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The extended gate problem: Intermodal hub location with multiple actors

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Abstract: Hinterland transportation has become increasingly critical for global container supply chain performance. However, the literature on the management of the container supply chain tends to overlook the issues related to hinterland transportation. The problem we consider here is faced by a deep-sea terminal operator who wants to open an inland terminal to facilitate its operations. When developed by a deep-sea terminal operator, such an inland terminal directly related to a deep-sea port is called an extended gate. In this setting, the shippers can decide to take their containers either directly at the deep-sea terminal or at the inland terminal. In this latter case, train transportation is used from the deep-sea terminal to the inland terminal. Intermodal hub location problems are usually solved by considering a single decision maker even though several actors often interact in practice. We analyze here the impact of having multiple actors involved by proposing a formulation of the problem based on game theory. We develop structural properties of the shippers' behavior. These properties enable us to identify the existing equilibria for the game and to solve the problem optimally. We apply the results to an example based on the features of the hinterland network in the Netherlands and we provide related insights. We show that the multiple actors feature of intermodal hinterland networks is critical and needs to be accounted for. Our results serve as a basis for appropriately taking multiple actors into account in hub location problems.

Keywords: hub location; intermodal transportation; hinterland supply chain; multiple actors; extended gate.

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1. Introduction

The vast majority of intercontinental freight transport takes place by sea and the amount of containerized goods is increasing. Containerization has indeed been the main technological revolution of the maritime industry in the past 30 years. We refer to Levinson (2010) for an analysis of the impact of containerization. This innovation has shaped current global supply chains by substantially reducing transportation costs. For example, the freight rate on a port-to-port basis between Shanghai and Rotterdam for a 40-foot container is €0.21 per km (OECD/ITF, 2009). This freight rate implies that the maritime transportation cost for 32-inch television screens from Asia to Europe is less than €3 per screen. As a result, global container traffic has been growing at almost three times world gross domestic product growth since the early 1990s (UN-ESCAP, 2005). The management of the container supply chain is consequently a key issue.

This article analyzes the efficiency of container transportation systems in the hinterland. Although the distance covered by the container in the hinterland is typically small, inland transportation costs are often substantial. For example, the freight rate for inland transportation by truck from the port of Rotterdam typically ranges from €1.50 per km to €4 per km, depending on the distance and weight (OECD/ITF, 2009); this is 7–19 times higher than the maritime transportation freight rate. Hinterland activities also include container handling operations. Summing up all the costs related to hinterland operations, Notteboom and Rodrigue (2005) estimate that the proportion of inland costs relative to the total transportation cost of a container shipping ranges from 40% to 80%. Thus, improving the efficiency of the hinterland supply chain could provide substantial benefits from a global supply chain perspective. Notteboom (2008a) accordingly mentions that the maritime industry have identified inland logistics as one of the most vital areas still left to cut costs.

When focusing on the hinterland supply chain, the main impact of containerization is the increasing role of intermodal transportation. Intermodal transportation involves the transportation of the load from origin to destination in the same transportation unit without handling of the goods themselves when changing modes (Crainic and Kim, 2007). The development of container based intermodal transportation leads to the development of inland terminals. This phase in the evolution of the hinterland supply chain is called “terminalization” and this is considered as a long-term trend (Rodrigue and Notteboom, 2009). The development of inland terminals is supported by several types of actors such as terminal operators, port authorities, rail operators, real estate promoters and/or local authorities. In addition to the actors directly involved in the development of inland terminals, the users of the designed service are of major importance for successful implementation. Indeed, the emergence of inland terminals can be seen as a compromise for which both operators and users find a valuable alternative to their constraints (Rodrigue and Notteboom, 2009). As a consequence, the performance of the hinterland network depends on the behavior of the different actors involved (De Langen and Chouly, 2004). This issue has been frequently emphasized and discussed in the maritime economics and transport geography literature (Notteboom, 2008b; Roso et al., 2009; Van Der Horst and De Langen, 2008). However, model-based research on hinterland network design with multiple actors is scarce. Accordingly, Fransoo and Lee (2013) recently argued that research from the operations management and transportation science community is lacking on container transportation systems despite its critical role in current global supply chains.

