that this procedure leads to good heuristic solutions when used for the optimization of the control limit. We also showed that the average costs can be reduced significantly when using both USDs and SDs instead of only SDs, only USDs, or no opportunities at all. Finally, we demonstrated that our model can also be used for solving multi-component systems within a reasonable computation time.

Acknowledgments

The authors thank TKI Dinalog for funding this research.

Appendices

A: Simulation procedure

In this appendix, we described the setup of the simulations to evaluate given maintenance policies.

A maintenance policy is described by the control limit C. Suppose that this control limit C is given. We simulate the unscheduled downs by a Poisson process with a rate λ and we take a given model with given parameters for the degradation of the CBM component. The simulation consists of $m = 100$ subruns, which are numbered as $1, \ldots, m$. Per subrun we simulate over a time horizon T_{max} that is sufficiently large to obtain accurate simulated results (e.g., 10^6 times larger than $L(C)$). In each subrun i, we have a finite number of maintenance cycles that we number as $1, \ldots, k_i$, and we keep track of the following variables:

$$
\begin{aligned}
I_{i,k}^{PM-USD} &= \begin{cases} 1 & \text{if a PM-USD action is taken at the end of the } k\text{-th cycle;} \\ 0 & \text{otherwise,} \end{cases} \\
I_{i,k}^{PM-SD} &= \begin{cases} 1 & \text{if a PM-SD action is taken at the end of the } k\text{-th cycle;} \\ 0 & \text{otherwise,} \end{cases} \\
I_{i,k}^{CM} &= \begin{cases} 1 & \text{if a CM action is taken at the end of the } k\text{-th cycle;} \\ 0 & \text{otherwise.} \end{cases} \\
L_i &= \text{total length of the } k_i \text{ cycles of subrun } i.
\end{aligned}
$$

The average costs in subrun i are then given by

$$\hat{Z}_i = \frac{1}{L_i} \sum_{k=1}^{k_i} \left(I_{i,k}^{PM-USD} c^{PM-USD} + I_{i,k}^{PM-SD} c^{PM-SD} + I_{i,k}^{CM} c^{CM} \right), \quad i = 1, \ldots, m.$$

Next, the average costs are estimated at $\hat{Z} = \frac{1}{m} \sum_{i=1}^{m} \hat{Z}_i$ with a $100(1-\alpha)\%$ confidence interval given by (cf. [16]):

$$\hat{Z} \pm t(1-\alpha/2, m-1) \sqrt{\frac{S^2}{m}},$$

where $S^2 = \frac{1}{m-1} \sum_{i=1}^{m} (\hat{Z}_i - \hat{Z})^2$ and $t(1-\alpha/2, m-1)$ is the upper $1-\alpha/2$ critical point for the t-distribution with $m-1$ degrees of freedom ($\alpha = 0.05$ for all reported results in this paper). The probabilities $\hat{P}_1$, $\hat{P}_2$, and $\hat{P}_3$ that an arbitrary cycle ends with a PM-USD, PM-SD, and CM action, respectively, are determined by

$$
\begin{aligned}
\hat{P}_1 &= \frac{1}{m} \sum_{i=1}^{m} \frac{1}{k_i} \sum_{k=1}^{k_i} I_{i,k}^{PM-USD}, \\
\hat{P}_2 &= \frac{1}{m} \sum_{i=1}^{m} \frac{1}{k_i} \sum_{k=1}^{k_i} I_{i,k}^{PM-SD}, \\
\hat{P}_3 &= \frac{1}{m} \sum_{i=1}^{m} \frac{1}{k_i} \sum_{k=1}^{k_i} I_{i,k}^{CM}.
\end{aligned}
$$

Finally, the expected cycle length $\hat{L}(C)$ is determined by $\hat{L} = \frac{1}{m} \sum_{i=1}^{m} (L_i/k_i)$.

B: Detail results for the Test beds 1 and 2

In this appendix, we give detailed results for the test beds 1 and 2. Tables 9 and 9 contain the detailed results for test bed 1 under the use of the random coefficient model. Tables 11 and 12 contain the detailed results for test bed 1 under the use of the Gamma process. Tables 13 and 14 contain detailed results for Test bed 2.

