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An age-based maintenance policy using the opportunities of scheduled and unscheduled system downs

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Abstract

The coordination of maintenance tasks for complex systems with a large number of components is a challenging optimization problem. Many opportunistic maintenance policies have been proposed in the literature to reduce fixed setup costs of maintenance and downtime costs. Most of the opportunistic maintenance policies treat the unscheduled downs from some components as the opportunities to perform preventive maintenance for other components that are approaching to failures. Yet the scheduled downs from periodic inspections may be used as the opportunities to perform preventive maintenance for components. Therefore, in this research we propose a new opportunistic maintenance model to determine the optimal age limits of components by considering not only the opportunities of unscheduled downs but also the opportunities of scheduled downs, which will further reduce the total cost rate of maintenance. An approximation method is proposed for evaluating the total cost rate for a single component, as well as an heuristic approach to optimize the multi-component model. A numerical study is provided to verify the accuracy of our approximation method. Moreover, the cost-saving potential of our model is demonstrated.

***keywords*:** Multi-component systems, opportunistic maintenance, approximation, heuristic optimization

1. Introduction

Complex engineering systems are widely used in the production of goods and delivery of services nowadays, e.g., lithography machines in the semiconductor industry, automatic cutting systems in the food-processing industry, baggage handling systems in the airports. These advanced capital goods increase the efficiencies of production or service dramatically. However, for systems with a large number of components and complex system structure, scheduling maintenance tasks is challenging. If no strong dependence exists among all the different components, single-component maintenance models can be independently applied to each component in order to obtain optimal replacement schedule. But most of the time, there are economic dependence, structural dependence or stochastic dependence [3, 7, 10, 18, 22] among the different components

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of a system, which make the optimization of maintenance schedule complicated. In this paper, we mainly focus on the economic dependence of components, for which the joint maintenance of several components can save the fixed setup cost of maintenance and total downtime. The fixed setup cost refers to a fixed cost that is incurred for a maintenance visit regardless of what maintenance actions are performed [25].

Opportunistic maintenance policies are one type of methods to solve this maintenance problem of multi-component systems with economic dependence [22]. However, most of the previous works on opportunistic maintenance policies only considered the use of one type of opportunities in the maintenance models, i.e., the unscheduled downs of a system. The scheduled downs of a system, e.g., periodic inspections, have rarely been treated as the opportunities for joint maintenance. This may be because the durations of scheduled downs are relatively short compared with the corrective maintenance periods, which makes it less attractive to use the scheduled downs as opportunities if we aim to reduce downtime. But under some circumstances, the fixed setup cost of maintenance can be significant. For example, some lithography machines require vacuum environments and the setups after system shutdowns are costly; some wind farms or solar farms are far away from the service centers and the costs of sending maintenance teams to the sites can be expensive. For these cases, it can be beneficial to take the opportunities of both the scheduled and unscheduled downs to jointly perform preventive maintenance tasks of components. In this research, we explore the potential of combining different types of opportunities for the joint maintenance of different components in a system and propose a new optimization model to determine the age limits of opportunistic maintenance.

There are many age/time-based models proposed for multi-component systems considering the economic dependence. Radner and Jorgenson [14] introduced an (n, N) policy, which distinguished two types of components, 0 and 1. The parameter n is the age threshold for opportunistic replacements of component 0 when component 1 fails and N is the preventive replacement threshold of component 0 when component 1 is good. Vergin [21] showed that the (n, N) policy is near-optimal with respect to a wide range of cost parameters. Some exact methods [8, 11] (e.g., via Markovian framework) for finding the optimal solution are intractable for systems with large amounts of components, due to the exponentially increasing state spaces. Hence, various heuristics were proposed to reduce the computational complexity [2, 19, 20]. As different approaches, one can make a decision on taking either the current opportunity or the next opportunity after X time units, based upon a marginal cost function and the distribution of time between maintenance opportunities [5, 6]. Dagpunar[4] pointed out these policies [5, 6] were not appropriate if the mean residual lifetimes of components are high. Instead a policy with a control limit on age is proposed, by assuming the opportunity process is Poisson. Moreover, Zheng [24] introduced a $(T - w, T)$ policy. If the ages of components exceed T, preventive maintenance actions will be taken, which are also considered to be opportunities to preventively replace other components with their ages between $T - w$ and T. This policy is similar to the (n, N) policy, but it was developed by renewal theory.

Wildeman *et al.* [23] developed a dynamic clustering method (often called the "group maintenance") to coordinate maintenance tasks at the system level, by considering the penalty cost of deviating from the optimal maintenance schedule of individual components. The optimal policy structure is proved by specifying the expected deterioration cost function based on a Weibull process, which reduced the complexity of the large-scale optimization problem. Unlike Wildeman *et al.* 's work, Laggoune *et al.* [9] developed a dynamic clustering model based on simulation. In this model, preventive maintenance is scheduled at each fixed time point $k\tau$, with an interval τ and $k \in \mathbb{N}$. Each component j can only be preventively replaced at a multiple of τ, $k_j\tau (k_j \in \mathbb{N})$. If unscheduled system downs occur, a decision on taking the opportunities or not will be made, according to the marginal costs. As an extension of age-based and block replacement policy, Berg and Epstein introduced a (b, t) model, where t is a fixed maintenance interval and b is a control limit in terms of age. At each point $nt, n \in \mathbb{N}$, preventive maintenance is performed on the components whose ages are larger than b.

Regarding k-out-of-n systems, Popova and Wilson [13] provided a (k, T) policy. This policy suggests the replacement of all components either at the time of the kth failure or time T, whichever occurs first. As an extension, Pham and Wang [12] proposed a (τ, T) policy. According to this policy, no preventive maintenance is taken and only minimum repairs are performed on failures in the period $(0, \tau]$. In the period $(\tau, T]$, if k components fail, those k components are replaced and all other components are preventively replaced; if the failed components are less than k, all components are preventively replaced at time point T. Similarly, a generalized group maintenance policy $(T, T + w, k)$ was introduced [16], which also includes k failures as a decision variable. In the period $(0, T]$, this policy distinguishes two types of failures: i) "minor" failures that will be fixed by minimum repair and ii) "catastrophic" failures that will be fixed by replacements. In the period $(T, T + w]$, if k "catastrophic" failures happen, all components are jointly replaced; otherwise, this joint maintenance will be delayed till $T + w$.

Recently, Taghipour and Banjevic [17] proposed a model that considers both scheduled inspection and unscheduled hard failures of systems as opportunities to perform inspections on soft-failure components. In their work, soft-failure components are under periodic and opportunistic inspections, while hard-failure components are under periodic inspections. A simulation-based algorithm was created to evaluate the expected cost per cycle.

Different from existing research on opportunistic maintenance policies, we consider both scheduled and unscheduled system downs as opportunities for preventive maintenance. The opportunities from unscheduled system downs are due to random failures of other components in the system, whereas the opportunities from scheduled downs are because of periodic inspections. It is important to consider both the scheduled and unscheduled system downs as opportunities for joint maintenance when the fixed setup costs of maintenance are high. The high setup costs of maintenance will be reduced further by considering multiple types of opportunities together in opportunistic maintenance models, compared with using only one type of opportunities. We develop an efficient and accurate approximation method for evaluating the total cost rate. In the literature, the cost rate evaluation using renewal theory is usually based on the assumption that scheduled downs will be planned by starting from the most recent renewal point. This assumption will simplify the modeling. However, in practice, once the scheduled downs are planned, it will not be rescheduled after maintenance actions. Hence, we relax this assumption and propose an iterative approximation method for the cost rate evaluation. The opportunistic maintenance model proposed by us is aimed at constructing a optimization model for multi-component systems. Via an iterative procedure, we demonstrate that the single-component model is a building block of the multi-component model and one can find a heuristic solution in a relatively short computation time for the multi-component model.

The outline of this paper is as follows. The description of the model and the assumptions are given in Section 2. The details of the approximation method are explained in Section 3. In Section 5.1, a numerical example is given, where our opportunistic maintenance policy is compared with a failure-based maintenance policy. Moreover, in Section 5, numerical experiments are performed to investigate the accuracy of our approximate evaluation and the cost-saving potential under various parameter settings. To demonstrate that our model is a building block to solve multi-component problems, we provide an example of a system consisting of 20 components in Section 6. Finally, the conclusions are given in Section 7.

2. Model description

Consider a system consisting of multiple different components, denoted by set $I = \{1, ..., |I|\}$. A single component $i \in I$ is subject to opportunistic maintenance policy in this multi-component system. The interval of scheduled downs (SDs) τ is given as a decision variable in this maintenance policy. The arrivals of unscheduled downs (USDs) are assumed to be a Poisson process with rate λ_i. The USDs are caused by other components $I \setminus i$ in the system. Preventive maintenance actions at USDs and SDs can be performed on this component i:

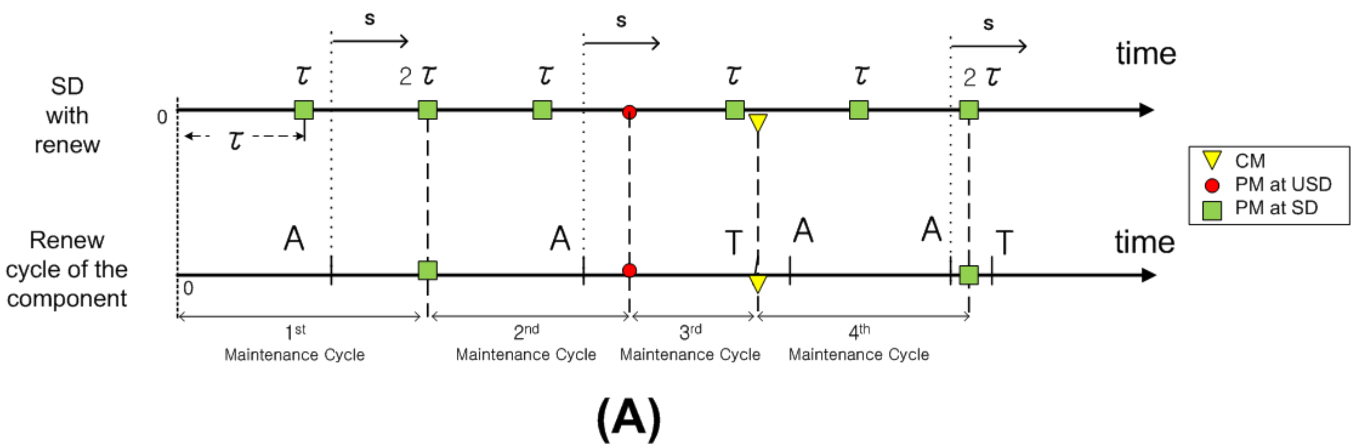
- *Preventive Maintenance at an USD (PM-USD)*: when the system stops due to an USD which is caused by other components $I \setminus i$ in the system, it is an opportunity for the component i to be maintained together with the failed components. If this opportunity is taken, a cost c_i^{PM-USD} will be incurred which includes the repair cost of the component i and the cost of the extra downtime caused by maintaining this component i. But the fixed setup cost and part of the downtime cost caused by this component i can be saved by conducting its maintenance action together with other components.

- *Preventive Maintenance at a SD (PM-SD)*: when the system stops due to a SD, it is an opportunity for the component i to be maintained without paying the fixed setup cost. If this opportunity is taken, a cost c_i^{PM-SD} will be incurred which includes the repair cost of the component i and the additional downtime cost caused by this component i.

Suppose the life time of this component T_i follows a certain distribution with a p.d.f. $f_i(t)$. If the random failures of this component i occur, corrective maintenance (CM) actions should be taken with a cost c_i^{CM}, which consists of not only the repair cost of the component i, but also the fixed setup cost and entire downtime cost. Notice that c_i^{PM-USD} and c_i^{PM-SD} are much smaller than c_i^{CM}. In other words, if the component is maintained at an opportunity, instead of correctively maintained by itself, the fixed setup cost and downtime cost of this component i can be partially saved from the system's viewpoint.