In this article, we consider the problem faced by a deep-sea terminal operator who wants to facilitate its operations by creating a direct rail connection between the deep-sea terminal and an inland terminal where containers can be sorted by destination. This solution may be implemented in areas where the possibilities of expansion inside the port are limited or too costly and/or where road terminal access is an issue due to congestion. This strategy enables moving a share of terminal operations more inland where land availability and road terminal access are less of an issue. In addition, this solution favors intermodal transportation as a large volume of containers can be transported to the same location (Zuidwijk and Veenstra, 2014). When the project is implemented by a terminal operator, such an inland terminal directly connected to a deep-sea port is called an extended gate (Veenstra et al., 2012). The concept is currently developed in Europe but this one may also be of interest in other regions. We refer to Rodrigue and Notteboom (2009) for a description of an extended gate operated in the region of Venlo in the Netherlands. We summarize the decision process as follows. The terminal operator first decides on where to locate the extended gate. Then the shippers decide if they want to take their containers at the deep-sea terminal or at the extended gate. In case they decide to use the extended gate, they pay for train transportation and extra handling costs. In this leader-follower setting, the terminal operator needs to take the shippers' reactions into account when deciding on where to locate the extended gate.

We consider two objectives for the location decision made by the terminal operator. Besides minimizing the total cost as classically done in the literature, we argue that maximizing extended gate utilization is of great importance for the terminal operator. We show that these two objectives may be conflicting and we apply multiobjective optimization to identify the set of efficient solutions (also called Pareto optimal solutions). The shippers are primarily focused on minimizing transportation and handling costs incurred from hinterland deliveries in the setting we consider. The model takes flow dependent economies of scale into account for intermodal transportation. Thus, the decision of each shipper depends on the other shippers' decisions. We formulate the shippers' route choice decisions as a non-cooperative game. We identify key structural properties that enable identifying the existing equilibria. We highlight that several equilibria may exist and that one of these equilibria often involves not having any shipper using the extended gate. Thus, the terminal operator often needs to convince some shippers to use the extended gate in order to obtain the base volume necessary for becoming competitive. We propose a way to quantify this effort and we show that the optimal extended gate location may depend on the effort the terminal operator is willing to make. Finally, we identify the system optimum solution (i.e., the solution that would be obtained under full centralization of decisions) in order to estimate the value of collaboration (as expressed by the so-called price of stability). We highlight that collaboration may be critical for successfully implementing an extended gate even if the value of collaboration may also be quite low. Overall, the results show that the multiple actors feature of intermodal hinterland networks is critical and needs to be accounted for.

We organize the rest of the article as follows: Section 2 analyzes the existing literature. Section 3 focuses on the description and the mathematical formulation of the model. Section 4 analyzes the structural properties of the problem. These properties enable us to identify the equilibria of the problem. Section 5 details the most important insights of our research by focusing on an example based on the features of the hinterland supply chain in the Netherlands. Finally, Section 6 offers conclusions.

2. Literature review

The maritime economics and transport geography literature related to the problem considered is presented in the introduction of the article. Therefore, we focus here on the model-based literature that serves as a basis for the development of the model. The problem we consider can be interpreted as a particular type of hub location problem. The first articles focusing on hub location can be traced back to 1986 (O'Kelly, 1986a, 1986b). The literature on hub location has since expanded rapidly. Alumur and Kara (2008), Campbell and O'Kelly (2012), and Farahani et al. (2013) all provide reviews. Several classical formulations of the hub location problem appear in the literature (e.g., hub median, hub center, hub covering), but the problem we consider here is related to the p-hub median problem. This problem consists of locating a given number of hubs and deciding how to allocate a set of origin/destination nodes to these hubs to minimize the total transportation cost of the system. The cost of transporting one unit of flow per unit distance is discounted on interhub arcs to represent the economies of scale achieved by such consolidation systems. This feature creates an incentive to route origin/destination flow through more than one hub because, though this increases the total distance traveled, it may lead to an overall cost reduction. The basic p-hub median model (and many of its extensions) assumes that economies of scale are somehow exogenous to the decisions made about hub location and origin/destination allocation. A fixed discount factor is typically applied to account for economies of scale on interhub arcs. This limitation was first addressed by O'Kelly and Bryan (1998), who account for flow-dependent economies of scale on interhub arcs by considering strictly increasing concave transportation cost functions. They prove that the optimal hub locations may differ greatly from the results obtained without taking flow-dependent economies of scale into account.

The p-hub median problem has been applied to study intermodal transportation networks. Bontekoning et al. (2004) present a review of the early research on intermodal transportation for hinterland supply chains. The literature on intermodal hub location has quickly expanded in the past decade (Alumur, Kara and Karasan, 2012; Alumur, Yaman and Kara, 2012; Arnold et al., 2004; Groothedde et al., 2005; Ishfaq and Sox, 2010, 2011, 2012; Jeong et al., 2007; Limbourg and Jourquin, 2009; Meng and Wang, 2011; Racunica and Wynter, 2005; Sörensen and Vanovermeire, 2013; Zhang et al., 2013). These articles primarily extend the classical p-hub median problem by taking classical features of intermodal container transportation into account. The most commonly considered aspects are flow-dependent economies of scale for transportation and transshipment activities, travel time, service constraints, and congestion in the system. These studies usually propose a new model that incorporates some features proven to be of practical importance. Then, they develop new solution techniques to solve the proposed problem and focus on assessing the efficiency of the considered solution technique.