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Table 9: Detailed results for the first 40 of the 81 instances of Test bed 1 under the use of the *random coefficient model*

Instance	Simulation $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$	Deviation $\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(C_1, \sigma_1, \lambda_1, \tau_1)$	$\{0.044, 0.956, 0.000, 0.344\}$	$\{0.000, 0.000, 0.000, 0.1\%\}$
$(C_1, \sigma_1, \lambda_1, \tau_2)$	$\{0.104, 0.896, 0.000, 0.403\}$	$\{0.000, -0.001, 0.000, 0.0\%\}$
$(C_1, \sigma_1, \lambda_1, \tau_3)$	$\{0.109, 0.890, 0.000, 0.409\}$	$\{0.003, -0.003, 0.000, 0.6\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_1)$	$\{0.085, 0.914, 0.000, 0.343\}$	$\{0.000, 0.000, 0.000, 0.2\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_2)$	$\{0.193, 0.807, 0.000, 0.397\}$	$\{-0.002, 0.002, 0.000, -0.1\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_3)$	$\{0.201, 0.799, 0.000, 0.401\}$	$\{0.008, -0.008, 0.000, 1.0\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_1)$	$\{0.125, 0.875, 0.000, 0.342\}$	$\{0.001, -0.001, 0.000, 0.2\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_2)$	$\{0.273, 0.727, 0.000, 0.391\}$	$\{-0.003, 0.003, 0.000, -0.3\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_3)$	$\{0.278, 0.722, 0.000, 0.393\}$	$\{0.016, -0.016, 0.000, 1.4\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_1)$	$\{0.048, 0.951, 0.000, 0.348\}$	$\{0.000, 0.000, 0.000, 0.0\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_2)$	$\{0.109, 0.891, 0.000, 0.409\}$	$\{-0.001, 0.001, 0.000, -0.3\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_3)$	$\{0.108, 0.891, 0.000, 0.408\}$	$\{0.002, -0.003, 0.000, 0.5\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_1)$	$\{0.092, 0.907, 0.000, 0.347\}$	$\{-0.001, 0.001, 0.000, 0.1\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_2)$	$\{0.201, 0.799, 0.000, 0.400\}$	$\{-0.005, 0.005, 0.000, -0.6\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_3)$	$\{0.202, 0.798, 0.000, 0.401\}$	$\{0.009, -0.009, 0.000, 1.1\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_1)$	$\{0.135, 0.864, 0.000, 0.346\}$	$\{0.000, 0.000, 0.000, 0.1\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_2)$	$\{0.279, 0.721, 0.000, 0.393\}$	$\{-0.011, 0.011, 0.000, -0.9\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_3)$	$\{0.280, 0.720, 0.000, 0.393\}$	$\{0.015, -0.015, 0.000, 1.3\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_1)$	$\{0.050, 0.950, 0.000, 0.350\}$	$\{0.000, 0.000, 0.000, 0.0\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_2)$	$\{0.109, 0.891, 0.000, 0.410\}$	$\{-0.002, 0.002, 0.000, -0.4\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_3)$	$\{0.108, 0.891, 0.000, 0.409\}$	$\{0.002, -0.002, 0.000, 0.5\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_1)$	$\{0.096, 0.903, 0.000, 0.348\}$	$\{0.000, 0.000, 0.000, 0.1\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_2)$	$\{0.201, 0.799, 0.000, 0.401\}$	$\{-0.008, 0.007, 0.000, -0.7\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_3)$	$\{0.202, 0.797, 0.000, 0.401\}$	$\{0.008, -0.008, 0.000, 0.9\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_1)$	$\{0.140, 0.860, 0.000, 0.346\}$	$\{0.000, 0.000, 0.000, -0.1\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_2)$	$\{0.279, 0.721, 0.000, 0.394\}$	$\{-0.014, 0.014, 0.000, -1.0\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_3)$	$\{0.282, 0.717, 0.000, 0.395\}$	$\{0.015, -0.015, 0.000, 1.5\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_1)$	$\{0.049, 0.950, 0.000, 0.549\}$	$\{0.000, -0.001, 0.000, 0.0\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_2)$	$\{0.107, 0.892, 0.000, 0.606\}$	$\{0.000, 0.000, 0.000, -0.2\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_3)$	$\{0.128, 0.871, 0.001, 0.627\}$	$\{0.004, -0.005, 0.001, 0.5\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_1)$	$\{0.096, 0.904, 0.000, 0.548\}$	$\{0.000, -0.001, 0.000, 0.1\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_2)$	$\{0.196, 0.803, 0.000, 0.598\}$	$\{-0.005, 0.004, 0.000, -0.3\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_3)$	$\{0.240, 0.759, 0.001, 0.620\}$	$\{0.011, -0.011, 0.001, 0.8\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_1)$	$\{0.138, 0.862, 0.000, 0.546\}$	$\{-0.001, 0.000, 0.000, 0.0\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_2)$	$\{0.272, 0.727, 0.000, 0.591\}$	$\{-0.010, 0.010, 0.000, -0.5\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_3)$	$\{0.334, 0.666, 0.001, 0.613\}$	$\{0.014, -0.015, 0.001, 1.0\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_1)$	$\{0.050, 0.950, 0.000, 0.548\}$	$\{0.001, -0.002, 0.000, 0.0\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_2)$	$\{0.097, 0.902, 0.000, 0.599\}$	$\{-0.002, 0.001, 0.000, -0.1\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_3)$	$\{0.139, 0.861, 0.001, 0.638\}$	$\{0.001, -0.002, 0.001, 0.2\%\}$
$(C_2, \sigma_2, \lambda_2, \tau_1)$	$\{0.095, 0.905, 0.000, 0.547\}$	$\{0.001, -0.002, 0.000, 0.0\%\}$