Although there are advantages of taking opportunities for preventive maintenance before unexpected failures, the maintenance cost rate can be increased if we take opportunities too frequently since the usage lifetime of the component can be wasted. Hence, we have to make decisions on the timing of taking opportunities for preventive maintenance actions, in order to minimize the total cost rate of the system. An age limit A_i is introduced as a decision variable to specify our opportunistic maintenance policy: if an opportunity at SD or USD appears at time $t < A_i$, do nothing at this opportunity; if an opportunity at SD or USD appears at time $t \geq A_i$, take this opportunity to do preventive maintenance (if a random failure of this component i occurs, perform corrective maintenance).

We assume that the maintenance actions can restore the component to a state as good as new, and the lifetime of the component is independent of the SDs and USDs caused by other components in the system. The service life of the system is assumed to be much longer than the lifetime of the component, and the time of maintenance is negligible compared with the lifetime of the component. Based on these assumptions, renewal theory is often applied to evaluate the long-run average cost rate of the component. The interval between two consecutive maintenance actions is a renewal cycle, which is also called *maintenance cycle* in Figure 1. However, in practice, the scheduled downs are planned at fixed time points in advance (see Figure 1-(B)), and can not be changed according to the maintenance actions of the component. The renewal property does not hold if scheduled downs (with interval τ) do not restart scheduling at the end of each maintenance cycle as shown in Figure 1-(A). Hence, the evaluation of average cost rate by renewal theory is not accurate from this perspective. We develop an approximation method to improve the cost rate evaluation for the case in which the scheduled downs are at fixed time points, as introduced in Section 3.



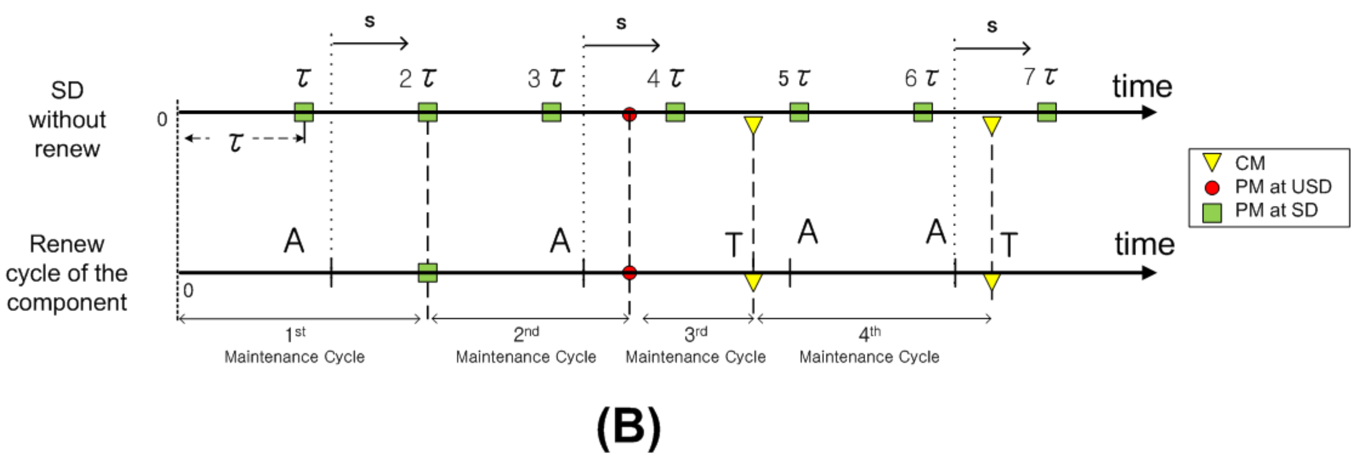


Figure 1: The three maintenance actions of a component in the context of renewal theory (A) and practical situation (B)

This single-component model for opportunistic maintenance can be used as a building block to construct an opportunistic maintenance model for multi-component systems. The system consists of multiple different components, denoted by set $I = \{1, ..., |I|\}$. The lifetime of component $i \in I, T_i$, has the p.d.f. $f_i(t)$. To specify the age limits $\boldsymbol{A} = \{A_1, ..., A_{|I|}\}$ for all the components in the system, we not only need to know the lifetime distributions of all the components, but also need to determine the interval of scheduled downs τ

and estimate the arrival rate of unscheduled downs $\lambda_i(\forall i \in I)$. The interval of scheduled downs τ can be a decision variable in the optimization model of opportunistic maintenance for multi-component systems. The arrival rate of unscheduled downs λ_i is determined by the processes of random failures from all the other components $I \setminus i$, which is dependent on the lifetimes of components $I \setminus i$ and the age limits $\boldsymbol{A} \setminus A_i$. It is interesting to notice that the changes of the age limits of components $I \setminus i$ will influence the processes of random failures, e.g., if the age limit of a component $j \in \{I \setminus i\}$ decreases, it is more likely that the component j will take a preventive maintenance action and thus less random failures will be generated by this component j. Therefore, the parameter $\lambda_i(\forall i \in I)$ should be estimated in an iterative manner while optimizing the age limits $\boldsymbol{A}$. We propose a heuristic approach in Section 4 to choose the age limits of components for minimizing the total cost rate.

The total cost rate of the system is thus given as

$$Z_{syst}(\tau, \mathbf{A}) = \frac{S^{SD}}{\tau} + \sum_{i \in I} Z_i(\tau, \mathbf{A}), \tag{1}$$

where S^{SD} is the fixed setup cost of maintenance, and $Z_i(\tau, \mathbf{A})$ is the cost rate of maintenance incurred by component i. The cost rate of component i, $Z_i(\tau, \mathbf{A})$, includes the opportunistic maintenance cost and corrective maintenance cost of component i. It is dependent on τ and all the age limits $\boldsymbol{A}$, as we discussed. The optimal solution $(\tau^*, \mathbf{A}^*)$ can be obtained by minimizing Equation 1. The difficulties of solving this optimization problem arise from the evaluation of $Z_i(\tau, \mathbf{A})$ and the optimization procedure coupled with it, which will be introduced in Section 3 and 4 respectively.

2.1 Notation

τ : interval of scheduled downs
λ : arrival rate of unscheduled downs
A : age limit for opportunistic maintenance
$Z(.)$: average cost rate of maintenance
c^{PM-USD} : PM cost at an USD
c^{PM-SD} : PM cost at a SD
c^{CM} : CM cost

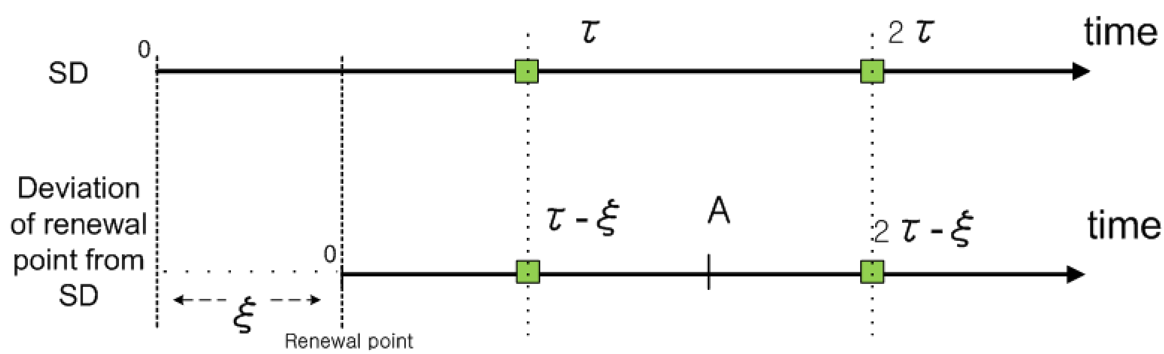


Figure 2: The deviation of the renewal point ξ

3. Approximation of single-component model

Suppose the interval of scheduled downs τ and the arrival rate of unscheduled downs λ_i are given. We propose an approximate method to evaluate the cost rate of component $i \in I$, $Z_i(.)$ (notice that since the arrival rate of unscheduled downs λ_i is given the cost rate of component i is not dependent on all the age limits $\boldsymbol{A}$ anymore). The renewal property is violated since scheduled downs do not restart scheduling at the end of each maintenance cycle as shown in Figure 1. A maintenance cycle will start at ξ time units away from the previous SD ($0 \leq \xi < \tau$), as shown in Figure 2. This deviation of a renewal point from SDs, ξ, is changing from maintenance cycle to maintenance cycle. But given this deviation ξ of a certain maintenance cycle, we can derive the cycle cost and cycle length by analyzing the renewal events. For instance, if a failure occurs before A_i, a corrective maintenance action will be taken. The corresponding probability is

$$Pr\Big\{T_i < A_i\Big\} = \int_0^{A_i} f_i(u)\ du. \tag{2}$$

If the lifetime of component i is larger than the age limit A_i but smaller than the arrival time of the first SD after A_i (i.e., $n_i\tau - \xi$, where $n_i = \lceil \frac{A_i+\xi}{\tau} \rceil$), the component will not be able to be preventively maintained at a SD. A preventive maintenance action will be taken upon the arrival of the first USD after A_i; or the component will be correctively maintained if there's no USDs before the component fails. In this case, the probability of PM-USD is

$$Pr\Big\{A_i < T_i < n_i\tau - \xi, \text{an USD appears before } T_i\Big\} = \int_{A_i}^{n_i\tau-\xi} (1 - e^{-\lambda_i(u-A_i)})\ f_i(u)\ du, \tag{3}$$

where $(1 - e^{-\lambda_i(u-A_i)})$ is the probability that the first USD after A_i appears before T_i. The probability of CM in this case is

$$Pr\Big\{A_i < T_i < n_i\tau - \xi, \text{an USD does not appear before } T_i\Big\} = \int_{A_i}^{n_i\tau-\xi} e^{-\lambda_i(u-A_i)}\ f_i(u)\ du. \tag{4}$$

If the lifetime of component i is larger than the arrival time of the first SD after $A_i (i.e., n_i\tau - \xi)$, the component will be preventively maintained at a SD if there's no USDs before $n_i\tau - \xi$. The probability of PM-SD in this case is

$$Pr\Big\{T_i > n_i\tau - \xi, \text{an USD does not appear before } n_i\tau - \xi\Big\} = \int_{n_i\tau-\xi}^{\infty} e^{-\lambda_i(n_i\tau-\xi-A_i)}\ f_i(u)\ du, \tag{5}$$

where $e^{-\lambda_i(n_i\tau-\xi-A_i)}$ is the probability that the first USD after A_i does not appear before $n_i\tau - \xi$. Or a preventive maintenance action will be taken upon the arrival of the first USD after A_i if the arrival of the first USD is before the arrival of the first SD. In this case, the probability of PM-USD is

$$Pr\Big\{T_i > n_i\tau - \xi, \text{an USD appears before } n_i\tau - \xi\Big\} = \int_{n_i\tau-\xi}^{\infty} (1 - e^{-\lambda_i(n_i\tau-\xi-A_i)})\ f_i(u)\ du. \tag{6}$$

To summarize, under our opportunistic maintenance policy for component i, the occurrence of the three possible maintenance actions at the end of a maintenance cycle depends on which event happens first, the failure of component i, the first SD after A_i, or the first USD after A_i. The probability of PM-USD conditioned on the deviation ξ, $\dot{P}_1(\xi)$, is the sum of Equation 3 and Equation 6 ; the probability of PM-SD conditioned on ξ, $\dot{P}_2(\xi)$, is equal to Equation 5 ; the probability of CM conditioned on ξ, $\dot{P}_3(\xi)$, is the sum of Equation 2 and Equation 4. (Notice that the sum of $\dot{P}_1(\xi)$, $\dot{P}_2(\xi)$ and $\dot{P}_3(\xi)$ is equal to one.)