To our knowledge, four articles consider multiple actors in intermodal hub location problems. Sirikijpanichkul et al. (2007) apply an agent-based modeling approach to optimize the location of intermodal freight hubs. They consider four types of agents, i.e., hub owners, transport network infrastructure provider, hub users and communities. Sörensen and Vanovermeire (2013) argue that, in general, the location and transportation costs typically included in intermodal hub location problems are not incurred by the same actors. Thus, they consider these two types of cost separately and develop a bi-objective optimization model to identify the existing trade-offs between both types of cost. Meng and Wang (2011) include user equilibrium constraints in a hub location problem to model the effects of having multiple stakeholders involved. They consider that a network planner designs the network with the objective of minimizing total costs and that the intermodal operators make their route choice decisions in order to minimize their own costs. The user equilibrium constraints, formulated as variational inequalities, ensure that no intermodal operator can reduce its transportation and transshipment cost by individually changing its routing strategy. Yamada et al. (2009) use bi-level programing and user equilibrium constraints to account for multiple actors in an intermodal network design problem. Bi-level programing provides results similar to a Stackelberg game in which the network planner first design the network and then the shippers make their routing decisions. The lower-level problem that focuses on routing decisions involves user equilibrium constraints as in Meng and Wang (2011) in order to account for multiple shippers.

These articles are particularly noteworthy here because they include a multiple-actors feature, which is often acknowledged as the most challenging aspect of container supply chains. However, they primarily focus on identifying effective procedures to solve the proposed models. Thus, the impact of having multiple actors involved in such supply chains cannot be easily assessed from their results. We attempt here to better understand the consequences of having multiple actors involved in hinterland supply chains. Our model is based on two main features. First, we consider that the hub location and the intermodal route choice decisions are not made by the same actors. We formulate a Stackelberg game in which the terminal operator first decides on where to open an extended gate, and shippers then allocate their demand flow in the network so as to minimize their individual transportation and handling cost. Second, we consider that the shippers do not cooperate in making their route choices. Instead of using user equilibrium constraints as in Yamada et al. (2009) and Meng and Wang (2011), we formulate the shippers' routing decisions problem as a non-cooperative game. This game may be viewed as a special type of atomic congestion game as firstly introduced by Rosenthal (1973). We show that several equilibria may exist for the shippers' routing decisions game. We analyze the consequences of having multiple equilibria and we show that this feature is of primary importance. Yamada et al. (2009) is the first article using traffic assignment techniques in intermodal network design problem. We propose to follow up and to go further by modeling the shippers' routing decisions problem as a non-cooperative game. We derive key structural properties of the game and we identify all the existing equilibria. Our article is, to our knowledge, the first to apply directly non-cooperative game theory to analyze routing decisions in a hub location problems.

Our main contributions are fourfold. First, this article is, to our knowledge, the first to model the extended gate location problem. The extended gate location problem possesses some particular aspects not covered by the models developed in the existing literature. For instance, we argue in the next section that maximizing the extended gate utilization is of great importance for the terminal operator. The concept of extended gate is also highly relevant in practice. Second, we formulate the routing decisions made by the shippers as a non-cooperative game and we derive key structural properties of the game. This allows us to identify all the existing equilibria for the game. Third, we study the impact of having multiple actors involved in the extended gate location problem. We show how to compute the minimum number of shippers who need to be convinced to reach a given equilibria and we measure the value of collaboration by computing the price of stability. Fourth, we apply the results to an example based on the features of the hinterland network in the Netherlands and provide related insights.

3. Model description

3.1 Context

The model we propose describes the extended gate location and shippers' routing decisions problem. For the sake of clarity, we present the model in terms of import flows for containerized cargo. The containers are unloaded to a deep-sea terminal and need to be delivered to various destinations. The problem could be reversed by considering export flows from various origins to the deep-sea terminal. Our results hold in that case as well. Because the dimensions of containers have been standardized (Agarwal and Ergun, 2008), the proposed model takes only one type of container into account. We consider that the containers must be delivered to a fixed set of destinations with constant and known demand as this is usually the case in the hub location literature. This deterministic demand assumption comes from the fact that the unplanned share of the demand requires urgent delivery. Consequently, the only feasible option in this case is to deliver by truck from the port. The problem under consideration does not take this share of the demand into account as this does not influence the decisions made and the results obtained. When arriving at their destination, the containers are unloaded. We do not explicitly take into account empty container management but consider this a parameter of the model.

