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Design approach to obtain a near-optimal generalized ABC classification for a multi-item inventory control problem

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Abstract

In this paper we consider a multi-item, single-location inventory control problem. We propose an approach on how to design a generalized ABC classification scheme to minimize the inventory investment costs while satisfying an aggregate fill rate constraint by considering four different aspects of a classification: the classification criteria, the number of classes, the class sizes, and the target fill rate per class. Using multiple alternatives per aspect and considering all combinations, we determine the best classification. The results are measured against the optimal solution, which allows a separate target fill rate for each SKU and therefore is more difficult to manage. By applying our approach, we are able to find near-optimal results for all three data sets, consisting of spare parts data from three different companies. The most important insight that we generate is that the best class sizes - which need to be carefully decided - depend on the target aggregate fill rate whereas the common used rule of thumb to determine class sizes may lead to solutions far from optimal.

Keywords: Inventory control, classification, SKUs, system-oriented service constraints, greedy heuristics

1. Introduction

A proper management of the inventory is of great importance for many companies as they have service contracts or want to provide a certain service to their customers, often measured by the aggregate fill rate (see Thonemann et al., 2002). For many practical cases, the number of stock keeping units (SKUs) is too large to apply SKU-specific inventory control methods (Buxey, 2006). Although some companies implement SKU-specific inventory control methods, there are many companies for which this is not possible due to a lack of knowledge about this topic within the company, the preference to implement simple inventory control policies, or due to limited functionality of their ERP systems (if any at all). As a practical solution, many companies resort to the use of ABC classifications (see e.g. Ramanathan, 2006; Hatefi and Torabi, 2015; Flores and Whybark, 1987; Chu et al., 2008; Eilon and Mallya, 1985; Liu et al., 2015). When applying an ABC classification, companies categorize their inventory into three, the so called ‘A’, ‘B’, and ‘C’ classes, and manage each SKU within a class in the same way, where class A represents the most important category receiving the closest attention and class C being the least important one. In this way companies are

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basically able to differentiate between different SKUs while still having a simple inventory control policy. However, the classification of SKUs is typically made using rules of thumb: E.g. 20% of SKUs that have the highest annual dollar volume are grouped under Class A. Such oversimplification is prone to significant suboptimization. In this paper we emphasize that this can be improved by a -still simple- ”generalized” ABC classification approach, in which the classification is made more carefully. In particular, we attempt to provide an answer to our main research question by making use of three data sets, stemming from three different companies: *How should one design a generalized ABC classification in order to obtain the lowest inventory investment costs while ensuring a target aggregate fill rate?*

When considering to use an ABC classifications, there are four aspects that together determine the performance. First of all, as different SKUs have different characteristics, and can be of different importance to the company, SKUs can be ranked based on one or more criteria. There have been many authors that propose different criteria to rank the SKUs. The most common criterion is the annual dollar volume (see e.g. Silver et al., 1998; Stevenson, 2007; Nahmias, 1997). However, this criterion does not set the relation with the resulting inventory costs. As inventory costs can be a large fraction of the total costs, there is a shift in the focus of ABC classifications where inventory costs are taken into consideration when deciding how to rank the SKUs (Ramanathan (2006), Bacchetti and Saccani (2012), Bacchetti et al. (2013), Zhou and Fan (2007), Teunter et al. (2010), Ng (2005), Babai et al. (2015), Hollier and Vrat (1978), Zhang et al. (2001) and Ernst and Cohen (1990)). Multicriteria classifications, where SKUs are classified into three classes based on multiple criteria, have also been proposed (see e.g. Flores et al., 1992). Moreover, in practice we commonly observe two-dimensional classifications where each SKU is classified twice based on each criterion and the final classification is obtained by taking the combination of the previous two classifications. In this way, often nine classes are being used (see e.g. Duchessi et al., 1988; Flores and Whybark, 1987). See Figure 1 for an example of a two-dimensional classification. Although these classifications are applied in practice, they have not been compared with respect to their resulting inventory costs before.

Price	High	C3	B3	A3
	Medium	C2	B2	A2
	Low	C1	B1	A1
		Low	Medium	High
		Demand rate		

Figure 1: *Example of Price and Demand classification*

The second aspect we consider is the number of classes used to classify the SKUs. Although most of the previously mentioned literature considers three classes only, it is possible to extend this to a larger number of classes. A larger number of classes would lead to a reduction of inventory investment costs and increase in performance as three classes may not capture the differences between SKUs sufficiently well (Lee, 2002). However, an increase of the number of classes also makes it more difficult to manage the SKUs. Based on our data, we show that increasing the number of classes results in lower inventory costs, and that six classes are sufficient to obtain near-optimal results for the data sets we examined.