Based on the above analysis of renewal events, the expected cycle length of component i conditioned on ξ can be obtained by

$$\begin{aligned}
\dot{L}_i(A_i|\xi) &= \int_0^{A_i} u f_i(u) du \\
&+ \int_{A_i}^{n_i\tau-\xi} \left(\int_0^{u-A_i} (A_i+s)\lambda_i e^{-\lambda_i s} ds + u e^{-\lambda_i (u-A_i)} \right) f_i(u) du \\
&+ \int_{n_i\tau-\xi}^{\infty} \left(\int_0^{n_i\tau-\xi-A_i} (A_i+s)\lambda_i e^{-\lambda_i s} ds + (n_i\tau-\xi) e^{-\lambda_i (n_i\tau-\xi-A_i)} \right) f_i(u) du, \qquad (7)
\end{aligned}$$

where $\lambda_i e^{-\lambda_i s}$ is the p.d.f. of the arrival time of the first USD after A_i. As mentioned previously, the deviation of ξ is changing from maintenance cycle to maintenance cycle. We use a random variable Δ to describe the distribution of the deviations among all the maintenance cycles. The distribution of Δ depends on the renewal events at the end of every maintenance cycle. If a PM-SD action is taken, the ξ for the next maintenance cycle will be equal to 0. If a PM-USD action or a CM action is taken, the ξ for the next maintenance cycle can be any possible value in $(0, \tau)$. Since the arrivals of USDs are assumed to follow a Poisson process, the ξ can take any value in $(0, \tau)$ with equal chances, given that a PM-USD action is taken at the end of the previous maintenance cycle. Furthermore, if we assume that the intervals of SDs are relatively small compared with the average value of the lifetime T_i, the ξ will also be approximately evenly-distributed over $(0, \tau)$, given that a CM action is taken at the end of the previous maintenance cycle. Hence, the following probability density function can be used to approximately describe the random variable Δ,

$$f_\Delta(\xi) = \begin{cases} q, & \text{if } \xi = 0, \\ \frac{(1-q)}{\tau}, & \text{if } 0 < \xi < \tau, \\ 0, & \text{otherwise.} \end{cases} \qquad (8)$$

We can see that Δ is uniformly-distributed on $(0, \tau)$ except that there is a positive probability mass q at $\xi = 0$. After specifying the probability density function $f_\Delta(\xi)$ (or equivalently the parameter q), the unconditional probabilities of PM-USD, PM-SD and CM (denoted by P_1, P_2, and P_3 respectively) can be obtained by the law of total probability,

$$\begin{aligned}
P_1 &= \dot{P}_1(0)q + \int_0^\tau \dot{P}_1(\xi)\frac{(1-q)}{\tau}\, d\xi, \\
P_2 &= \dot{P}_2(0)q + \int_0^\tau \dot{P}_2(\xi)\frac{(1-q)}{\tau}\, d\xi, \\
P_3 &= \dot{P}_3(0)q + \int_0^\tau \dot{P}_3(\xi)\frac{(1-q)}{\tau}\, d\xi.
\end{aligned} \qquad (9)$$

Obviously, the accuracy of the approximation method is dependent on the selection of the probability density function $f_\Delta(\xi)$ or the parameter q. Notice that the parameter q represents the probability that a PM-SD action is taken at the end of the previous maintenance cycle, which should be equal to the unconditional probability of PM-SD, P_2. Hence, the parameter q can be determined by solving the second equation in

Equation 9, which is given as

$$q = \frac{\int_0^\tau \dot{P}_2(\xi)\ d\xi}{\tau - \tau\dot{P}_2(0) + \int_0^\tau \dot{P}_2(\xi)\ d\xi}. \tag{10}$$

The expected cycle length of component i without the condition can be derived by

$$L_i(A_i) = \dot{L}_i(A_i|\xi = 0)q + \int_0^\tau \dot{L}_i(A_i|\xi)\frac{(1-q)}{\tau}\ d\xi. \tag{11}$$

The expected cycle cost of component i can be obtained by

$$K_i(A_i) = P_1\ c_i^{PM-USD} + P_2\ c_i^{PM-SD} + P_3\ c_i^{CM}. \tag{12}$$

According to renewal theory [15], the expected maintenance cost rate of component i, $Z_i(A_i)$, is equal to $K_i(A_i)/L_i(A_i)$. Therefore, given the interval of scheduled downs τ and the arrival rate of unscheduled downs λ_i, we can approximately evaluate the expected cost rate of component i,$Z_i(A_i)$, by Equation 11 and 12.

The expected cost rate $Z_i(A_i)$ is not a continuous function with respect to A_i, since P_1, P_2, P_3 and L_i are not continuous functions with respect to A_i. This is due to the fact that in Equation 3,4,5,6 and 7, the variable n_i is the ceiling function of $(A_i + \xi)/\tau$, i.e., $n_i = \lceil \frac{A_i+\xi}{\tau} \rceil$, which is discontinuous at the values of A_i that are in the set $\{A_i|(A_i + \xi)/\tau \in \mathbb{N}\}$, for a given ξ; then $\dot{P}_2(0)$ and $\dot{P}_2(\xi)$ are discontinuous with respect to A_i; since the unconditional probability of PM-SD action P_2 (or equivalently q) is obtained by solving the second equation in Equation 9, q or P_2 is discontinuous with respect to A_i (notice that the integrals in Equation 9 are improper integrals [1]); therefore, P_1, P_3, L_i are not continuous functions with respect to A_i. Difficulties thus arise in the search of optimal solutions for A_i.

4. Heuristic approach for optimization

The optimal solution $(\tau^*, \mathbf{A}^*)$ for a system consisting of multiple components $\{1, ..., |I|\}$ can be obtained by minimizing Equation 1. But there are two difficulties for this optimization problem. One is that the evaluation of the cost rate $Z_i(.), \forall i \in I$, needs an approximation method as discussed in Section 3. The other one is that the optimization problem is not decomposable with respect to $\sum_{i\in I} Z_i(\tau, \mathbf{A})$. This is because the evaluation of $Z_i(.)$ needs the arrival rate of unscheduled downs λ_i (see Section 3), which is dependent on the age limits $\boldsymbol{A} \setminus A_i$. As discussed in Section 2, the age limits $\boldsymbol{A}$ together with the lifetimes $\{T_1, ..., T_{|I|}\}$ determine the arrival process of USDs which will be used in the approximation method (proposed in Section 3) to evaluate the cost rate of $Z_i(.), \forall i \in I$. Hence, it is not possible to minimize $\sum_{i\in I} Z_i(\tau, \mathbf{A})$ by minimizing $Z_i(\tau, \mathbf{A})$ $(\forall i \in I)$ separately.

We develop a heuristic approach based on our approximation method for optimization. In this heuristic approach, we choose the values of $(\tau, \mathbf{A})$ in a nested way. For a given τ, we search for the age limits $\tilde{\mathbf{A}}$ that minimize $\sum_{i\in I} Z_i(\tau, \mathbf{A})$ by an iterative procedure. Then we specify the value of τ to minimize $Z_{syst}(\tau, \tilde{\mathbf{A}})$ by enumeration.

Algorithm

Step 1 For every $\tau \in (\tau^{LB}, \tau^{UB})$, evaluate and minimize the cost rate of $Z_i(\tau, \tilde{\mathbf{A}})$.($(\tau^{LB}, \tau^{UB})$ is the range of search for τ)

Step 1.1 Initiation ($k = 1$): $\forall i \in I$, set $A_i = \infty$, and calculate the arrival rate of USDs from component i at the first iteration, $\theta_{i,1} = 1/L_{i,1}$, where $L_{i,k}$ is the expected cycle length of component i at the first iteration. ($L_{i,1}$ equals the expected lifetime of component i when $A_i = \infty$ since there's no preventive maintenance actions ; $\theta_{i,1}$ refers to the USDs generated by component i, whereas λ_i refers to the USDs generated by components $I \setminus i$)

Step 1.2 Repeat the following iteration with the updated $\theta_{i,k}$ until $|\theta_{i,k} - \theta_{i,k-1}| < \varepsilon$ for all components :

- For all $i \in I$: By using the rate of USDs $\lambda_i = \sum_{j \in \{I \setminus i\}} \theta_{j,k}$, we can find optimal age limit $A^*_{i,k}$ to minimize $Z_i(.)$ in a certain range $({A_i}^{LB}, {A_i}^{UB})$ (by enumeration), and update

$$\theta_{i,k+1} = P^3_{i,k}(\tau, A^*_{i,k})/L_{i,k}(\tau, A^*_{i,k}),$$

where $P^3_{i,k}(\tau, A^*_{i,k})$ is the probability of CM (see Equation 9 and 11 in Section 3).

- Let $k := k + 1$.

Step 1.3 Obtain the $\tilde{\mathbf{A}} = \{A^*_{i,k}, \ \forall i \in I\}$ for each τ ; $Z_i(\tau, \tilde{\mathbf{A}})$ is minimized for each τ.

Step 2 Optimize τ with respect to $Z_{syst}(\tau, \mathbf{A})$. Given the optimal interval τ^*, $\tilde{\mathbf{A}}$ can be obtained from the results of **Step 1**.

Recall that for the approximate evaluation of single component in Section 3, the arrival process of USDs from all other components is assumed to be a Poisson process. However, the aggregated failure occurrences of all other components is a superposition of the failure processes of all other components, which does not have i.i.d. exponential distributions as interoccurrence time distribution. Thus in this heuristic approach, the modeling of the arrival processes of USDs is an approximation for the real situation.

This heuristic algorithm will be guaranteed to terminate if the arrival rates of USDs generated by all the components, $\theta_{\mathbf{k}} = \{\theta_{\mathbf{1},\mathbf{k}}, \theta_{\mathbf{2},\mathbf{k}}, ..., \theta_{\mathbf{i},\mathbf{k}}, ..., \theta_{|\mathbf{I}|,\mathbf{k}}\}$, converges to a fixed point $\theta_{\mathbf{0}} = \{\theta_{\mathbf{1},\mathbf{0}}, \theta_{\mathbf{2},\mathbf{0}}, ..., \theta_{\mathbf{i},\mathbf{0}}, ..., \theta_{|\mathbf{I}|,\mathbf{0}}\}$.

5. Numerical experiments

The approximation of single-component model determines the accuracy of our evaluation procedure, as well as the quality of the heuristic solutions for the optimization of multi-component systems. Therefore, in this section, we will first give a numerical example of the approximation method proposed in Section 3, in order to have the first impression about our opportunistic maintenance policy and the accuracy of the approximation method. To further validate our approximation method under various parameter settings, we conduct numerical experiments on full factorial test beds in Subsection 5.2 and 5.3. We also evaluate the cost reduction potential of our proposed policy in comparison with other maintenance policies in Subsection 5.4.

5.1 Numerical example

To demonstrate the usage of the model, we first show a numerical example of the single-component model proposed in Section 3. The lifetime distribution of the component is a Weibull distribution with a scale

Table 1: The parameter setting

Parameter	Explanation
$c^{PM-SD} = 1$	Preventive maintenance due to scheduled downs [thousand Euro]
$c^{PM-USD} = 2$	Preventive maintenance due to unscheduled downs [thousand Euro]
$c^{CM} = 10$	Corrective maintenance [thousand Euro]
$\tau = 0.2$	The interval of scheduled downs [year]
$\alpha = 1.129$	Scale parameter of Weibull distribution
$\beta = 2.101$	Shape parameter of Weibull distribution
$\lambda = 2$	Poisson arrival rate of unscheduled downs [per year]

parameter α and a shape parameter β. In this case, according to Equation 2 - 6, the probabilities of PM-USD, PM-SD and CM conditioned on the deviation ξ are (notice that we exclude the index i, because we are dealing with the single-component model in this section)

$$\begin{aligned}
\dot{P}_1(\xi) &= \int_A^{n\tau-\xi} \left(1 - e^{-\lambda(u-A)}\right)\left(\frac{\beta u^{(\beta-1)}}{\alpha^\beta} e^{-(\frac{u}{\alpha})^\beta}\right) du \ + \ \left(1 - e^{-\lambda(n\tau-\xi-A)}\right)\left(1 - F(n\tau-\xi)\right), \\
\dot{P}_2(\xi) &= \left(e^{-\lambda(n\tau-\xi-A)}\right)\left(1 - F(n\tau-\xi)\right), \\
\dot{P}_3(\xi) &= F(A) \ + \ \int_A^{n\tau-\xi} \left(e^{-\lambda(u-A)}\right)\left(\frac{\beta u^{(\beta-1)}}{\alpha^\beta} e^{-(\frac{u}{\alpha})^\beta}\right) du,
\end{aligned} \tag{13}$$

where $F(u) = 1 - e^{-(\frac{u}{\alpha})^\beta}$ is the c.d.f. of Weibull distribution.