Table 10: Detailed results for the last 41 of the 81 instances of Test bed 1 under the use of the random coefficient model

Instance	Simulation $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$	Deviation $\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(C_2, \sigma_2, \lambda_2, \tau_2)$	$\{0.184, 0.816, 0.000, 0.592\}$	$\{-0.002, 0.001, 0.000, -0.2\%\}$
$(C_2, \sigma_2, \lambda_2, \tau_3)$	$\{0.255, 0.745, 0.001, 0.627\}$	$\{0.003, -0.003, 0.001, 0.2\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_1)$	$\{0.136, 0.863, 0.000, 0.546\}$	$\{0.001, -0.001, 0.000, 0.1\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_2)$	$\{0.255, 0.745, 0.000, 0.586\}$	$\{-0.006, 0.006, 0.000, -0.1\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_3)$	$\{0.351, 0.648, 0.001, 0.618\}$	$\{0.003, -0.004, 0.001, 0.2\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_1)$	$\{0.049, 0.951, 0.000, 0.549\}$	$\{0.001, -0.001, 0.000, 0.2\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_2)$	$\{0.094, 0.906, 0.000, 0.595\}$	$\{0.000, 0.000, 0.000, 0.1\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_3)$	$\{0.145, 0.855, 0.001, 0.645\}$	$\{-0.001, 0.000, 0.001, 0.0\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_1)$	$\{0.094, 0.906, 0.000, 0.547\}$	$\{0.001, -0.001, 0.000, 0.0\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_2)$	$\{0.176, 0.824, 0.000, 0.588\}$	$\{-0.001, 0.001, 0.000, 0.0\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_3)$	$\{0.263, 0.736, 0.001, 0.632\}$	$\{-0.002, 0.002, 0.001, -0.2\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_1)$	$\{0.136, 0.863, 0.000, 0.546\}$	$\{0.001, -0.001, 0.000, 0.1\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_2)$	$\{0.245, 0.754, 0.000, 0.583\}$	$\{-0.003, 0.003, 0.000, 0.1\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_3)$	$\{0.360, 0.640, 0.001, 0.620\}$	$\{-0.005, 0.005, 0.001, -0.3\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_1)$	$\{0.048, 0.952, 0.001, 0.748\}$	$\{0.000, 0.000, 0.001, 0.0\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_2)$	$\{0.098, 0.902, 0.001, 0.797\}$	$\{0.000, -0.001, 0.001, 0.0\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_3)$	$\{0.145, 0.762, 0.093, 0.847\}$	$\{-0.005, 0.014, -0.01, -0.3\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_1)$	$\{0.093, 0.907, 0.001, 0.747\}$	$\{-0.001, 0.000, 0.001, 0.0\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_2)$	$\{0.181, 0.819, 0.001, 0.791\}$	$\{-0.001, 0.001, 0.001, 0.0\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_3)$	$\{0.263, 0.670, 0.067, 0.832\}$	$\{-0.009, 0.021, -0.012, -0.5\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_1)$	$\{0.137, 0.863, 0.001, 0.745\}$	$\{0.001, -0.002, 0.001, 0.0\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_2)$	$\{0.254, 0.746, 0.001, 0.785\}$	$\{-0.003, 0.002, 0.001, -0.1\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_3)$	$\{0.354, 0.599, 0.048, 0.818\}$	$\{-0.017, 0.029, -0.013, -0.6\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_1)$	$\{0.048, 0.952, 0.001, 0.749\}$	$\{-0.001, 0.000, 0.001, 0.1\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_2)$	$\{0.092, 0.908, 0.001, 0.792\}$	$\{0.000, -0.001, 0.001, 0.1\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_3)$	$\{0.132, 0.790, 0.079, 0.833\}$	$\{-0.002, 0.006, -0.004, -0.1\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_1)$	$\{0.093, 0.906, 0.001, 0.748\}$	$\{0.000, 0.000, 0.001, 0.2\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_2)$	$\{0.172, 0.827, 0.001, 0.786\}$	$\{0.000, -0.001, 0.001, 0.0\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_3)$	$\{0.240, 0.702, 0.058, 0.822\}$	$\{-0.003, 0.008, -0.005, 0.1\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_1)$	$\{0.136, 0.863, 0.001, 0.746\}$	$\{0.000, 0.000, 0.001, 0.0\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_2)$	$\{0.244, 0.755, 0.001, 0.782\}$	$\{0.002, -0.002, 0.001, 0.2\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_3)$	$\{0.332, 0.623, 0.045, 0.811\}$	$\{-0.001, 0.005, -0.004, -0.1\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_1)$	$\{0.048, 0.951, 0.001, 0.749\}$	$\{0.000, -0.001, 0.001, 0.1\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_2)$	$\{0.092, 0.906, 0.002, 0.792\}$	$\{0.000, -0.001, 0.001, 0.1\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_3)$	$\{0.128, 0.802, 0.070, 0.829\}$	$\{0.000, 0.002, -0.001, 0.0\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_1)$	$\{0.094, 0.906, 0.001, 0.747\}$	$\{0.000, -0.001, 0.001, 0.0\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_2)$	$\{0.172, 0.826, 0.002, 0.787\}$	$\{0.000, -0.001, 0.001, 0.2\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_3)$	$\{0.237, 0.708, 0.055, 0.818\}$	$\{0.002, -0.002, 0.000, 0.1\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_1)$	$\{0.135, 0.864, 0.001, 0.745\}$	$\{-0.001, 0.000, 0.001, 0.0\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_2)$	$\{0.243, 0.755, 0.001, 0.782\}$	$\{0.000, -0.001, 0.001, 0.1\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_3)$	$\{0.325, 0.632, 0.043, 0.809\}$	$\{0.002, -0.003, 0.001, 0.2\%\}$