Thirdly, one needs to decide on the class sizes and thus how many SKUs should be put in each class. For ABC classifications with three classes, a common rule of thumb is to have about 20% of the SKUs in class A, 30% in class B and 50% in class C, considering three classes (see e.g. Jacobs et al., 2009; Slack et al., 2007; Stevenson, 2007; Nahmias, 1997). However, when classes are managed by setting different service levels per class, there are no clear guidelines on how to manage each class of SKUs. Van Kampen et al. (2012) state that it is likely to obtain a better performance with more classes, which does bring a higher complexity, but that is still unclear how to manage the class sizes in combination with the number of classes. There are authors that argue that class A SKUs should have the highest service target to avoid frequent backlogs (see e.g. Armstrong, 1985; Stock and Lambert, 2001), whereas other authors claim that class C SKUs should have the highest service as dealing with stockouts is not worth the effort (see e.g. Knod and Schonberger, 2001). Moreover, these suggestions are not considering the relative size of each class together with the desired service or classification criteria used. To the best of our knowledge, besides this rule of thumb, which is also used by Teunter et al. (2010), there is no literature on this topic in relation with inventory costs and service measures. In this paper we show that this rule of thumb can give far from optimal results and that the class size should depend on the aggregate fill rate target.

Finally, one needs to decide on how to manage each class of parts. In this paper this is done by setting a target fill rate for each class. Although most literature only considers the classification criteria used to classify the SKUs and does not relate to the resulting inventory costs, there are a few authors that do consider the resulting inventory costs (see e.g. Teunter et al., 2010; Babai et al., 2015). The first authors to combine target setting with a classification are Teunter et al. (2010). However, they assume that each SKU is put on stock as they only consider strictly positive target fill rates, which is common in some fields, but not common for spare parts. As our focus is solely on the overall performance from an inventory cost perspective, and because we make use of spare parts data, stocking all SKUs may lead to additional costs, and thus suboptimal results. Especially when there are large cost differences between the SKUs it is not desirable to stock all SKUs (Sherbrooke, 1968).

As these four aspects cannot be considered independently of the others (e.g. the target service level of a class depends on how many classes there are), we examine each combination of all four aspects together, which to the best of our knowledge has not been done before. We consider three possible classification criteria, including two-dimensional classifications which have not been compared before, vary the number of classes, use enumeration over possible class sizes, and set the

optimal target fill rate per class. This approach is applied on three different data sets consisting of real life spare parts data of three different companies, with different characteristics. We compare the outcome of the classification with the optimal solution, which stands for having a separate class for each SKU (see e.g. Sherbrooke, 1968; Van Houtum and Kranenburg, 2015).

The main contribution of this paper is our approach to obtain a generalized and simple ABC classification that takes the above four aspects jointly into account. Using a structured scheme, we are able to design a classification which minimizes inventory investment costs while meeting the aggregate fill rate constraint. Using real life spare parts data and by comparing multiple classification criteria, we show that when these four aspects are combined, the resulting inventory investment costs can be within one percent of the optimal results, which brings the expense of more complicated inventory control. Our approach is not restricted to the chosen criteria and classes, but can be widely applied to other criteria, demand distributions, and even inventory policies. Beside our approach, we show the importance of having the right class sizes, which in relation with the inventory costs is to the best of our knowledge not studied before. Using the wrong class sizes can have a big impact on the performance and result in far from optimal solutions. We compared the best class sizes with class sizes determined using the common rule of thumb where about 20% of the SKUs are in class A, 30% in class B and 50% in class C (cf. Teunter et al., 2010). Cost differences up to 290% are possible and more likely to occur for higher aggregate fill rate targets. Because we use enumeration to determine the optimal target per class, our approach can become time consuming when the number of classes increases. For testing and sensitivity analysis it may be of interest to have a faster solution and therefore we also propose an algorithm that significantly decreases the calculation time for our classification scheme, whereas the cost increase on average is below 2%.

The remainder of the paper is organized as follows: In Section 2 we introduce the model description. In Section 3 we give our approach on how to design a generalized ABC classification scheme that considers all of the four aspects jointly. In Section 4 we first explain our real life data set of spare parts, which we use for our numerical experiment. We then present the optimality gap of the alternatives that we consider and we conduct some analysis to obtain more insights regarding how to set the right class sizes and propose an algorithm to set the target fill rates per class in a fast way. We conclude our paper and give some recommendations for future research in Section 5.

2. Model

We consider a single warehouse in which several SKUs are kept on stock. Whenever a SKU is requested, it is immediately delivered from stock on hand, and backordered and delivered as soon as possible when there is no stock on hand. The set of SKUs is denoted by I ; the number of SKUs is $|I| \in \mathbb{N} := \{1, 2, \dots\}$. For notational convenience, the SKUs are numbered as $1, \dots, |I|$. For each SKU the stock is controlled by a continuous-review basestock policy, with basestock level S_i for SKU i . For each SKU $i \in I$, we assume that demand follows an independent Poisson process

with a constant rate $\lambda_i (\geq 0)$, which is particularly plausible in the maintenance and spare parts environment, but also in some retail environments. The total demand rate for all SKUs is denoted by $\Lambda = \sum_{i \in I} \lambda_i$. The replenishment lead time $L_i (> 0)$ is assumed to be constant. Each SKU has a price of c_i , and the total inventory investment costs are given by

$$C(\mathbf{S}) = \sum_{i \in I} c_i S_i, \quad (1)$$

where $\mathbf{S} = (S_1, \dots, S_{|I|})$ denote the vector consisting of all basestock levels. The fill rate for item i , in steady state, is denoted by $\beta_i(S_i)$. The (aggregate) fill rate in steady state is as follows:

$$\beta(\mathbf{S}) = \sum_{i \in I} \frac{\lambda_i}{\Lambda} \beta_i(S_i). \quad (2)$$