The input parameters are given in Table 1. We set $\beta = 2.101$ and $\alpha = 1.129$ for the Weibull distribution, so that the expected life time of the component $E[T]$ is equal to 1 year, which normalizes the time unit.

The optimal age limit A^* can be found by minimizing the average cost rate $Z(A)$, which can be approximately evaluated by the method proposed in Section 3. As a comparison, we simulate the average cost rate $\hat{Z}$ (see Appendix A under different A. Figure 3 illustrates the changes of the average cost rate over the age limit A via the approximate evaluation and the simulation with a 95% confidence interval.

The first observation is that the curves of the average cost rate obtained via the approximate evaluation and the simulation method are very close, which means our approximate evaluation is relatively accurate. The optimal maintenance policy via the approximate evaluation has a age limit $A^* = 0.400$ year and a minimum cost rate $Z(A^*) = 5.189$ thousand euro per year (see Figure 3), which is slightly different from the simulation results $\hat{A}^* = 0.380$ and $\hat{Z}(\hat{A}^*) = 5.185 \pm 0.006$ in Table 2. Moreover, the confidence interval is very small in Figure 3 (More details in Appendix A).

Table 2 shows i) the optimal policy via the approximate evaluation, including the optimal age limit A^*, its minimum cost rate $Z(A^*)$, its probabilities of three maintenance actions $\left\{P_1, P_2, P_3\right\}$ and its expected cycle length L; ii) the simulation results under the optimal age limit A^* obtained via the approximate evaluation, where $\hat{Z}(A^*)$ denotes the average cost rate, $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$ denotes the probabilities of three maintenance actions and $\hat{L}$ denotes the mean cycle length; iii) the optimal age limit $\hat{A}^*$ obtained via simulation-based optimization with its minimum cost rate $\hat{Z}(\hat{A}^*)$, its probabilities of three maintenance actions $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$

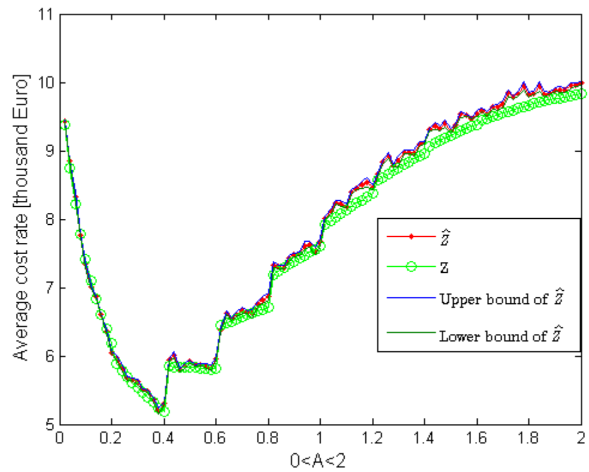


Figure 3: Average cost rate [thousand euro per year] over A [year]. The approximate result Z is compared with the simulated result $\hat{Z}$ in a 95% confidence interval with a lower and upper bound

and its mean cycle length $\hat{L}$.

Table 2: The optimal maintenance policies under the parameter setting in Table 1 ($\{P_1, P_2, P_3\}$ and $\{\hat{P}_1, \hat{P}_2, \hat{P}_3\}$ are the probabilities of taking PM-USD, PM-SD and CM actions by approximate evaluation and simulation respectively)

Approximation Result	Simulation Result 1	Simulation Result 2
$Z(A^*) = 5.189$ [K euro per year] $A^* = 0.400$ [year] $\left\{P_1, P_2, P_3\right\} = \{0.0269, 0.8570, 0.1161\}$ $L = 0.3993$ [year]	$\hat{Z}(A^*) = 5.289 \pm 0.008$ [K euro per year] $A^* = 0.400$ [year] $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.0601, 0.8132, 0.1267\}$ $\hat{L} = 0.4161$ [year]	$\hat{Z}(\hat{A}^*) = 5.185 \pm 0.006$ [K euro per year] $\hat{A}^* = 0.380$ [year] $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\} = \{0.0485, 0.8420, 0.1095\}$ $\hat{L} = 0.3923$ [year]
$\|Gap1\|$: $\|\big(\hat{Z}(A^*) - Z(A^*)\big)/\hat{Z}(A^*)\| = 1.88\%$; $\|Gap2\|$: $\|\big(\hat{Z}(\hat{A}^*) - \hat{Z}(A^*)\big)/\hat{Z}(\hat{A}^*)\| = 1.96\%$		

Based on the results in Table 2, we observe that the absolute relative deviation $|\big(\hat{Z}(A^*) - Z(A^*)\big)/\hat{Z}(A^*)|$, denoted as Gap 1, is only 1.88%, which shows that the average cost rate evaluated through our approximate method is very close to the simulation result in this numerical example, under the same A^* value. The absolute relative deviation $|\big(\hat{Z}(\hat{A}^*) - \hat{Z}(A^*)\big)/\hat{Z}(\hat{A}^*)|$, denoted as Gap 2, is only 1.96%. This implies that the deviation of A^* from $\hat{A}^*$ does not lead to a large deviation on the simulated cost rate, which is due to the fact that the cost rate $\hat{Z}(A)$ is relatively flat in the neighborhood of its minimum. Hence, in practice, the optimal maintenance policy based on our approximate evaluation will result in an optimal solution having an average cost rate that is very close to the true minimum cost rate. Also notice that the values of P_2, P_3 and L via the approximate evaluation are very close to the simulated values of $\hat{P}_2, \hat{P}_3$ and $\hat{L}$ (the value of P_1 is relatively less accurate due to the fact that the probability of the occurrence of USDs is small in this numerical example). Therefore, we can conclude that the gaps are small and our approximate evaluation is relatively accurate in this numerical example.

It is very interesting to observe that the average cost rates of approximate evaluation and simulation are not

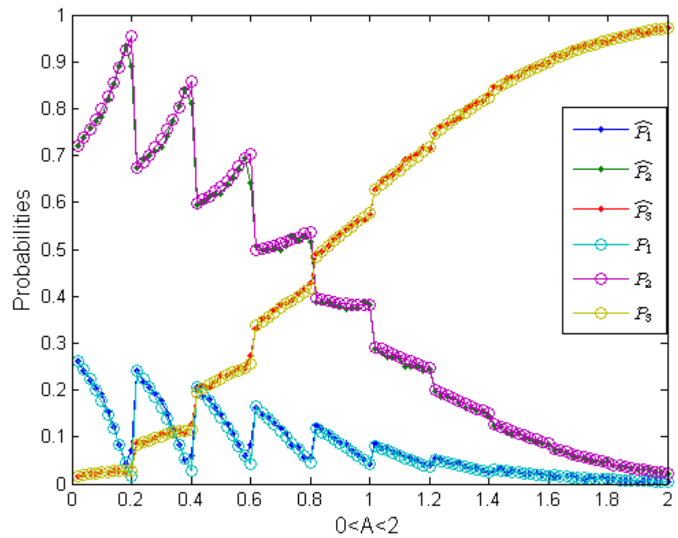


Figure 4: The probabilities of three maintenance actions (i.e., PM-USD, PM-SD and CM) over A [year]. The approximate result $\left\{P_1, P_2, P_3\right\}$ is compared with the simulated result $\left\{\hat{P}_1, \hat{P}_2, \hat{P}_3\right\}$

smooth curves. To have insights further on the spikes of the curves, we plot the probabilities P_1, P_2 and P_3 in Figure 4 and the expected cycle length L in Figure 5.

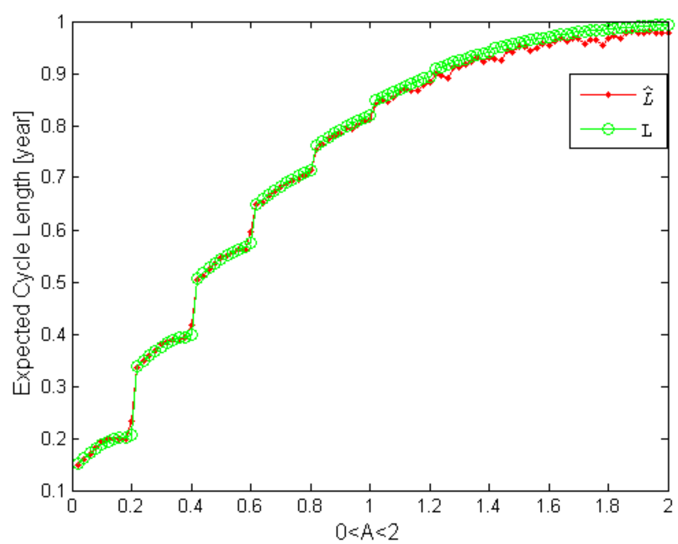


Figure 5: Expected cycle length [year] over A [year]. The approximate result L is compared with the simulated result $\hat{L}$

The first observation is that the differences between the values of P_1, P_2 and P_3 and L obtained via the approximate evaluation and the simulation method are very small, which verifies the accuracy of our approximate evaluation. The spikes in Figure 4 and the jumps in Figure 5 appear at scheduled downs $n\tau$, where $\tau = 0.2$ and $n \in \mathbb{N}$. The reason is that our model has a strict age limit for taking opportunities.

Consider two cases: 1) when A is just before $n\tau$, or $A + \epsilon = n\tau$ where ϵ is infinitely small and positive, the next opportunity after A is almost certain to be a scheduled down; the probability of PM-SD is thus relatively large compared with the other settings of A in $((n-1)\tau, n\tau)$. 2) However, if A is just after $n\tau$ or $A = n\tau + \epsilon$, then the next SD opportunity is at $(n+1)\tau$, instead of $n\tau$; therefore, P_2 is much smaller in this case compared with the previous case. At the same time, P_1 is much smaller in Case 1 compared with Case 2, because for case 2 there is longer period of waiting for the USD opportunities before the SD opportunity comes at $(n+1)\tau$. When A becomes larger, much less USD and SD opportunities are taken and the probability of CM becomes larger. This means that the renewal cycles end with failures more often. Therefore, P_1 and P_2 are approaching to zero in Figure 4 and the spikes are becoming less sharp when A increases. The jumps of the expected cycle length in Figure 5 is due to the delayed SD opportunity in Case 2. The magnitude of the jumps in Figure 5 becomes less at larger values of A since the probability of CM becomes larger and the impact of the probability of PM-SD is less apparent.

5.2 Accuracy of the approximate evaluation

The accuracy of our approximate evaluation is assessed based on the gap between the simulation result $\hat{Z}(A)$ and the approximation result $Z(A)$. We vary four factors in our test bed: the variable A and three parameters τ, λ and σ [1]. Notice that τ and λ are varied, because they determine the frequencies of the SD and USD opportunities(see Section 2). Moreover, to show the impact of the variance of the life time distribution, we choose the standard deviation σ as a varying parameter. Three different levels of the age limit A, $\{0.5, 1.0, 1.5\}$, are chosen. For each of the other three parameters, three different levels are obtained by multiplying a base value with a set of coefficients, $\{50\%, 100\%, 150\%\}$ (see Table 3). The base values are the same as the parameter setting given in Table 1. Hence, in Table 3, a full factorial test bed is set up and a space of instances Λ is defined as $\{(A_j, \sigma_l, \lambda_k, \tau_m) \in \Lambda | \forall j, l, k, m \in \{1, 2, 3\}\}$, which leads to $|\Lambda| = 81$ instances in the test bed.

Table 3: Parameter setting of the test bed

Parameter	Explanation
$\{\tau_1, \tau_2, \tau_3\} = 0.2 * \{50\%, 100\%, 150\%\}$	The interval of scheduled downs [year]
$\{\lambda_1, \lambda_2, \lambda_3\} = 2 * \{50\%, 100\%, 150\%\}$	Poisson arrival rate of unscheduled downs [per year]
$\{\sigma_1, \sigma_2, \sigma_3\} = 1/2 * \{50\%, 100\%, 150\%\}$	Standard deviation of component life time [year]
$\{A_1, A_2, A_3\} = \{0.5, 1.0, 1.5\}$	Age limit values [year]
$E[T] = 1$	Expected component life time [year]

We set the expected life time $E[T]$ of the component to be equal to 1, which normalizes the time unit. By fitting the two moments of the component life time, the shape and scale parameters of the Weibull distribution can be calculated.