Table 11: Detailed results for the first 40 of the 81 instances of Test bed 1 under the use of the Gamma process

Instance	Simulation $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$	Deviation $\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(C_1, \sigma_1, \lambda_1, \tau_1)$	$\{0.047, 0.953, 0.000, 0.360\}$	$\{-0.001, 0.001, 0.000, 0.0\%\}$
$(C_1, \sigma_1, \lambda_1, \tau_2)$	$\{0.095, 0.905, 0.000, 0.406\}$	$\{0.001, -0.001, 0.000, 0.0\%\}$
$(C_1, \sigma_1, \lambda_1, \tau_3)$	$\{0.132, 0.868, 0.000, 0.446\}$	$\{-0.001, 0.001, 0.000, 0.3\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_1)$	$\{0.092, 0.908, 0.000, 0.358\}$	$\{-0.002, 0.001, 0.000, -0.1\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_2)$	$\{0.176, 0.824, 0.000, 0.400\}$	$\{-0.001, 0.001, 0.000, -0.1\%\}$
$(C_1, \sigma_1, \lambda_2, \tau_3)$	$\{0.241, 0.759, 0.000, 0.432\}$	$\{-0.001, 0.001, 0.000, -0.1\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_1)$	$\{0.138, 0.861, 0.000, 0.357\}$	$\{0.002, -0.002, 0.000, 0.1\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_2)$	$\{0.246, 0.753, 0.000, 0.393\}$	$\{-0.003, 0.003, 0.000, -0.4\%\}$
$(C_1, \sigma_1, \lambda_3, \tau_3)$	$\{0.330, 0.669, 0.000, 0.421\}$	$\{-0.001, 0.001, 0.000, -0.2\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_1)$	$\{0.051, 0.949, 0.000, 0.371\}$	$\{0.002, -0.002, 0.000, -0.1\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_2)$	$\{0.092, 0.907, 0.000, 0.416\}$	$\{-0.001, 0.001, 0.000, 0.0\%\}$
$(C_1, \sigma_2, \lambda_1, \tau_3)$	$\{0.131, 0.868, 0.001, 0.457\}$	$\{-0.002, 0.003, -0.001, 0.1\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_1)$	$\{0.092, 0.908, 0.000, 0.370\}$	$\{-0.002, 0.001, 0.000, 0.2\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_2)$	$\{0.176, 0.824, 0.000, 0.412\}$	$\{0.000, 0.000, 0.000, 0.4\%\}$
$(C_1, \sigma_2, \lambda_2, \tau_3)$	$\{0.244, 0.755, 0.001, 0.446\}$	$\{0.001, 0.000, -0.001, 0.3\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_1)$	$\{0.137, 0.863, 0.000, 0.368\}$	$\{0.000, -0.001, 0.000, 0.1\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_2)$	$\{0.244, 0.756, 0.000, 0.406\}$	$\{-0.004, 0.004, 0.000, 0.1\%\}$
$(C_1, \sigma_2, \lambda_3, \tau_3)$	$\{0.338, 0.661, 0.001, 0.435\}$	$\{0.003, -0.003, 0.000, 0.2\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_1)$	$\{0.049, 0.951, 0.000, 0.385\}$	$\{0.001, -0.001, 0.000, 0.5\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_2)$	$\{0.094, 0.905, 0.001, 0.430\}$	$\{0.000, 0.001, -0.001, 0.4\%\}$
$(C_1, \sigma_3, \lambda_1, \tau_3)$	$\{0.136, 0.861, 0.003, 0.469\}$	$\{0.001, 0.003, -0.004, -0.1\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_1)$	$\{0.094, 0.906, 0.000, 0.382\}$	$\{0.000, 0.000, 0.000, 0.1\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_2)$	$\{0.177, 0.822, 0.001, 0.423\}$	$\{0.001, 0.000, -0.001, 0.0\%\}$
$(C_1, \sigma_3, \lambda_2, \tau_3)$	$\{0.251, 0.747, 0.003, 0.457\}$	$\{0.004, 0.000, -0.004, -0.2\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_1)$	$\{0.134, 0.865, 0.000, 0.380\}$	$\{-0.002, 0.002, 0.000, 0.0\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_2)$	$\{0.252, 0.747, 0.001, 0.417\}$	$\{0.003, -0.003, -0.001, -0.1\%\}$
$(C_1, \sigma_3, \lambda_3, \tau_3)$	$\{0.345, 0.653, 0.002, 0.448\}$	$\{0.005, -0.002, -0.003, 0.1\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_1)$	$\{0.045, 0.954, 0.000, 0.555\}$	$\{-0.003, 0.003, 0.000, -0.2\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_2)$	$\{0.095, 0.905, 0.000, 0.599\}$	$\{0.001, -0.001, 0.000, -0.4\%\}$
$(C_2, \sigma_1, \lambda_1, \tau_3)$	$\{0.131, 0.868, 0.001, 0.643\}$	$\{-0.003, 0.007, -0.003, 0.0\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_1)$	$\{0.093, 0.906, 0.000, 0.556\}$	$\{0.000, 0.000, 0.000, 0.2\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_2)$	$\{0.178, 0.821, 0.001, 0.595\}$	$\{0.002, -0.003, 0.000, -0.2\%\}$
$(C_2, \sigma_1, \lambda_2, \tau_3)$	$\{0.249, 0.751, 0.001, 0.632\}$	$\{0.003, 0.000, -0.003, 0.2\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_1)$	$\{0.135, 0.864, 0.001, 0.554\}$	$\{-0.001, 0.000, 0.001, 0.2\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_2)$	$\{0.240, 0.759, 0.001, 0.593\}$	$\{-0.008, 0.008, 0.000, 0.3\%\}$
$(C_2, \sigma_1, \lambda_3, \tau_3)$	$\{0.339, 0.661, 0.001, 0.622\}$	$\{0.001, 0.001, -0.002, 0.3\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_1)$	$\{0.049, 0.950, 0.001, 0.565\}$	$\{0.001, -0.001, 0.000, 0.0\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_2)$	$\{0.094, 0.906, 0.001, 0.607\}$	$\{0.000, 0.004, -0.004, -0.4\%\}$
$(C_2, \sigma_2, \lambda_1, \tau_3)$	$\{0.134, 0.865, 0.001, 0.653\}$	$\{-0.002, 0.019, -0.017, 0.2\%\}$
$(C_2, \sigma_2, \lambda_2, \tau_1)$	$\{0.093, 0.907, 0.001, 0.562\}$	$\{-0.001, 0.001, 0.000, -0.2\%\}$