Note that the calculation for the fill rate is as follows (see also Van Houtum and Kranenburg, 2015)

$$\beta_i(S_i) = \sum_{x=0}^{S_i-1} P\{X_i = x\},$$

$$P\{X_i = x\} = \frac{(\lambda_i L_i)^x}{x!} e^{-\lambda_i L_i}, x \in \mathbb{N}_0.$$

The target aggregate fill rate for $\beta(\mathbf{S})$ is given by β^o . The objective is to minimize the total costs subject to an aggregate fill rate constraint. In mathematical terms the optimization problem is as follows:

$$\begin{aligned} \text{(P)} \quad & \min && C(\mathbf{S}) \\ & \text{subject to} && \beta(\mathbf{S}) \geq \beta^o \\ & && S_i \in \mathbb{N}_0, \forall i \in I \end{aligned}$$

As managing each SKU individually can be very complex, especially for a large number of SKUs, companies often resort to the use of ABC classifications. In the next section, we show how we design a generalized ABC classification to obtain minimum inventory investment costs against the aggregate fill rate target.

Although we made some assumptions on the demand process and the inventory control policy our approach can be easily applied with other demand processes and/or inventory control policies as this merely changes the evaluation of the fill rate and costs. The same steps of our approach can still be applied to design the ABC classification.

3. Design aspects of a generalized ABC classification

In this section we propose a design of a generalized ABC classification scheme. As the four different aspects cannot be considered separately, we consider each combination and their resulting inventory investment costs. We describe the four aspects in the same order as we apply our scheme. We first describe the different classification criteria we consider in this research. Then we explain the number of classes we consider for each of the four different classification criteria. Thirdly, we explain how we set the class sizes. Finally, we describe how we set the target service level for each class.

3.1 Classification criteria

When considering a classification for SKUs, we first need to define the classification criteria that enable us to rank the SKUs. Once we have ranked the SKUs we can take the other three aspects into consideration. There has been extensive studies on how to classify SKUs, and recently there were also authors that relate the classification with the inventory control (see e.g. Babai et al., 2015; Teunter et al., 2010; Ng, 2005). The annual dollar volume - i.e. annual demand times price - (ADV) criterion, is the traditional criterion for ABC classifications and found in many textbooks (see e.g. Silver et al., 1998; Nahmias, 1997). However, if we follow the idea of a system approach (Sherbrooke, 1986), parts with a high price end up with a lower service level, whereas parts with a high demand end up with a higher service level. Therefore, ADV is a poor construction in terms of inventory investment costs as demand and price work in opposite directions as under the system approach. This was also observed by Teunter et al. (2010) and they proposed a classification based on Demand/Price (D/P), which captures the idea of a higher service for cheap and fast moving SKUs. They found that this classification criterion was the best performing criterion among the criteria they considered.

In practice, two-dimensional classifications based on price and demand are often seen to manage the inventory (see e.g. Duchessi et al., 1988; Flores and Whybark, 1987), but they have not been compared before. This two-dimensional classification classifies SKUs based on both price and demand. We compare this two-dimensional classification to the previous mentioned methods. Note that we also compared some other classification criteria; leadtime demand, price, and leadtime and price. We omit them in our exposition as they did not give better results or further insights. Therefore, we present the results and findings of the following three criteria:

1. **Annual Dollar volume (ADV)** (one-dimensional)

This single criterion classification is the most common criterion. The demand rate of each SKU is multiplied with the price of each SKU and then ordered based on this annual dollar volume. The SKUs are ranked in descending order, where SKUs with the largest ADV are considered class A SKUs.

2. **Demand/Price (D/P)** (one-dimensional)

This classification is proposed by Teunter et al. (2010), and shown to give the best performance related to inventory costs. The SKUs are classified based on their demand rate divided by their price. They are then ranked in descending order, where the top group of SKUs, consisting of fast moving and cheap SKUs, are considered class A SKUs which should get the highest service level.

3. **Price and Demand (P&D)** (two-dimensional)

This classification is often found in practice: Each SKU is classified based on both price and demand. See Figure 1 for an example of this classification.

3.2 Number of classes

When considering ABC classifications, often three classes per dimension are considered: Class A, class B, and class C. However, it is also possible to consider different numbers of classes when classifying SKUs. As less classes will most likely end up with higher investment costs, it makes the problem less complex. Increasing the number will most likely give better results because there are more options available, but this will increase the complexity of the problem. Therefore, there is a trade-off between performance and manageability. Graham (1987) and Silver et al. (1998) mention that up to six classes can be used as more classes increases the complexity. Using six classes, Teunter et al. (2010) showed that good results can already be obtained. For the two-dimensional classification, a matrix approach is common with 9 classes Duchessi et al. (1988). Having a 4 by 4 matrix would lead to 16 classes which becomes too difficult and costly to manage.