Notice that no cost parameters are chosen as factors in this test bed, since the approximate evaluation of the objective function is fully determined by the estimations of the probabilities of the three maintenance

[1] $\sigma^2 = E[T^2] - E[T]^2$, where $E[T]$ and $E[T^2]$ are the 1^{st} and 2^{nd} moment of the component life time. σ is the standard deviation of the component life time distribution T

actions and the expected cycle length. This also helps to reduce the size of the test bed. We compare the probabilities of PM-USD, PM-SD and CM and the expected cycle length obtained by the approximate evaluation method $\big(P_1, P_2, P_3, L\big)$ and the simulation method $\big(\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}\big)$, which is similar to Table 2 in Section 5.1. To see how much the approximation results deviate from the simulation results, we define a deviation vector $\big[\delta_1, \delta_2, \delta_3, \delta_4\big] = \big[\hat{P}_1 - P_1, \hat{P}_2 - P_2, \hat{P}_3 - P_3, (\hat{L} - L)/\hat{L}\big]$. The deviation vectors of 81 instances are shown in Table 9 and 10 of Appendix B. There are three levels for each factor in the test bed Λ. We categorize the instances that have the same level of a certain factor into a subset. For example, a subset of instances containing A_1 is defined as $\Lambda_{A_1} = \big\{(A_1, \sigma_l, \lambda_k, \tau_m) | \forall l, k, m \in \{1, 2, 3\}\big\}$. For each of these subsets, the average of the absolute deviations (denoted by AAD) and the maximum of the absolute deviations (denoted by MAD) are summarized in Table 4.

Table 4: The average absolute difference (AAD) and the maximum absolute difference (MAD) between the simulation results and the approximation results

	$\lvert\delta_1\rvert$ $\{AAD, MAD\}$	$\lvert\delta_2\rvert$ $\{AAD, MAD\}$	$\lvert\delta_3\rvert$ $\{AAD, MAD\}$	$\lvert\delta_4\rvert$ $\{AAD, MAD\}$
Λ_{A_1}	$\{0.0058, 0.0224\}$	$\{0.0086, 0.0268\}$	$\{0.0037, 0.0089\}$	$\{0.32\%, 0.96\%\}$
Λ_{A_2}	$\{0.0017, 0.0044\}$	$\{0.0031, 0.0071\}$	$\{0.0029, 0.0100\}$	$\{0.92\%, 1.73\%\}$
Λ_{A_3}	$\{0.0011, 0.0040\}$	$\{0.0034, 0.0085\}$	$\{0.0036, 0.0102\}$	$\{1.51\%, 2.65\%\}$
Λ_{σ_1}	$\{0.0029, 0.0131\}$	$\{0.0042, 0.0183\}$	$\{0.0028, 0.0063\}$	$\{0.88\%, 2.04\%\}$
Λ_{σ_2}	$\{0.0027, 0.0191\}$	$\{0.0052, 0.0253\}$	$\{0.0034, 0.0089\}$	$\{0.87\%, 2.65\%\}$
Λ_{σ_3}	$\{0.0030, 0.0224\}$	$\{0.0057, 0.0268\}$	$\{0.0040, 0.0102\}$	$\{1.01\%, 2.65\%\}$
Λ_{λ_1}	$\{0.0016, 0.0076\}$	$\{0.0041, 0.0134\}$	$\{0.0030, 0.0089\}$	$\{0.93\%, 2.65\%\}$
Λ_{λ_2}	$\{0.0028, 0.0131\}$	$\{0.0043, 0.0197\}$	$\{0.0027, 0.0102\}$	$\{0.93\%, 2.04\%\}$
Λ_{λ_3}	$\{0.0041, 0.0224\}$	$\{0.0068, 0.0268\}$	$\{0.0044, 0.0100\}$	$\{0.90\%, 2.65\%\}$
Λ_{τ_1}	$\{0.0010, 0.0032\}$	$\{0.0024, 0.0084\}$	$\{0.0027, 0.0102\}$	$\{0.77\%, 2.65\%\}$
Λ_{τ_2}	$\{0.0048, 0.0224\}$	$\{0.0078, 0.0268\}$	$\{0.0038, 0.0089\}$	$\{1.13\%, 2.65\%\}$
Λ_{τ_3}	$\{0.0027, 0.0105\}$	$\{0.0048, 0.0194\}$	$\{0.0037, 0.0100\}$	$\{0.85\%, 2.60\%\}$
Λ	$\{0.0028, 0.0224\}$	$\{0.0050, 0.0268\}$	$\{0.0034, 0.0102\}$	$\{0.92\%, 2.65\%\}$

The first insight from Table 4 is that the AADs and MADs of δ_1, δ_2, δ_3 and δ_4 are small, which implies that our approximate evaluation is accurate under all parameter settings (including the age limit A). The AADs and MADs of δ_1, δ_2 and δ_3 are at the magnitude of 10^{-3} and 10^{-2} respectively. The AADs of δ_4 are around 1% and the MAD of δ_4 are less than 3%.

5.3 Heuristic optimization based on the approximate evaluation

The results in Table 2 of Section 5.1 show that the optimal policies via the approximate evaluation method and the simulation method are close to each other. In this subsection, we intend to verify these results further under various parameter settings. Similar to Table 2, we evaluate two gaps: i) Gap 1, $\big(\hat{Z}(A^*) - Z(A^*)\big)/\hat{Z}(A^*)$, shows how much the true cost rate deviates from the cost rate of approximate evaluation, while using the optimal age limit that is obtained from our approximate evaluation model; and ii) Gap 2, $\big(\hat{Z}(\hat{A}^*) - \hat{Z}(A^*)\big)/\hat{Z}(\hat{A}^*)$, shows how much the optimal maintenance policy of our approximate evaluation deviates from the true optimal policy.

The space of instances Ω has $(\sigma_l, \lambda_k, \tau_m) \in \Omega, \forall l, k, m \in \{1, 2, 3\}$; which leads to $|\Omega| = 27$ instances in

Table 5: The average absolute difference (AAD) and the maximum absolute difference (MAD) between the simulation results and the approximation results for Gap 1, Gap 2 and $(\hat{A}^* - A^*)/\hat{A}^*$

	$(\hat{A}^* - A^*)/\hat{A}^*$ $\{AAD, MAD\}$	$Gap1$ $\{AAD, MAD\}$	$Gap2$ $\{AAD, MAD\}$
Ω_{σ_1}	$\{4.07\%, 6.67\%\}$	$\{1.83\%, 3.21\%\}$	$\{1.63\%, 3.04\%\}$
Ω_{σ_2}	$\{4.35\%, 6.67\%\}$	$\{2.31\%, 3.13\%\}$	$\{2.28\%, 3.01\%\}$
Ω_{σ_3}	$\{10.6\%, 25.0\%\}$	$\{3.19\%, 4.79\%\}$	$\{2.67\%, 5.21\%\}$
Ω_{λ_1}	$\{6.57\%, 25.0\%\}$	$\{1.99\%, 4.38\%\}$	$\{2.19\%, 5.21\%\}$
Ω_{λ_2}	$\{6.20\%, 25.0\%\}$	$\{2.50\%, 3.84\%\}$	$\{2.15\%, 3.10\%\}$
Ω_{λ_3}	$\{6.20\%, 25.0\%\}$	$\{2.84\%, 4.79\%\}$	$\{2.23\%, 4.82\%\}$
Ω_{τ_1}	$\{10.6\%, 25.0\%\}$	$\{2.16\%, 3.21\%\}$	$\{1.59\%, 2.63\%\}$
Ω_{τ_2}	$\{3.61\%, 5.00\%\}$	$\{2.80\%, 4.79\%\}$	$\{2.71\%, 5.21\%\}$
Ω_{τ_3}	$\{4.81\%, 6.67\%\}$	$\{2.37\%, 3.82\%\}$	$\{2.26\%, 3.10\%\}$
Ω	$\{6.33\%, 25.0\%\}$	$\{2.44\%, 4.79\%\}$	$\{2.19\%, 5.21\%\}$

the test bed. The cost parameters are the same as the settings in Table 1. The deviation vectors of 27 instances are shown in Table 11 of Appendix B. For each factor, we categorize the instances that have the same level of a certain factor into a subset. For example, a subset of instances containing σ_1 is defined as $\Omega_{\sigma_1} = \{(\sigma_1, \lambda_k, \tau_m) | \forall k, m \in \{1, 2, 3\}\}$. For each of these subsets, the average of the absolute deviations is denoted by AAD and the maximum of the absolute deviations is denoted by MAD. These results are provided in Table 5.

The first insight from Table 5 is that the AADs and MADs of Gap 1 and Gap 2 are relatively small, even though the AADs and MADs of $(\hat{A}^* - A^*)/A^*$ are larger. This implies two points: 1) the neighborhood of the minimum $\hat{Z}$ is flat and it is robust to the deviations of age limits; 2) our approximate evaluation is accurate in the neighborhood of the optimal solution. Comparing Gap 1 with Gap 2, we observe that the AADs of Gap 2 are smaller than Gap 1. This implies that the deviations of A^* from $\hat{A}^*$ do not lead to big differences on the simulated cost rates under various parameter settings; even though the approximated results (or Gap 1) are slightly less accurate. Hence, in practice, the optimal maintenance policy of our approximate evaluation will lead to an average cost rate that is very close to the true minimum cost rate.

5.4 Cost reduction potential

To show the cost benefits of including the opportunities at USDs and SDs for the age-based maintenance policy, three policies are considered:1) an *only-SD-opportunistic policy*, which means that only SDs are considered as opportunities and no opportunistic preventive maintenance actions are taken at USDs; 2) an *only-USD-opportunistic policy*, which means that only USDs are considered as opportunities and no opportunistic preventive maintenance actions are taken at SDs; and 3) a *failure-based policy*, which means that neither USDs nor SDs are considered as opportunities for preventive maintenance. Notice that Policy 1 can be analyzed as a special case of our policy, where $\lambda = 0$; Policy 2 can be analyzed as a special case of our policy, where $\tau = \infty$; and Policy 3 can be analyzed as a special case of our policy, where $A = \infty$.

To show the cost benefits under various parameter settings, we use the same test bed and parameter settings as in Section 5.3. The minimum average cost rate of our policy that includes opportunities at both USDs and SDs is denoted by Z. The minimum average cost rates of Policy 1 and 2 are denoted by $\tilde{Z}_1$ and $\tilde{Z}_2$

respectively. Notice that no opportunity is considered in Policy 3; then its cost rate $\tilde{Z}_3$ remains unchanged under different parameter settings, which is 10 thousand euro per year.

Table 6: Summary of the cost saving percentages by using opportunities at USDs and SDs; including the mean, minimum and maximum values of $\triangle_A$, $\triangle_B$ and $\triangle_C$ respectively.