Table 12: Detailed results for the last 41 of the 81 instances of Test bed 1 under the use of the Gamma process

Instance	Simulation $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}(C)\}$	Deviation $\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(C_2, \sigma_2, \lambda_2, \tau_2)$	$\{0.176, 0.824, 0.001, 0.605\}$	$\{0.000, 0.003, -0.003, 0.2\%\}$
$(C_2, \sigma_2, \lambda_2, \tau_3)$	$\{0.246, 0.753, 0.001, 0.641\}$	$\{-0.001, 0.015, -0.014, 0.2\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_1)$	$\{0.136, 0.863, 0.001, 0.561\}$	$\{0.000, 0.000, 0.000, -0.1\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_2)$	$\{0.250, 0.749, 0.000, 0.600\}$	$\{0.003, 0.000, -0.003, 0.2\%\}$
$(C_2, \sigma_2, \lambda_3, \tau_3)$	$\{0.343, 0.656, 0.001, 0.630\}$	$\{0.004, 0.008, -0.012, 0.1\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_1)$	$\{0.048, 0.952, 0.000, 0.572\}$	$\{-0.001, 0.003, -0.002, -0.3\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_2)$	$\{0.095, 0.904, 0.001, 0.618\}$	$\{0.002, 0.010, -0.012, 0.1\%\}$
$(C_2, \sigma_3, \lambda_1, \tau_3)$	$\{0.132, 0.863, 0.005, 0.660\}$	$\{-0.002, 0.031, -0.029, 0.1\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_1)$	$\{0.094, 0.906, 0.001, 0.571\}$	$\{0.000, 0.002, -0.002, -0.2\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_2)$	$\{0.176, 0.823, 0.001, 0.614\}$	$\{0.001, 0.009, -0.010, 0.3\%\}$
$(C_2, \sigma_3, \lambda_2, \tau_3)$	$\{0.244, 0.752, 0.004, 0.650\}$	$\{0.000, 0.025, -0.025, 0.4\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_1)$	$\{0.135, 0.864, 0.000, 0.570\}$	$\{-0.001, 0.003, -0.002, -0.1\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_2)$	$\{0.246, 0.753, 0.001, 0.608\}$	$\{-0.001, 0.010, -0.009, 0.1\%\}$
$(C_2, \sigma_3, \lambda_3, \tau_3)$	$\{0.339, 0.658, 0.003, 0.635\}$	$\{0.003, 0.018, -0.022, -0.2\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_1)$	$\{0.048, 0.951, 0.001, 0.754\}$	$\{0.000, 0.001, 0.000, 0.1\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_2)$	$\{0.091, 0.908, 0.000, 0.797\}$	$\{-0.002, 0.022, -0.020, -0.1\%\}$
$(C_3, \sigma_1, \lambda_1, \tau_3)$	$\{0.133, 0.826, 0.041, 0.840\}$	$\{0.001, 0.050, -0.051, 0.4\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_1)$	$\{0.091, 0.908, 0.001, 0.754\}$	$\{-0.003, 0.003, 0.000, 0.3\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_2)$	$\{0.176, 0.823, 0.001, 0.791\}$	$\{0.002, 0.015, -0.017, -0.1\%\}$