In our numerical experiment we vary the number of classes between 2 and 6 for the one-dimensional classifications. For the two-dimensional classification we consider a 2 by 2 matrix and a 3 by 3 matrix. Moreover, we add the results of using only a single class to manage all SKUs, thus not applying any differentiation between the SKUs at all. By considering these different class sizes, we obtain in total 13 combinations of ABC classification criteria and number of classes. Given the number of classes, we introduce a set of classes K where classes are numbered $(1, \dots, |K|)$.

3.3 Class sizes

For each of the above mentioned combinations, we need to decide on the size of each class, which in turn determines the class each SKU will be placed in. There has not been any research about the best class sizes for ABC classifications, when considering inventory costs. Typically a rule of thumb for the ADV classification is used (see e.g. Jacobs et al., 2009), in which 20% of the SKUs are class A, 30% class B, and 50% class C. Teunter et al. (2010) also use this rule of thumb for their D/P classification. One of our goals in this paper is to find out whether different class sizes have an impact on the inventory investment costs.

Therefore, we use threshold values to determine which class each SKU is placed into. For example, in the case we have three classes and SKUs are ranked based on their price and the price of a SKU is larger than all threshold values, it will be placed in class C (unlike the logic of the ADV classification). If the price is smaller than the smallest threshold value it will be placed in class A. When there are more than three classes, or multiple dimensions, there are also more threshold values but the logic remains the same.

We then use enumeration over the threshold values, which we set equal to the x -th percentile. With $x \in \{\Delta, 2\Delta, \dots, \lceil 100/\Delta \rceil \Delta\}$, for each threshold value, where Δ is chosen sufficiently small. Based on the threshold values, we determine k_i , $i \in I$, representing the class to which SKU i belongs.

Note that the threshold values are sorted in ascending order as they must be increasing in size, and that if two threshold values are equal, this is the same as having fewer classes.

3.4 Target setting

Given the set of classes K and the class to which each SKU $i \in I$ belongs, noted by k_i , we need to decide on how to manage each class of SKUs. It is common to set different service targets for each class, θ_k , to manage each class (Teunter et al., 2010). Although there are also other ways to manage the inventory of each SKU in a class, for instance by choosing S_i such that the distance to the service level target of that SKU's class, θ_{k_i} , is minimized, we follow the more common approach of Teunter et al. (2010), where the fill rate has to be greater than or equal to the target for each SKU in that class. We thus need to decide on θ_k , $k \in K$, the target fill rate per class. We set the value of S_i , $i \in I$, equal to the lowest integer that satisfies this fill rate target, θ_{k_i} , for SKU i .

The corresponding problem (C) is then as follows:

$$\begin{aligned}
 \text{(C)} \quad & \min && C(\mathbf{S}) \\
 & \text{subject to} && \beta(\mathbf{S}) \geq \beta^o \\
 & && S_i = \min\{x | \beta_i(x) \geq \theta_{k_i}, x \in \mathbb{N}_0\}, \quad \forall i \in I \\
 & && \theta_k \in [0, 1), \quad \forall k \in K
 \end{aligned}$$

Different than what Teunter et al. (2010) does, we allow to have zero stock of SKUs. Sherbrooke (1968) already showed that when we are mainly concerned with the overall performance it is sometimes better to take more risk for the very expensive SKUs because their inventory investment costs are very high, and reduce the risk on many other cheap SKUs.

To obtain the solution that minimizes the inventory investment costs of problem (C), we first look at each class separately, as the classes are independent due to the fact that the class to which each SKU belongs is known at this stage. Let

$$\mathbf{S}^k = \{S_i\}_{i \in I, k_i=k},$$

be the vector of basestock levels for each SKU i in class k , and

$$\Lambda^k = \sum_{i \in I, k_i=k} \lambda_i,$$

be the total demand rate for class k . The aggregate fill rate for class k , $\beta^k(\mathbf{S})$, and the inventory investment costs for class k , $C^k(\mathbf{S}^k)$, are calculated as follows:

$$\begin{aligned}
 \beta^k(\mathbf{S}^k) &= \sum_{i \in I, k_i=k} \frac{\lambda_i \beta_i(S_i)}{\Lambda^k} \\
 C^k(\mathbf{S}^k) &= \sum_{i \in I, k_i=k} c_i S_i.
 \end{aligned}$$

The following is applied for each class $k \in K$, with ϵ being a small positive value close to zero:

1. Set $\theta_k := 0$
 $S_i = 0 \forall k_i = k, i \in I$
 $\mathbf{S}^k = \{S_i\}_{i \in I, k_i=k}$
 Compute $C^k(\mathbf{S}^k)$ and $\beta^k(\mathbf{S}^k)$
 $\mathcal{E}_k := \{(\mathbf{S}^k, C^k(\mathbf{S}^k), \beta^k(\mathbf{S}^k), \theta_k)\}$

2. $\theta_k := \min(\beta_i(S_i) : i \in I, k_i = k)$
 For all $i \in I, k_i = k$: If $\beta_i(S_i) = \theta_k$, then $S_i := S_i + 1$
 $\mathbf{S}^k = \{S_i\}_{i \in I, k_i = k}$
3. Compute $C^k(\mathbf{S}^k)$ and $\beta^k(\mathbf{S}^k)$;
 $\mathcal{E}_k = \mathcal{E}_k \cup \{(\mathbf{S}^k, C^k(\mathbf{S}^k), \beta^k(\mathbf{S}^k), \theta_k)\}$
4. If $\beta^k(\mathbf{S}^k) \geq 1 - \epsilon$, then stop, else go to 2

After this procedure, we have a solution vector per class, \mathcal{E}_k , consisting of the base-stock levels, costs, fill rate, and target fill rate. We then enumerate over all different combinations of solutions per class, θ_k , and look for the combination that gives the lowest inventory investment costs while satisfying the aggregate fill rate target.