	$\triangle_A$			$\triangle_B$			$\triangle_C$		
	mean	min	max	mean	min	max	mean	min	max
Ω_{σ_1}	63.7%	60.5%	65.4%	58.8%	55.9%	60.3%	28.7%	19.6%	36.1%
Ω_{σ_2}	47.8%	45.9%	48.9%	41.0%	39.7%	41.7%	19.7%	14.1%	24.3%
Ω_{σ_3}	32.7%	30.7%	33.9%	24.2%	23.0%	25.0%	12.2%	9.1%	14.6%
Ω_{λ_1}	48.5%	32.2%	65.4%	41.3%	23.0%	60.3%	14.3%	9.1%	19.6%
Ω_{λ_2}	48.1%	31.4%	65.3%	41.3%	23.0%	60.3%	21.4%	12.9%	30.4%
Ω_{λ_3}	47.7%	30.7%	65.2%	41.3%	23.0%	60.3%	25.0%	14.6%	36.1%
Ω_{τ_1}	49.2%	33.4%	65.4%	42.3%	25.0%	60.3%	20.2%	9.1%	36.1%
Ω_{τ_2}	48.7%	32.6%	65.2%	42.1%	24.7%	60.2%	20.2%	9.1%	36.1%
Ω_{τ_3}	46.3%	30.7%	61.4%	39.5%	23.0%	55.9%	20.2%	9.1%	36.1%
Ω	48.1%	30.7%	65.4%	41.3%	23.0%	60.3%	20.2%	9.1%	36.1%

We use $\tilde{Z}_3$ as the basis of the comparison. In total, we have three comparisons: A) the cost saving percentage of including opportunities at both USDs and SDs, denoted by $\triangle_A = (\tilde{Z}_3 - Z)/\tilde{Z}_3$; B) the cost saving percentage of using only opportunities at SDs (i.e., Policy 1), denoted by $\triangle_B = (\tilde{Z}_3 - \tilde{Z}_1)/\tilde{Z}_3$; C) the cost saving percentage of using only opportunities at USDs (i.e., Policy 2), denoted by $\triangle_C = (\tilde{Z}_3 - \tilde{Z}_2)/\tilde{Z}_3$. Similar to Subsection 5.3, we categorize the instances that have the same level of a certain factor into a subset. For example, a subset of instances containing σ_1 is defined as $\Omega_{\sigma_1} = \big\{(\sigma_1, \lambda_k, \tau_m)|\forall k, m \in \{1,2,3\}\big\}$. The means, minimums and maximums of the cost saving percentages of these 9 subsets are provided in Table 6. The results of all instances are shown in Table 12 in Appendix B.

The first observation from Table 6 is that our policy (using opportunities at both SDs and USDs) is better than Policy 1 (using opportunities at SDs only), Policy 2 (using opportunities at USDs only) and Policy 3 (using no opportunities) from the perspective of cost savings. The mean values of $\triangle_A$ are bigger than $\triangle_B$ and $\triangle_C$, because more opportunities for preventive maintenance (cheaper than corrective maintenance) are included in our policy compared with Policy 1 and 2. The mean values of $\triangle_B$ are bigger than $\triangle_C$, because the cost of preventive maintenance at a SD is cheaper than at an USD. Regarding the variation of the mean values under various parameter settings, we observe that i) $\triangle_A$ is inversely proportional to σ, λ and τ; ii) $\triangle_B$ is inversely proportional to σ and τ (it remains unchanged on different λ, because no USD opportunities is considered in Policy 1); iii) $\triangle_C$ is proportional to λ and inversely proportional to σ (it remains unchanged on different τ, because no SD opportunities is considered in Policy 2). A higher σ means a higher variance in the lifetime distribution of the component, which leads to a higher probability of having corrective maintenance (more expensive than preventive maintenance). Hence, the mean values of $\triangle_A$, $\triangle_B$ and $\triangle_C$ decrease when σ increases. Moreover, the cost of taking opportunities at SDs is cheaper than at USDs. A higher λ leads to more opportunities at expensive USDs. Hence, $\triangle_A$ decreases when λ increases. A higher τ leads to less opportunities at cheaper SDs. Hence, $\triangle_A$ decreases when τ increases. For the same reason, a higher τ leads to a lower $\triangle_B$. However, $\triangle_C$ increases at a higher λ, because only opportunities at USDs are considered in Policy 2. In this case, a higher λ leads to more opportunities to take, so that higher cost saving percentages can be observed.

6. Demonstration in a case of multi-component systems

The high accuracy of our approximate evaluation has been shown in Section 5. Hence, we can use this model for a single component as a building block to construct a model for a multi-component system, as mentioned in Section 2. To be able to solve the maintenance optimization problem for multi-component systems, we develop an heuristic approach with an iterative procedure in Section 4. We give a simple example of a system consisting of 20 components with their lifetime distributions modeled by Weibull distribution in this section. The input parameters are given in Table 7, where 1) α_i and β_i are the scale and shape parameters of the Weibull distribution respectively, for component i; and 2) c_i^{PM-USD}, c_i^{PM-SD} and c_i^{CM} are the costs of PM-USD, PM-SD and CM actions respectively, for component i. The fixed setup cost of maintenance S^{SD} is set at 2 thousand euros.

Table 7: The parameter setting for a system consisting of 20 components

Component	Input parameters	α_i	β_i	c_i^{PM-USD}	c_i^{PM-SD}	c_i^{CM}
1		1.13	2.10	2.00	1.00	10.0
2		1.15	2.15	2.05	1.03	10.3
3		1.18	2.19	2.11	1.05	10.5
4		1.20	2.23	2.16	1.08	10.8
5		1.22	2.28	2.21	1.11	11.1
6		1.25	2.32	2.26	1.13	11.3
7		1.27	2.37	2.32	1.16	11.6
8		1.30	2.41	2.37	1.18	11.8
9		1.32	2.46	2.42	1.21	12.1
10		1.34	2.50	2.47	1.24	12.4
11		1.37	2.54	2.53	1.26	12.6
12		1.39	2.59	2.58	1.29	12.9
13		1.41	2.63	2.63	1.32	13.2
14		1.44	2.68	2.68	1.34	13.4
15		1.46	2.72	2.74	1.37	13.7
16		1.49	2.76	2.79	1.39	13.9
17		1.51	2.81	2.84	1.42	14.2
18		1.53	2.85	2.89	1.45	14.5
19		1.56	2.90	2.95	1.47	14.7
20		1.58	2.94	3.00	1.50	15.0

Via the heuristic approach, we find a heuristic solution for this multi-component system. The solution of τ^* is 0.35 year. For all components, we obtain the age limits $\tilde{\mathbf{A}}$, as shown in Table 8. The average cost rate of the system Z_{syst} under this heuristic solution is approximated to be 99.66 thousand euros per year. To check the accuracy of the approximation through the heuristic approach, we also simulate our multi-component maintenance policy by using the same heuristic solution $\{\tau^*, \tilde{\mathbf{A}}\}$; the average cost rate Z_{sim} of the simulation method can be obtained, together with a 95% confidence interval. Figure 6 illustrates the changes of the average cost rates over the decision variable τ via the heuristic approach and the simulation method. The curves of the average cost rates obtained via the heuristic approach and simulation are very close, especially in the range that is around the heuristic solution $\tau^* = 0.35$. Notice that the average cost rate obtained via the heuristic approach Z_{syst} deviates more from the simulation result Z_{sim} in the range that is around $\tau = 0.7$. This is due to the fact that our approximation is based on the assumption that the intervals of SDs are relatively small compared with the average values of the lifetimes of components; when $\tau = 0.7$, the intervals of SDs (0.7) are comparable to the average values of the components' lifetimes ($\{1, ..., 1.41\}$).

Moreover, in Table 8, the heuristic results for the cost rate of maintenance incurred by each component in the system, $\{Z_i(\tau^*, \tilde{\mathbf{A}}), \forall i \in I\}$, are provided; the simulated results for the cost rate of maintenance incurred by each component in the system, $\{\hat{Z}_i(\tau^*, \tilde{\mathbf{A}}), \forall i \in I\}$, are also provided, as well as the gaps between the simulated results and the heuristic results $(|(\hat{Z}_i(\tilde{A}_i) - Z_i(\tilde{A}_i))/\hat{Z}_i(\tilde{A}_i)|)$. Based on the results in Table 8, we observe that the gaps between the simulated results and the heuristic results are relatively small for most of the components, under the same decision variables $\{\tau^*, \tilde{\mathbf{A}}\}$.

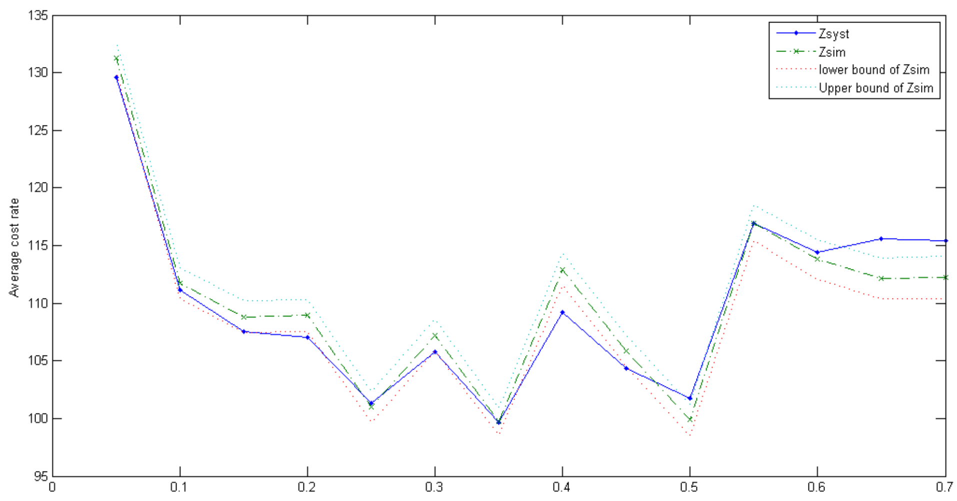


Figure 6: Average cost rate [thousand euro per year] of a multi-component system over τ [year]. The approximate result Z_{syst} is compared with the simulated result Z_{sim}

7. Conclusions

In this paper, we proposed a new opportunistic maintenance policy for multi-component systems with age information. In order to further minimize the downtime cost and setup cost of maintenance we included both the scheduled and unscheduled system downs as opportunities for preventive maintenance. The age limits were introduced to determine the timing of taking opportunities to preventively maintain a component together with other failed components in the system. We discovered that if we want to model the situation that the scheduled downs are set at fixed time points, the average cost rate needs to be approximately evaluated since the renewal property does not hold for this situation. Hence, we proposed an approximation

Table 8: The heuristic and simulated results of a system with 20 components

Component	$\tilde{A}_i$	$Z_i(\tilde{A}_i)$	$\hat{Z}_i(\tilde{A}_i)$	Gap
1	0.35	5.25	5.21 ± 0.06	0.85%
2	0.35	5.16	5.11 ± 0.08	0.91%
3	0.35	5.08	5.03 ± 0.06	0.97%
4	0.35	5.00	5.02 ± 0.08	0.24%
5	0.35	4.94	4.91 ± 0.09	0.47%
6	0.35	4.88	4.80 ± 0.06	1.66%
7	0.35	4.83	4.79 ± 0.04	0.90%
8	0.35	4.78	4.77 ± 0.05	0.39%
9	0.35	4.75	4.74 ± 0.05	0.17%
10	0.35	4.72	4.74 ± 0.05	0.32%
11	0.35	4.70	4.66 ± 0.04	0.88%
12	0.35	4.68	4.69 ± 0.04	0.04%
13	0.35	4.68	4.66 ± 0.04	0.24%
14	0.35	4.67	4.70 ± 0.04	0.59%
15	0.35	4.56	4.62 ± 0.08	1.27%
16	0.70	4.45	4.58 ± 0.05	2.84%
17	0.70	4.35	4.34 ± 0.08	0.12%
18	0.70	4.25	4.26 ± 0.07	0.39%
19	0.70	4.16	4.26 ± 0.07	2.42%
20	0.70	4.07	4.15 ± 0.08	2.06%

method to evaluate the average cost rate of a single component. An heuristic approach was also developed to solve the optimization problem of multi-component systems based on the approximate evaluation.

To validate our model, we compared our approximation results with the simulation results under various parameter settings. In the extensive numerical experiments, our model showed a good accuracy and a considerable cost-saving potential. We also demonstrated the usage of our model for a multi-component system. By comparing the heuristic results with the simulated results, we observed that our heuristic approach is relatively accurate.

It is also interesting to observe the spikes and jumps of the average cost rate when the age limit is a multiple of the interval of scheduled downs, which is unexpected. This is due to the fact that we have strict age limits and it is more beneficial to take cheap SD opportunities for preventive maintenance. Thus it is more beneficial to allow the component to take opportunities for preventive maintenance right before the SD opportunity comes and set the age limit approximately equal to the multiple of the intervals of SDs. This observation was also verified by the demonstration case of multi-component systems.