$(C_3, \sigma_1, \lambda_2, \tau_3)$	$\{0.244, 0.721, 0.035, 0.829\}$	$\{0.002, 0.039, -0.041, 0.4\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_1)$	$\{0.133, 0.866, 0.001, 0.748\}$	$\{-0.003, 0.003, 0.000, -0.3\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_2)$	$\{0.247, 0.752, 0.001, 0.787\}$	$\{0.000, 0.015, -0.015, 0.1\%\}$
$(C_3, \sigma_1, \lambda_3, \tau_3)$	$\{0.340, 0.637, 0.023, 0.816\}$	$\{0.007, 0.033, -0.040, 0.1\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_1)$	$\{0.048, 0.951, 0.001, 0.761\}$	$\{0.000, 0.008, -0.009, 0.4\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_2)$	$\{0.095, 0.904, 0.001, 0.805\}$	$\{0.003, 0.043, -0.047, 0.5\%\}$
$(C_3, \sigma_2, \lambda_1, \tau_3)$	$\{0.136, 0.802, 0.063, 0.843\}$	$\{0.008, 0.052, -0.059, 0.7\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_1)$	$\{0.091, 0.908, 0.001, 0.757\}$	$\{-0.002, 0.010, -0.008, 0.1\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_2)$	$\{0.182, 0.817, 0.002, 0.799\}$	$\{0.010, 0.032, -0.042, 0.4\%\}$
$(C_3, \sigma_2, \lambda_2, \tau_3)$	$\{0.253, 0.703, 0.044, 0.832\}$	$\{0.019, 0.041, -0.060, 0.6\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_1)$	$\{0.135, 0.865, 0.001, 0.754\}$	$\{-0.001, 0.008, -0.008, -0.1\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_2)$	$\{0.240, 0.759, 0.001, 0.793\}$	$\{-0.003, 0.041, -0.037, 0.2\%\}$
$(C_3, \sigma_2, \lambda_3, \tau_3)$	$\{0.342, 0.621, 0.037, 0.825\}$	$\{0.018, 0.034, -0.052, 0.9\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_1)$	$\{0.050, 0.949, 0.001, 0.764\}$	$\{0.002, 0.017, -0.019, 0.2\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_2)$	$\{0.094, 0.901, 0.006, 0.806\}$	$\{0.004, 0.060, -0.064, 0.2\%\}$
$(C_3, \sigma_3, \lambda_1, \tau_3)$	$\{0.133, 0.786, 0.081, 0.846\}$	$\{0.007, 0.053, -0.061, 0.7\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_1)$	$\{0.090, 0.909, 0.001, 0.763\}$	$\{-0.002, 0.021, -0.018, 0.3\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_2)$	$\{0.177, 0.818, 0.005, 0.798\}$	$\{0.007, 0.051, -0.058, -0.2\%\}$
$(C_3, \sigma_3, \lambda_2, \tau_3)$	$\{0.243, 0.693, 0.064, 0.839\}$	$\{0.012, 0.046, -0.059, 1.1\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_1)$	$\{0.136, 0.864, 0.001, 0.762\}$	$\{0.001, 0.016, -0.018, 0.3\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_2)$	$\{0.245, 0.751, 0.004, 0.799\}$	$\{0.005, 0.048, -0.053, 0.5\%\}$
$(C_3, \sigma_3, \lambda_3, \tau_3)$	$\{0.333, 0.616, 0.050, 0.828\}$	$\{0.015, 0.042, -0.057, 0.8\%\}$

Table 13: Detailed results for the 27 instances of Test bed 2 under the use of the random coefficient model (RCM) and Gamma process (GP).