4. Numerical experiment

In order to find which classification gives the best result for problem (P), we apply our approach and perform a numerical experiment using empirical data. In Section 4.1 we give an overview of the data we use for our numerical experiment. We show that the D/P classifications gives near-optimal results with only a few classes in Section 4.2. Then in Section 4.3 we show the impact of the class sizes on the performance. Finally we introduce a greedy ABC algorithm that reduces the calculation time significantly in Section 4.4.

4.1 Data

To apply our approach to design a generalized ABC classification we use three spare parts data sets from three different companies. Data set 1 consists of spare parts usage for the maintenance of Ground Support Equipment (GSE) at an airport. The GSE consists of a wide range of vehicles consisting of passenger cars up to specialised equipment such as de-icing vehicles used to remove ice off the wings of airplanes. Although there are many expensive parts that are not frequently used, the data also contains fast moving spare parts that are not expensive. The largest majority of parts can be delivered within 3 weeks. Data set 2 consists of spare parts usage for a company involved in producing and maintaining lithography machines. Compared to the other two companies, the demand for the spare parts are much lower. Moreover, many parts are expensive, with costs up to over a million euro. Besides these high costs and low demand rates, the time it takes to obtain the part from the supplier are relatively high compared to the other two companies, with an average leadtime of 110 days. Data set 3 consists of spare parts used for the maintenance of professional printing machines. The average leadtime and price of spare parts are in between the company 1 and 2. These parts have the highest usage of the three different companies on average, there are also many parts that are both expensive, as well as fast moving. See Figure 2, of the price and demand relationship of company 2 and 3. From this figure, we clearly see that company 3 has parts

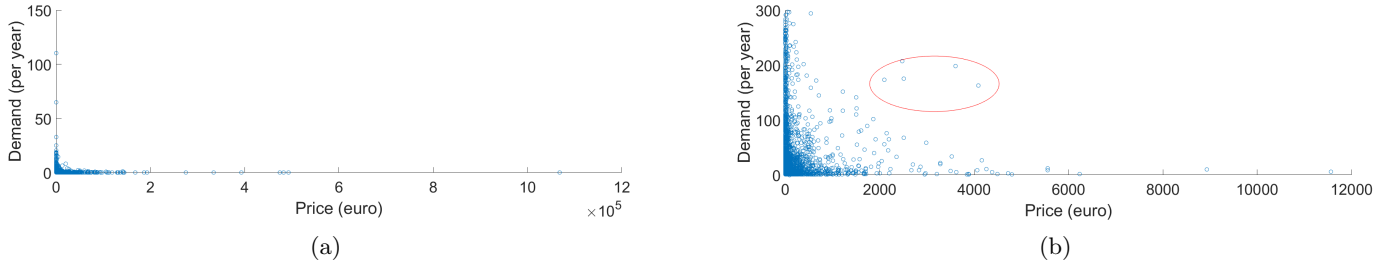


Figure 2: Relation between price and demand for data set 2 (a) and 3 (b)

that are both expensive, as well as fast moving (the parts within the red circle), whereas this is not the case for company 2, which has much more slower moving parts, which can be very expensive.

An overview of the characteristics of the data of all three companies is presented in Table 1.

Table 1: Data characteristics

	Data set 1			Data set 2			Data set 3		
	Demand per year	Price (€)	Leadtime (days)	Demand per year	Price (€)	Leadtime (days)	Demand per year	Price (€)	Leadtime (days)
Min	0.10	0.01	1.00	0.0003	0.01	5.18	1	0.01	14.02
Average	8.53	78.61	5.97	0.4329	4,819.89	106.84	26.82	142.51	28.18
Max	610.50	10,950	111.00	110.29	1,068,444	320.29	297	4,381.59	42.01

4.2 Choosing the best classification criteria

In this section we present our findings regarding the inventory investment costs for the different classifications. For our numerical study, we choose to look at service level targets of 80%, 90%, and 97%, representing a low, medium and high service. We have also looked at different targets in between, but the results were similar and therefore we only present the results for these three different service level targets. Note that our methodology is not restricted with a certain service level or measure. In Table 2 we present our findings regarding the inventory investment costs for the different classifications of all three data sets. We represent the costs as a percentage of the system approach, which is considered the optimal solution. To apply the system approach, we use the algorithm as described in Section 2.4.2 of Van Houtum and Kranenburg (2015). The results regarding the ADV are in line with the results of Teunter et al. (2010): we clearly see that the ADV classification gives very poor results. We did not show the findings for the ADV classification for 5 and 6 classes as they only give slightly better results, but still far from optimal. An explanation for this is that both fast moving SKUs as well as expensive SKUs can be put in the same class of SKUs according to this criterion. However, from the inventory investment cost perspective, we prefer to stock less expensive SKUs over expensive SKUs, because expensive SKUs bring higher inventory investment costs. Moreover, it is more interesting to stock fast moving SKUs over slow