8. Appendices

A. Simulation procedures

To evaluate the accuracy of the approximation, we run a simulation to compare with the approximate evaluation results in Section 5. There are m runs in the simulation (e.g., $m = 100$). For each run $i \in$

$\{1, 2, ..., m\}$, we generate 1) a Poisson process with a rate λ and random arrival time points $D = \{d_1, d_2, ...\}$; 2) a set of random failure times $T = \{t_1, t_2, ...\}$; and 3) a constant set $B = \{\tau, 2\tau, ...\}$, on a time horizon $(0, T_{max})$ that is sufficiently large(e.g., 10^6 times larger than the mean value of the lifetime). Then by using the method of discrete event simulation, we record the number of PM-USD actions, PM-SD actions and CM actions separately for each run, as well as the average cycle length. The probabilities of PM-USD actions, PM-SD actions and CM actions can be calculated based on these results for each run. We also can compute the expected cost rate for each run $\hat{Z}_i$ after obtaining the probabilities of PM-USD actions, PM-SD actions and CM actions and the average cycle length. The final result of the simulation is the mean value of the m runs, with a $100(1-\alpha)\%$ confidence interval

$$\hat{Z} \pm t(1-\alpha/2, m-1)\sqrt{\frac{S^2}{m}}$$

where $S = \sum_{i=1}^{m} \frac{(\hat{Z}_i - \hat{Z})^2}{m-1}$ and $t(1-\alpha/2, m-1)$ is the upper $1-\alpha/2$ critical point for the t-distribution with $(m-1)$ degrees of freedom (in our case, $m = 100$ and $\alpha = 5\%$).

B. Detail results of Test bed 1 and 2

Detail results of Tables 4, 5 and 6 are given in the following tables

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Table 9: A full factorial test bed including $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}\}$ from the simulation and the deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$ from the approximation

	Simulation $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}\}$	Deviation $\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(A_1, \sigma_1, \lambda_1, \tau_1)$	$\{0.003, 0.915, 0.082, 0.493\}$	$\{-0.001, 0.0000.000 - 0.2\%\}$
$(A_1, \sigma_1, \lambda_1, \tau_2)$	$\{0.090, 0.775, 0.135, 0.577\}$	$\{0.006, -0.009, 0.003, 0.6\%\}$
$(A_1, \sigma_1, \lambda_1, \tau_3)$	$\{0.093, 0.766, 0.142, 0.583\}$	$\{-0.001, 0.0000.001, -0.1\%\}$
$(A_1, \sigma_1, \lambda_2, \tau_1)$	$\{0.006, 0.914, 0.080, 0.493\}$	$\{-0.001, 0.003, -0.002, -0.2\%\}$
$(A_1, \sigma_1, \lambda_2, \tau_2)$	$\{0.169, 0.697, 0.133, 0.574\}$	$\{0.011, -0.015, 0.004, 0.8\%\}$
$(A_1, \sigma_1, \lambda_2, \tau_3)$	$\{0.184, 0.677, 0.139, 0.582\}$	$\{0.004, -0.003, -0.001, 0.4\%\}$
$(A_1, \sigma_1, \lambda_3, \tau_1)$	$\{0.009, 0.912, 0.079, 0.493\}$	$\{-0.001, 0.004, -0.003, -0.2\%\}$
$(A_1, \sigma_1, \lambda_3, \tau_2)$	$\{0.250, 0.617, 0.133, 0.569\}$	$\{0.026, -0.032, 0.006, 0.8\%\}$
$(A_1, \sigma_1, \lambda_3, \tau_3)$	$\{0.275, 0.585, 0.140, 0.579\}$	$\{0.016, -0.019, 0.003, 0.5\%\}$
$(A_1, \sigma_2, \lambda_1, \tau_1)$	$\{0.007, 0.824, 0.169, 0.478\}$	$\{0.0000.002, -0.002, -0.1\%\}$
$(A_1, \sigma_2, \lambda_1, \tau_2)$	$\{0.081, 0.693, 0.226, 0.548\}$	$\{0.006, -0.004, -0.003, 0.2\%\}$
$(A_1, \sigma_2, \lambda_1, \tau_3)$	$\{0.085, 0.678, 0.238, 0.558\}$	$\{-0.001, 0.004, -0.003, 0.0\%\}$
$(A_1, \sigma_2, \lambda_2, \tau_1)$	$\{0.014, 0.815, 0.171, 0.477\}$	$\{0.0000.0000.000 - 0.400\%\}$
$(A_1, \sigma_2, \lambda_2, \tau_2)$	$\{0.151, 0.618, 0.231, 0.546\}$	$\{0.010, -0.016, 0.006, 0.5\%\}$
$(A_1, \sigma_2, \lambda_2, \tau_3)$	$\{0.172, 0.589, 0.239, 0.557\}$	$\{0.007, -0.008, 0.001, 0.4\%\}$
$(A_1, \sigma_2, \lambda_3, \tau_1)$	$\{0.018, 0.815, 0.166, 0.479\}$	$\{-0.002, 0.006, -0.004, -0.1\%\}$
$(A_1, \sigma_2, \lambda_3, \tau_2)$	$\{0.214, 0.562, 0.223, 0.541\}$	$\{0.015, -0.016, 0.002, 0.4\%\}$
$(A_1, \sigma_2, \lambda_3, \tau_3)$	$\{0.257, 0.513, 0.230, 0.553\}$	$\{0.021, -0.017, -0.004, 0.3\%\}$
$(A_1, \sigma_3, \lambda_1, \tau_1)$	$\{0.009, 0.748, 0.243, 0.462\}$	$\{0.000 - 0.008, 0.008, -0.4\%\}$
$(A_1, \sigma_3, \lambda_1, \tau_2)$	$\{0.072, 0.634, 0.294, 0.526\}$	$\{0.004, -0.007, 0.003, 0.3\%\}$
$(A_1, \sigma_3, \lambda_1, \tau_3)$	$\{0.076, 0.611, 0.313, 0.533\}$	$\{-0.005, -0.004, 0.009, -0.6\%\}$
$(A_1, \sigma_3, \lambda_2, \tau_1)$	$\{0.015, 0.752, 0.233, 0.462\}$	$\{-0.002, 0.004, -0.002, -0.3\%\}$
$(A_1, \sigma_3, \lambda_2, \tau_2)$	$\{0.139, 0.569, 0.292, 0.523\}$	$\{0.009, -0.014, 0.005, 0.5\%\}$
$(A_1, \sigma_3, \lambda_2, \tau_3)$	$\{0.154, 0.541, 0.305, 0.531\}$	$\{-0.001, -0.004, 0.004, -0.3\%\}$
$(A_1, \sigma_3, \lambda_3, \tau_1)$	$\{0.024, 0.746, 0.230, 0.463\}$	$\{-0.002, 0.007, -0.005, -0.1\%\}$
$(A_1, \sigma_3, \lambda_3, \tau_2)$	$\{0.199, 0.514, 0.287, 0.517\}$	$\{0.016, -0.018, 0.003, 0\%\}$
$(A_1, \sigma_3, \lambda_3, \tau_3)$	$\{0.228, 0.473, 0.299, 0.529\}$	$\{0.008, -0.010, 0.002, -0.1\%\}$
$(A_2, \sigma_1, \lambda_1, \tau_1)$	$\{0.012, 0.451, 0.537, 0.861\}$	$\{0.000 - 0.003, 0.003, -0.9\%\}$
$(A_2, \sigma_1, \lambda_1, \tau_2)$	$\{0.023, 0.416, 0.562, 0.871\}$	$\{0.0000.000 - 0.001, -1.0\%\}$
$(A_2, \sigma_1, \lambda_1, \tau_3)$	$\{0.061, 0.279, 0.660, 0.908\}$	$\{0.003, -0.005, 0.002, -0.8\%\}$
$(A_2, \sigma_1, \lambda_2, \tau_1)$	$\{0.022, 0.447, 0.531, 0.864\}$	$\{-0.001, 0.004, -0.003, -0.5\%\}$
$(A_2, \sigma_1, \lambda_2, \tau_2)$	$\{0.041, 0.395, 0.564, 0.871\}$	$\{-0.003, 0.0000.003, -0.9\%\}$
$(A_2, \sigma_1, \lambda_2, \tau_3)$	$\{0.111, 0.249, 0.640, 0.902\}$	$\{0.005, -0.002, -0.003, -0.9\%\}$
$(A_2, \sigma_1, \lambda_3, \tau_1)$	$\{0.035, 0.435, 0.529, 0.857\}$	$\{0.001, 0.003, -0.004, -1.4\%\}$
$(A_2, \sigma_1, \lambda_3, \tau_2)$	$\{0.062, 0.377, 0.561, 0.873\}$	$\{-0.002, 0.0000.002, -0.7\%\}$
$(A_2, \sigma_1, \lambda_3, \tau_3)$	$\{0.138, 0.219, 0.643, 0.894\}$	$\{-0.008, -0.004, 0.011, -1.3\%\}$
$(A_2, \sigma_2, \lambda_1, \tau_1)$	$\{0.011, 0.431, 0.558, 0.804\}$	$\{-0.001, 0.002, -0.001, -0.7\%\}$
$(A_2, \sigma_2, \lambda_1, \tau_2)$	$\{0.023, 0.397, 0.580, 0.813\}$	$\{0.001, -0.002, 0.002, -1.0\%\}$
$(A_2, \sigma_2, \lambda_1, \tau_3)$	$\{0.061, 0.291, 0.648, 0.849\}$	$\{0.003, -0.008, 0.005, -0.9\%\}$
$(A_2, \sigma_2, \lambda_2, \tau_1)$	$\{0.022, 0.420, 0.557, 0.806\}$	$\{-0.001, 0.002, -0.001, -0.6\%\}$
$(A_2, \sigma_2, \lambda_2, \tau_2)$	$\{0.043, 0.377, 0.580, 0.812\}$	$\{0.000 - 0.003, 0.003, -1.0\%\}$

Table 10: (continued) A full factorial test bed including $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}\}$ from the simulation and the deviation $[\delta_1, \delta_2, \delta_3, \delta_4]$ from the approximation