Instance	RCM $\{(\hat{C}^* - C^*)/H, Gap1, Gap2\}$	GP $\{(\hat{C}^* - C^*)/H, Gap1, Gap2\}$
$(\sigma_1, \lambda_1, \tau_1)$	$\{-0.03\%, 0.06\%, 0.01\%\}$	$\{1.79\%, 2.36\%, 1.69\%\}$
$(\sigma_1, \lambda_1, \tau_2)$	$\{0.28\%, 0.25\%, 0.05\%\}$	$\{-0.26\%, 4.70\%, 0.07\%\}$
$(\sigma_1, \lambda_1, \tau_3)$	$\{0.35\%, 0.30\%, 0.01\%\}$	$\{-3.82\%, 4.80\%, 0.72\%\}$
$(\sigma_1, \lambda_2, \tau_1)$	$\{-0.03\%, 0.11\%, 0.02\%\}$	$\{1.54\%, 1.69\%, 1.20\%\}$
$(\sigma_1, \lambda_2, \tau_2)$	$\{0.28\%, 0.24\%, 0.24\%\}$	$\{-1.12\%, 4.25\%, 0.04\%\}$
$(\sigma_1, \lambda_2, \tau_3)$	$\{6.57\%, 0.09\%, 0.41\%\}$	$\{-1.83\%, 4.86\%, 0.54\%\}$
$(\sigma_1, \lambda_3, \tau_1)$	$\{-0.03\%, 0.24\%, 0.07\%\}$	$\{1.29\%, 1.43\%, 1.08\%\}$
$(\sigma_1, \lambda_3, \tau_2)$	$\{0.28\%, 0.26\%, 0.10\%\}$	$\{-0.64\%, 3.44\%, 0.60\%\}$
$(\sigma_1, \lambda_3, \tau_3)$	$\{-0.88\%, 0.27\%, 0.17\%\}$	$\{-3.57\%, 4.93\%, 0.19\%\}$
$(\sigma_2, \lambda_1, \tau_1)$	$\{0.58\%, 0.06\%, 0.19\%\}$	$\{1.32\%, 1.75\%, 0.39\%\}$
$(\sigma_2, \lambda_1, \tau_2)$	$\{-0.50\%, 0.01\%, 0.10\%\}$	$\{-1.47\%, 4.61\%, 1.35\%\}$
$(\sigma_2, \lambda_1, \tau_3)$	$\{0.35\%, 0.11\%, 0.04\%\}$	$\{-4.59\%, 6.76\%, 1.74\%\}$
$(\sigma_2, \lambda_2, \tau_1)$	$\{-0.20\%, 0.03\%, 0.09\%\}$	$\{1.02\%, 1.05\%, 0.36\%\}$
$(\sigma_2, \lambda_2, \tau_2)$	$\{-0.50\%, 0.08\%, 0.15\%\}$	$\{-2.39\%, 4.27\%, 0.96\%\}$
$(\sigma_2, \lambda_2, \tau_3)$	$\{-0.96\%, 0.63\%, 0.06\%\}$	$\{-5.14\%, 6.34\%, 1.44\%\}$
$(\sigma_2, \lambda_3, \tau_1)$	$\{1.36\%, 0.12\%, 0.11\%\}$	$\{0.74\%, 0.88\%, 0.40\%\}$
$(\sigma_2, \lambda_3, \tau_2)$	$\{0.28\%, 0.12\%, 0.16\%\}$	$\{-1.94\%, 3.65\%, 1.17\%\}$
$(\sigma_2, \lambda_3, \tau_3)$	$\{-0.18\%, 0.64\%, 0.05\%\}$	$\{-5.45\%, 6.52\%, 1.50\%\}$
$(\sigma_3, \lambda_1, \tau_1)$	$\{-0.20\%, 0.01\%, 0.10\%\}$	$\{0.29\%, -0.55\%, 0.38\%\}$
$(\sigma_3, \lambda_1, \tau_2)$	$\{3.72\%, 0.24\%, 0.33\%\}$	$\{-4.30\%, 3.18\%, 2.73\%\}$
$(\sigma_3, \lambda_1, \tau_3)$	$\{1.48\%, 0.25\%, 0.20\%\}$	$\{-6.65\%, 7.13\%, 3.93\%\}$
$(\sigma_3, \lambda_2, \tau_1)$	$\{-0.20\%, 0.35\%, 0.21\%\}$	$\{-1.32\%, -0.26\%, 0.20\%\}$
$(\sigma_3, \lambda_2, \tau_2)$	$\{-0.50\%, 0.14\%, 0.16\%\}$	$\{-3.92\%, 2.45\%, 2.45\%\}$
$(\sigma_3, \lambda_2, \tau_3)$	$\{-0.96\%, 0.25\%, 0.05\%\}$	$\{-4.52\%, 6.90\%, 2.79\%\}$
$(\sigma_3, \lambda_3, \tau_1)$	$\{-0.20\%, 0.02\%, 0.10\%\}$	$\{-1.61\%, 0.06\%, 0.43\%\}$
$(\sigma_3, \lambda_3, \tau_2)$	$\{0.28\%, 0.02\%, 0.09\%\}$	$\{-3.47\%, 1.80\%, 2.58\%\}$
$(\sigma_3, \lambda_3, \tau_3)$	$\{0.60\%, 0.29\%, 0.04\%\}$	$\{-7.33\%, 6.33\%, 1.76\%\}$