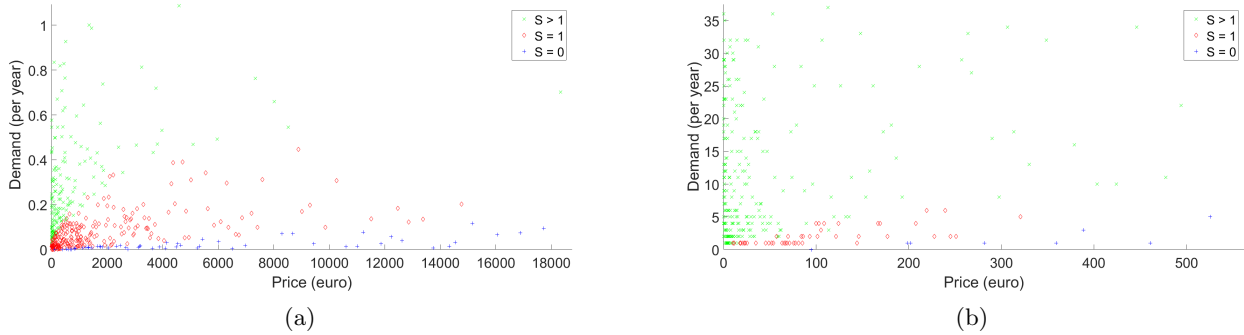


Figure 3: Base stock levels for data set 2 (a) and 3 (b) for optimal solution

moving SKUs because the contribution to the the aggregate fill rate of fast moving SKUs is larger.

The classification that leads to the lowest inventory investment costs is the D/P classification. The D/P classification captures the trade-off between the contribution to the aggregate fill rate and costs very well, therefore the SKUs in the same class will end up with similar fill rate targets as if we apply a system approach. Figure 3 shows the inventory levels for each SKU for data set 2 and 3 for the optimal solution and confirms this thought. We find that by applying the D/P classification with six classes, and setting the best class sizes and target fill rates, the resulting inventory investment costs are within a few percent of the optimal solution for all three data sets. The P&D classification is also able to capture this trade-off, although it does not directly capture this trade-off for each SKU. Although we see that the costs for the Price and Demand classification are higher compared to the D/P classification, the resulting costs are still very reasonable, especially compared to using no classification or the ADV classification. For data set 3 it even gets within 6 percent of the optimal solution. This is mainly due to the fact that this data set has parts that are both fast moving as well as expensive. Using the Price and Demand classification, these parts can be handled separately, thereby providing more flexibility in this region.

Table 2: Inventory investment costs compared to optimal solution

Inventory investment costs (% over optimal)		Data set 1			Data set 2			Data set 3		
Classification	Number of classes	80%	90%	97%	80%	90%	97%	80%	90%	97%
Annual Dollar Volume	2	3,098	1,278	330	1,595	670	229	87	79	38
	3	2,753	1,217	329	1,595	665	224	79	70	35
	4	2,749	1,216	327	1,595	665	223	77	69	34
Demand/Price	2	19	38	47	44	43	53	7	42	17
	3	12	4	6	9	15	6	2	6	12
	4	9	2	3	4	10	2	1	4	10
	5	9	1	1	4	7	1	1	3	5
	6	7	1	1	2	7	0	1	2	2
Price and Demand	4 (2x2)	40	26	26	74	50	25	5	25	21
	9 (3x3)	23	12	22	23	16	14	2	6	6
One class		3,300	1,294	342	1,628	711	328	132	120	61

When we look at the number of classes used for the D/P classifications, we clearly see that when only three classes are considered, already a significant gain with respect to two classes can be obtained. Whenever the number of classes is increased further, the performance can only increase as the same solution as with less classes is always possible. As an increase in the number of classes makes it more difficult and costly to control the inventory policy, a company should make the trade-off between costs and complexity.