	Simulation $\{\hat{P}_1, \hat{P}_2, \hat{P}_3, \hat{L}\}$	Deviation $\{\delta_1, \delta_2, \delta_3, \delta_4\}$
$(A_2, \sigma_2, \lambda_2, \tau_3)$	$\{0.110, 0.264, 0.626, 0.849\}$	$\{0.004, 0.002, -0.007, -0.3\%\}$
$(A_2, \sigma_2, \lambda_3, \tau_1)$	$\{0.035, 0.408, 0.557, 0.800\}$	$\{0.002, 0.000, -0.002, -1.2\%\}$
$(A_2, \sigma_2, \lambda_3, \tau_2)$	$\{0.062, 0.366, 0.572, 0.811\}$	$\{-0.002, 0.006, -0.004, -1.1\%\}$
$(A_2, \sigma_2, \lambda_3, \tau_3)$	$\{0.146, 0.225, 0.629, 0.830\}$	$\{0.001, -0.006, 0.004, -2\%\}$
$(A_2, \sigma_3, \lambda_1, \tau_1)$	$\{0.011, 0.408, 0.581, 0.762\}$	$\{0.000, -0.002, 0.002, -0.8\%\}$
$(A_2, \sigma_3, \lambda_1, \tau_2)$	$\{0.023, 0.382, 0.595, 0.771\}$	$\{0.001, -0.001, 0.000, -1.0\%\}$
$(A_2, \sigma_3, \lambda_1, \tau_3)$	$\{0.058, 0.296, 0.646, 0.809\}$	$\{0.001, -0.001, 0.000, -0.5\%\}$
$(A_2, \sigma_3, \lambda_2, \tau_1)$	$\{0.022, 0.390, 0.588, 0.758\}$	$\{0.000, -0.009, 0.009, -1.2\%\}$
$(A_2, \sigma_3, \lambda_2, \tau_2)$	$\{0.039, 0.365, 0.596, 0.762\}$	$\{-0.004, 0.002, 0.002, -2.0\%\}$
$(A_2, \sigma_3, \lambda_2, \tau_3)$	$\{0.108, 0.256, 0.635, 0.800\}$	$\{0.006, -0.003, -0.003, -0.9\%\}$
$(A_2, \sigma_3, \lambda_3, \tau_1)$	$\{0.034, 0.389, 0.577, 0.749\}$	$\{0.001, 0.001, -0.001, -2.5\%\}$
$(A_2, \sigma_3, \lambda_3, \tau_2)$	$\{0.059, 0.343, 0.598, 0.764\}$	$\{-0.004, -0.001, 0.005, -1.7\%\}$
$(A_2, \sigma_3, \lambda_3, \tau_3)$	$\{0.144, 0.226, 0.630, 0.799\}$	$\{0.003, -0.003, -0.001, -0.5\%\}$
$(A_3, \sigma_1, \lambda_1, \tau_1)$	$\{0.004, 0.068, 0.929, 0.982\}$	$\{0.000, 0.003, -0.004, -0.8\%\}$
$(A_3, \sigma_1, \lambda_1, \tau_2)$	$\{0.005, 0.049, 0.946, 0.983\}$	$\{-0.001, 0.001, -0.001, -0.9\%\}$
$(A_3, \sigma_1, \lambda_1, \tau_3)$	$\{0.007, 0.043, 0.950, 0.991\}$	$\{0.000, 0.003, -0.004, -0.2\%\}$
$(A_3, \sigma_1, \lambda_2, \tau_1)$	$\{0.005, 0.057, 0.938, 0.979\}$	$\{-0.001, -0.004, 0.005, -1.0\%\}$
$(A_3, \sigma_1, \lambda_2, \tau_2)$	$\{0.011, 0.042, 0.947, 0.980\}$	$\{0.001, -0.003, 0.002, -1.2\%\}$
$(A_3, \sigma_1, \lambda_2, \tau_3)$	$\{0.014, 0.035, 0.951, 0.982\}$	$\{0.001, -0.001, 0.000, -1.0\%\}$
$(A_3, \sigma_1, \lambda_3, \tau_1)$	$\{0.008, 0.053, 0.938, 0.974\}$	$\{-0.001, -0.006, 0.007, -1.5\%\}$
$(A_3, \sigma_1, \lambda_3, \tau_2)$	$\{0.015, 0.038, 0.947, 0.981\}$	$\{0.000, -0.004, 0.004, -1.1\%\}$
$(A_3, \sigma_1, \lambda_3, \tau_3)$	$\{0.020, 0.032, 0.949, 0.990\}$	$\{0.002, -0.002, 0.000, -0.2\%\}$
$(A_3, \sigma_2, \lambda_1, \tau_1)$	$\{0.005, 0.138, 0.857, 0.948\}$	$\{-0.001, -0.004, 0.005, -0.5\%\}$
$(A_3, \sigma_2, \lambda_1, \tau_2)$	$\{0.013, 0.114, 0.873, 0.945\}$	$\{0.000, -0.003, 0.003, -1.6\%\}$
$(A_3, \sigma_2, \lambda_1, \tau_3)$	$\{0.015, 0.111, 0.873, 0.940\}$	$\{0.000, 0.004, -0.004, -2.4\%\}$
$(A_3, \sigma_2, \lambda_2, \tau_1)$	$\{0.013, 0.136, 0.851, 0.954\}$	$\{0.001, 0.001, -0.002, 0.1\%\}$
$(A_3, \sigma_2, \lambda_2, \tau_2)$	$\{0.023, 0.102, 0.874, 0.941\}$	$\{0.000, -0.005, 0.006, -2.0\%\}$
$(A_3, \sigma_2, \lambda_2, \tau_3)$	$\{0.027, 0.094, 0.879, 0.948\}$	$\{-0.001, -0.003, 0.004, -1.4\%\}$
$(A_3, \sigma_2, \lambda_3, \tau_1)$	$\{0.018, 0.133, 0.849, 0.940\}$	$\{0.001, 0.002, -0.003, -1.3\%\}$
$(A_3, \sigma_2, \lambda_3, \tau_2)$	$\{0.036, 0.092, 0.872, 0.954\}$	$\{0.003, -0.008, 0.005, -0.5\%\}$
$(A_3, \sigma_2, \lambda_3, \tau_3)$	$\{0.042, 0.085, 0.873, 0.947\}$	$\{0.003, -0.003, 0.000, -1.4\%\}$
$(A_3, \sigma_3, \lambda_1, \tau_1)$	$\{0.007, 0.175, 0.818, 0.905\}$	$\{0.000, -0.001, 0.001, -1.2\%\}$
$(A_3, \sigma_3, \lambda_1, \tau_2)$	$\{0.016, 0.150, 0.834, 0.917\}$	$\{0.001, 0.000, -0.001, -0.9\%\}$
$(A_3, \sigma_3, \lambda_1, \tau_3)$	$\{0.021, 0.146, 0.833, 0.924\}$	$\{0.003, 0.004, -0.007, -0.4\%\}$
$(A_3, \sigma_3, \lambda_2, \tau_1)$	$\{0.013, 0.167, 0.820, 0.910\}$	$\{0.000, -0.002, 0.003, -0.7\%\}$
$(A_3, \sigma_3, \lambda_2, \tau_2)$	$\{0.032, 0.137, 0.831, 0.899\}$	$\{0.002, 0.000, -0.002, -2.8\%\}$
$(A_3, \sigma_3, \lambda_2, \tau_3)$	$\{0.035, 0.124, 0.842, 0.915\}$	$\{0.000, -0.004, 0.004, -1.3\%\}$
$(A_3, \sigma_3, \lambda_3, \tau_1)$	$\{0.020, 0.170, 0.810, 0.915\}$	$\{0.000, 0.007, -0.007, -0.1\%\}$
$(A_3, \sigma_3, \lambda_3, \tau_2)$	$\{0.043, 0.121, 0.836, 0.902\}$	$\{0.001, -0.006, 0.005, -2.4\%\}$
$(A_3, \sigma_3, \lambda_3, \tau_3)$	$\{0.052, 0.117, 0.831, 0.916\}$	$\{0.004, 0.001, -0.005, -1.0\%\}$

Table 11: The simulation results and the approximation results for Gap 1, Gap 2 and $(\hat{A}^* - A^*)/\hat{A}^*$

Ω	$\{(\hat{A}^* - A^*)/\hat{A}^*, Gap1, Gap2\}$
$(\sigma_1, \lambda_1, \tau_1)$	$\{5.00\%, 2.07\%, 2.63\%\}$
$(\sigma_1, \lambda_1, \tau_2)$	$\{2.50\%, 0.51\%, 0.33\%\}$
$(\sigma_1, \lambda_1, \tau_3)$	$\{6.67\%, 1.40\%, 1.34\%\}$
$(\sigma_1, \lambda_2, \tau_1)$	$\{2.50\%, 0.10\%, 0.30\%\}$
$(\sigma_1, \lambda_2, \tau_2)$	$\{2.50\%, 2.75\%, 3.04\%\}$
$(\sigma_1, \lambda_2, \tau_3)$	$\{3.33\%, 2.41\%, 2.27\%\}$
$(\sigma_1, \lambda_3, \tau_1)$	$\{2.50\%, 3.21\%, 2.17\%\}$
$(\sigma_1, \lambda_3, \tau_2)$	$\{5.00\%, 2.29\%, 1.46\%\}$
$(\sigma_1, \lambda_3, \tau_3)$	$\{6.67\%, 1.76\%, 1.12\%\}$
$(\sigma_2, \lambda_1, \tau_1)$	$\{2.50\%, 1.96\%, 2.54\%\}$
$(\sigma_2, \lambda_1, \tau_2)$	$\{2.50\%, 1.61\%, 1.67\%\}$
$(\sigma_2, \lambda_1, \tau_3)$	$\{6.67\%, 1.83\%, 3.01\%\}$
$(\sigma_2, \lambda_2, \tau_1)$	$\{5.00\%, 2.81\%, 2.14\%\}$
$(\sigma_2, \lambda_2, \tau_2)$	$\{5.00\%, 1.89\%, 1.96\%\}$
$(\sigma_2, \lambda_2, \tau_3)$	$\{6.67\%, 2.74\%, 2.83\%\}$
$(\sigma_2, \lambda_3, \tau_1)$	$\{2.50\%, 2.24\%, 0.93\%\}$
$(\sigma_2, \lambda_3, \tau_2)$	$\{5.00\%, 3.13\%, 2.94\%\}$
$(\sigma_2, \lambda_3, \tau_3)$	$\{3.33\%, 2.54\%, 2.47\%\}$
$(\sigma_3, \lambda_1, \tau_1)$	$\{-25.0\%, 2.74\%, 1.27\%\}$
$(\sigma_3, \lambda_1, \tau_2)$	$\{5.00\%, 4.38\%, 5.21\%\}$
$(\sigma_3, \lambda_1, \tau_3)$	$\{3.33\%, 1.42\%, 1.69\%\}$
$(\sigma_3, \lambda_2, \tau_1)$	$\{-25.00\%, 2.12\%, 0.76\%\}$
$(\sigma_3, \lambda_2, \tau_2)$	$\{2.50\%, 3.84\%, 3.00\%\}$
$(\sigma_3, \lambda_2, \tau_3)$	$\{3.33\%, 3.82\%, 3.10\%\}$
$(\sigma_3, \lambda_3, \tau_1)$	$\{-25.0\%, 2.21\%, 1.61\%\}$
$(\sigma_3, \lambda_3, \tau_2)$	$\{2.50\%, 4.79\%, 4.82\%\}$
$(\sigma_3, \lambda_3, \tau_3)$	$\{3.33\%, 3.40\%, 2.56\%\}$

Table 12: The cost saving potential of including opportunities at USDs and SDs: $\triangle_A$, $\triangle_B$ and $\triangle_C$.

Ω	$\triangle_A$	$\triangle_B$	$\triangle_C$
$(\sigma_1, \lambda_1, \tau_1)$	65.4%	60.3%	19.6%
$(\sigma_1, \lambda_1, \tau_2)$	65.2%	60.2%	19.6%
$(\sigma_1, \lambda_1, \tau_3)$	61.4%	55.9%	19.6%
$(\sigma_1, \lambda_2, \tau_1)$	65.3%	60.3%	30.4%
$(\sigma_1, \lambda_2, \tau_2)$	64.9%	60.2%	30.4%
$(\sigma_1, \lambda_2, \tau_3)$	61.0%	55.9%	30.4%
$(\sigma_1, \lambda_3, \tau_1)$	65.2%	60.3%	36.1%
$(\sigma_1, \lambda_3, \tau_2)$	64.8%	60.2%	36.1%
$(\sigma_1, \lambda_3, \tau_3)$	60.5%	55.9%	36.1%
$(\sigma_2, \lambda_1, \tau_1)$	48.9%	41.7%	14.1%
$(\sigma_2, \lambda_1, \tau_2)$	48.5%	41.4%	14.1%
$(\sigma_2, \lambda_1, \tau_3)$	47.1%	39.7%	14.1%
$(\sigma_2, \lambda_2, \tau_1)$	48.7%	41.7%	20.8%
$(\sigma_2, \lambda_2, \tau_2)$	48.1%	41.3%	20.8%
$(\sigma_2, \lambda_2, \tau_3)$	46.4%	39.7%	20.8%
$(\sigma_2, \lambda_3, \tau_1)$	48.5%	41.7%	24.3%
$(\sigma_2, \lambda_3, \tau_2)$	47.7%	41.4%	24.3%
$(\sigma_2, \lambda_3, \tau_3)$	45.9%	39.7%	24.3%
$(\sigma_3, \lambda_1, \tau_1)$	33.9%	25.0%	9.1%
$(\sigma_3, \lambda_1, \tau_2)$	33.6%	24.7%	9.1%
$(\sigma_3, \lambda_1, \tau_3)$	32.2%	23.0%	9.1%
$(\sigma_3, \lambda_2, \tau_1)$	33.7%	25.0%	12.9%
$(\sigma_3, \lambda_2, \tau_2)$	33.1%	24.7%	12.9%
$(\sigma_3, \lambda_2, \tau_3)$	31.4%	23.0%	12.9%
$(\sigma_3, \lambda_3, \tau_1)$	33.4%	25.0%	14.6%
$(\sigma_3, \lambda_3, \tau_2)$	32.6%	24.7%	14.6%
$(\sigma_3, \lambda_3, \tau_3)$	30.7%	23.0%	14.6%

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