Table 14: Detailed results for the cost savings percentages $\triangle_0$, $\triangle_1$, $\triangle_2$ for all 27 instances of Test bed 2 and under two fitting options for the degradation process

Ω	$\triangle_0$		$\triangle_1$		$\triangle_2$	
	RCM	GP	RCM	GP	RCM	GP
$(\sigma_1, \lambda_1, \tau_1)$	35.3%	31.4%	31.6%	25.9%	5.1%	8.8%
$(\sigma_1, \lambda_1, \tau_2)$	30.9%	27.4%	27.0%	20.6%	5.1%	8.8%
$(\sigma_1, \lambda_1, \tau_3)$	25.0%	24.0%	11.6%	16.5%	5.1%	8.8%
$(\sigma_1, \lambda_2, \tau_1)$	35.0%	31.2%	31.6%	25.9%	8.9%	11.8%
$(\sigma_1, \lambda_2, \tau_2)$	30.2%	27.1%	27.0%	20.6%	8.9%	11.8%
$(\sigma_1, \lambda_2, \tau_3)$	23.0%	23.7%	11.6%	16.5%	8.9%	11.8%
$(\sigma_1, \lambda_3, \tau_1)$	34.7%	31.0%	31.6%	25.9%	11.7%	14.0%
$(\sigma_1, \lambda_3, \tau_2)$	29.6%	26.9%	27.0%	20.6%	11.7%	14.0%
$(\sigma_1, \lambda_3, \tau_3)$	23.1%	23.6%	11.6%	16.5%	11.7%	14.0%
$(\sigma_2, \lambda_1, \tau_1)$	34.7%	30.1%	29.7%	26.4%	5.0%	7.5%
$(\sigma_2, \lambda_1, \tau_2)$	29.0%	26.2%	23.0%	20.7%	5.0%	7.5%
$(\sigma_2, \lambda_1, \tau_3)$	24.0%	23.1%	14.6%	15.4%	5.0%	7.5%
$(\sigma_2, \lambda_2, \tau_1)$	34.4%	29.9%	29.7%	26.4%	8.7%	10.7%
$(\sigma_2, \lambda_2, \tau_2)$	28.6%	26.0%	23.0%	20.7%	8.7%	10.7%
$(\sigma_2, \lambda_2, \tau_3)$	22.7%	23.0%	14.6%	15.4%	8.7%	10.7%
$(\sigma_2, \lambda_3, \tau_1)$	34.1%	29.8%	29.7%	26.4%	11.5%	13.1%
$(\sigma_2, \lambda_3, \tau_2)$	28.2%	25.9%	23.0%	20.7%	11.5%	13.1%
$(\sigma_2, \lambda_3, \tau_3)$	22.7%	23.0%	14.6%	15.4%	11.5%	13.1%
$(\sigma_3, \lambda_1, \tau_1)$	34.3%	29.7%	29.5%	27.9%	4.8%	6.4%
$(\sigma_3, \lambda_1, \tau_2)$	28.4%	25.8%	20.2%	21.9%	4.8%	6.4%
$(\sigma_3, \lambda_1, \tau_3)$	23.7%	22.9%	16.2%	12.2%	4.8%	6.4%
$(\sigma_3, \lambda_2, \tau_1)$	34.0%	29.6%	29.5%	27.9%	8.5%	9.8%
$(\sigma_3, \lambda_2, \tau_2)$	28.1%	25.7%	20.2%	21.9%	8.5%	9.8%
$(\sigma_3, \lambda_2, \tau_3)$	23.1%	22.9%	16.2%	12.2%	8.5%	9.8%
$(\sigma_3, \lambda_3, \tau_1)$	33.7%	29.5%	29.5%	27.9%	11.3%	12.4%
$(\sigma_3, \lambda_3, \tau_2)$	27.8%	25.6%	20.2%	21.9%	11.3%	12.4%
$(\sigma_3, \lambda_3, \tau_3)$	23.0%	23.0%	16.2%	12.2%	11.3%	12.4%