4.3 The impact of class sizes on performance

When considering the size of each class, we would like to see how sensitive the results are with respect to the class sizes. As the D/P criterion gives the best overall results, we consider the impact of the class sizes for this criterion only from now on. There has not been much literature about the proper class sizes, besides the common proposal to have around 20% of the SKUs in class A, 30% in class B and 50% in class C (Teunter et al., 2010). However, this rule of thumb makes use of the pareto rule and does not take into consideration the impact on the resulting inventory costs. Based on our numerical experiment, a key observation is that there are large inventory investment cost differences by only changing the class sizes. We also noticed that for the best solutions found, there is always at least one class for which SKUs are not stocked. This is always the class for which the value of D/P is the smallest, thus consisting of the expensive and/or slow moving SKUs. Because we consider spare parts data these SKUs have very high individual costs, and thus a very large impact on the total inventory costs as soon as the company chooses to stock these SKUs. Taking this into consideration, and given the aggregate fill rate target, we know the fraction of demand that does not have to be satisfied directly from stock on hand. Whenever the fraction of total demand in the class of expensive SKUs is larger than the fraction of total demand that does not have to be satisfied from stock on hand directly, it is impossible to find a feasible solution without stocking all SKUs in that class. We call this a “forced presence” of stock for these SKUs. Whenever a forced presence of stock occurs, the inventory costs are significantly larger than the best solution possible. This impact will be smaller if the cost differences are also smaller. Still we expect that using our approach instead of the commonly used rule of thumb, where the fraction of demand of this class of SKUs is set to 50% (see e.g. Teunter et al., 2010), leads to a better solution in terms of costs. Figure 4 shows the fraction of total demand in this class using the best target setting for different aggregate fill rate targets when there are three classes. We find that the fraction of demand in this class is always close to the maximum allowed fraction of demand that does not result in a forced presence of the SKUs. We clearly observe the jump in inventory costs due to the forced presence of stock. Thus, for higher aggregate fill rate targets, the total fraction of demand in this class should be smaller than for lower targets.

In Table 3 we present our findings when we would use the commonly used rule of thumb, as suggested by Teunter et al. (2010), to divide the SKUs over the classes. We compare these findings with the findings when we set the best class sizes. Based on our data, even if the target fill rates are set optimally, but class sizes are not considered properly, huge cost differences are found, with

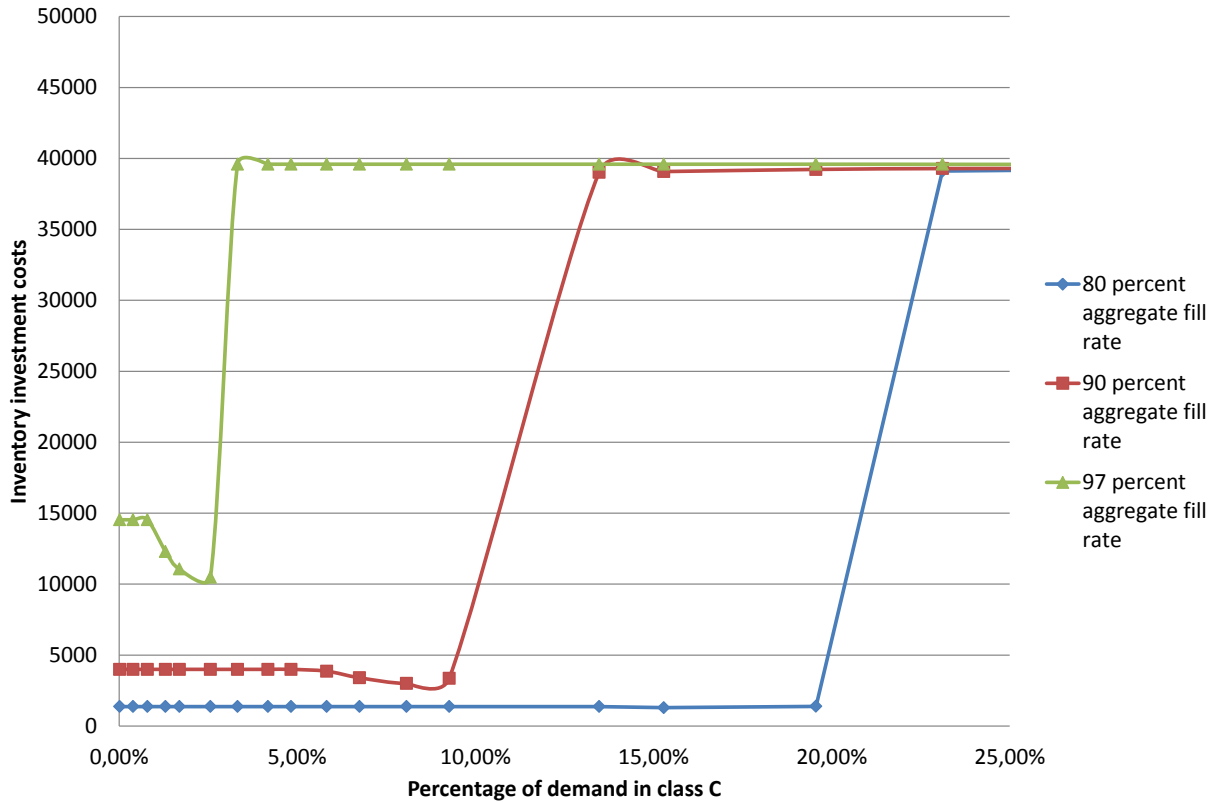


Figure 4: Impact of different class sizes on investment costs for data set 1

costs over 300% higher than the optimal solution. This difference is mainly explained by the forced presence of class C SKUs. However, we should note that the cost differences between SKUs play an important role in the magnitude of the impact, which is common in the case of spare parts. If there would be no cost difference, it would make less benefit to optimize the class size, but at the same time there would be less benefit to differentiate parts as well. Moreover, if it is required to have strictly positive basestock levels (so that all SKUs are really kept on stock), we do always have forced presence of stock and thus setting the best class size would have less impact, although gains should still be possible.

4.4 ABC classification target setting

In order to apply our approach, we make use of enumeration to determine the optimal target setting for each class, as described in Section 3.4. Unfortunately, when the number of classes is increasing, the run time is increasing exponentially; for the D/P classification with five classes and an aggre-

Table 3: Inventory investment costs over optimal for data set 1 based on D/P classification

Number of classes	Target aggregate fill rate		
	80%	90%	97%
3 best	12	4	6
3 rule of thumb	180	22	302
4 best	9	2	3
4 rule of thumb	14	60	301

gate fill rate target of 90% the run times are around 45 minutes. Although these calculations are not done on a regular basis, and thus may not have a big impact, companies may be interested in sensitivity analysis to see how the total inventory investment costs change with respect to the aggregate fill rate in order to set their goals. Therefore, we propose the greedy ABC algorithm to set the target fill rates for each class based on getting the “biggest bang for the buck”. We increase θ_k with a small step size, θ^{step} , for the class where we get the biggest increase in aggregate fill rate per euro invested. This ratio, Γ_k , is defined as the difference in aggregate fill rate after increasing θ_k , divided by the difference in inventory investment costs. Because we do not have to calculate all solutions for each class and use enumeration, this results in significantly reduced run times. The greedy ABC algorithm is as follows:

The run-times using this algorithm are around 0.15 seconds when there are two classes and up to around 980 seconds for six classes on a Pentium Intel(R) Core(TM) i5-2520M CPU with 4GB memory. We find that savings in calculation time of around factor 20 are obtained for five classes, and increasing for larger numbers of classes. The increase in costs compared to the optimal target setting, over all different targets per class with more than three classes, is less than two percent on average. Thus for a larger number of classes or a larger number of SKUs, the greedy ABC algorithm can significantly speed up the calculation time of our approach whereas the cost increase is limited.

5. Conclusions and suggestions for further research

In this paper we propose an approach on how to design a generalized ABC classification to minimize the inventory investment costs by combining four different key aspects: Classification criteria, number of classes, class sizes, target fill rates per class.

Using our structured approach on empirical data we show that it is possible to obtain a solution within a few percent of the optimal solution by using D/P as classification criterion. We show that this classification criteria captures the idea of the system approach very well, and that even though the Demand and Price classification is commonly used in practice and gives decent results, especially for one of the data sets, it is still outperformed by the D/P classification.

Another key insight is that it is of great importance to have the proper class sizes, as using

Algorithm 1 Greedy ABC algorithm

1. Set $\theta_k := 0$ for all $k \in K$
 $S_i = 0$ for all $i \in I$
 $\mathbf{S} = (S_1, S_2, \dots, S_{|I|})$
Compute $C(\mathbf{S})$ and $\beta(\mathbf{S})$
 2. For all $k \in K$:
 - (a) For all $i \in I$:
If $k_i = k$: $v_i = \min \{x | \beta_i(x) \geq \theta_{k_i} + \theta^{step}, x \in \mathbb{N}_0\} - S_i$
else, $v_i = 0$
 - (b) $\mathbf{v}_k = (v_1, v_2, \dots, v_{|I|})$
 $\Gamma_k := (\beta(\mathbf{S} + \mathbf{v}_k) - \beta(\mathbf{S}_k)) / (C(\mathbf{S} + \mathbf{v}_k) - C(\mathbf{S}))$
 3. $m := \arg \max \{\Gamma_k : k \in K\}$
 $\mathbf{S} := \mathbf{S} + \mathbf{v}_m$
 $\theta_k := \min \{\beta_i(\mathbf{S}) : i \in I, k_i = k\}$ for all $k \in K$
 4. Compute $C(\mathbf{S})$ and $\beta(\mathbf{S})$;
If $\beta^k(\mathbf{S}) \geq \beta^o$, then stop, else go to 2
-

a rule of thumb as proposed by Teunter et al. (2010) leads to costs that are far from optimal, with costs differences of over 280% compared to the optimal costs. This large difference is mainly explained by the forced presence of stock of expensive and slow moving SKUs, although there are also significant gains when there is no forced presence of stock. When using a simple rule of thumb, it is important that the target aggregate fill rate is taken into consideration when determining the class sizes.

We also propose a greedy ABC algorithm which significantly reduces run times, whereas the cost increase of using this algorithm is only limited to a few percent on average. Although the design of a classification is not something done often in practice, it does allow for fast sensitivity analysis to support decision making using our approach.

For further research it may be interesting to find simple rules to determine proper class sizes instead of enumeration, which may further reduce the run time of our approach. Moreover, we used a basestock policy for each class to manage the inventory, whereas it might be interesting to apply different policies for different classes. Especially if transport costs are involved for each order. By including these costs and batching for one or more of the classes, it would be interesting to see whether the D/P classification would still perform best. Note that although we did not investigate this problem, our methodology can still be applied as the difference lies only in the evaluation of the performance and costs. Another interesting point for further research is the target setting. We assumed that each SKU at least has to meet the target for its class. However, it might be

interesting to see how the performance of the different classifications would change if we would set the basestock levels such that the difference with the target is minimized instead. This may also reduce the impact of forced presence of stock as not all parts have to be stocked at once in this case.

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