The destinations are considered individual companies, such as retailers, requesting a transportation service in the model (merchant haulage setting). We will consequently refer to the shippers as the destinations in what follows. We consider that the destinations choose among two possible options for being delivered. Either they decide to take their containers at the port and have them delivered by truck, or they decide to use the extended gate. In that case, we assume that the containers are loaded on a train from the port to the extended gate. We focus on train transportation because this is the most developed intermodal transportation solution worldwide and because most of the current examples of extended gates are connected to the deep-sea terminal by rail. However, the model is also valid for barge and short sea intermodal transportation systems. After arriving at the extended gate, the containers are transshipped to trucks to reach their final destination. In case this latter option is chosen, the destinations are charged for the train transportation cost from the deep-sea terminal to the extended gate as well as for the extra handling costs. They also incur truck transportation cost for the final leg of the containers' journey.

We refer to the routing decisions made by the destinations as the shippers' allocation problem, by analogy with the hub location literature. This problem consists of deciding, for each destination, which share of the total flow should be allocated to the extended gate, i.e., shipped by intermodal transportation. In the literature on intermodal transportation, the decision of using intermodal transportation instead of direct truck delivery is mainly based on two attributes, the total cost and the leadtime. Indeed, intermodal transportation is traditionally perceived as less flexible and consequently implies longer leadtime compared to truck. In the setting considered here, we argue that the key attribute for explaining the destinations' decisions is the total cost for two main reasons. First, the increase in leadtime when using intermodal transportation compared to direct truck delivery is mainly due to the waiting time for train transportation at the port. However, the average dwell times at the main European ports are equivalent for vessel/truck and vessel/train transshipment (Rodrigue and Notteboom, 2009). This implies that the difference in leadtime is small. This small difference in leadtime would fall below the threshold of indifference from a shipping time perspective which is often evaluated to 3 days (Rodrigue and Guan, 2009). We recall here that we exclude urgent deliveries from the model. Second, the high dwell times at the port got increasingly associated with deliberate decisions of the destinations who use the terminals as low cost locations for keeping inventory. When using the extended gate, the containers can be directly shipped and stored at the extended gate and the remaining leadtime can even be lower than when using direct truck delivery from the port. Consequently, we assume here that the destinations decide on whether or not to use the extended gate by evaluating only the costs incurred for the two options. This feature has been validated with industrials.

Our model takes into account flow-dependent economies of scale for train transportation and transshipment operations at the extended gate. We model the train transportation cost and the transshipment cost as general non-negative function, non-increasing in the total number of containers shipped. As soon as these functions are non-constant, a cost interdependency among the destinations occurs, leading to some noteworthy allocation issues. In the classical single-actor vision of hub location problems, O'Kelly and Bryan (1998) point out that "some origin-destination pairs may be routed via a path that is not their least-cost path because doing so will minimize total network travel cost" (p. 608). This statement does not hold in a multiple-actor setting. The situation is similar to a classical problem in the traffic assignment literature. Because of congestion, the solution that minimizes the total travel time in the system is not equivalent to the solution that minimizes the travel times of each individual users. This leads to two extreme behaviors that Wardrop (1952) describes as user equilibrium (each user minimizes its own travel time) versus system optimum (the total travel time of the system is minimized). In our model, the destinations seek to minimize their own transportation and handling costs. Therefore, we refer to an allocation of the destinations' demand flow as a user equilibrium (UE) if no destination can decrease its transportation and handling cost by individually changing its routing decision and if intermodal transportation is used by a destination only if this strictly reduces its cost. In addition, we consider a system optimum (SO) allocation (i.e., an allocation that minimizes to total transportation and handling cost) in order to assess the value of collaboration.

The terminal operator decides where to locate an extended gate given a set of predetermined sites. The problem can be formulated as a p-hub median problem with a single origin and a single hub to locate (the extended gate). In the p-hub median problem, the objective is to minimize the total cost of the system. Our model assumes that the terminal operator and the different destinations are autonomous actors who make their decisions separately. By direct transposition, we could consider that the terminal operator aims at minimizing the total cost given the reactions of the destinations as in Yamada et al. (2009) and Meng and Wang (2011). Considering the total cost as the location objective makes sense as we may expect the extended gate to offer a competitive service if the total cost are minimized. On the other hand, the terminal operators also aim at maximizing the utilization of the designed service, given the reaction of the destinations. Indeed, such projects are very capital intensive and may be beneficial only if well utilized. In addition, one of the key motivation for the terminal operator to start such project is to transfer a share of terminal operations outside the port in order to increase its throughput. Indeed, the main customers of such deep-sea terminals are the shipping lines who are mainly focused on reducing the waiting time of their ships at the port. Moreover, shipping lines are charged by the terminal per container movement (loading or unloading). By increasing its throughput, the terminal operator can attract more traffic and increase its revenue.

Consequently, from a terminal operator's point of view, maximizing the utilization of the extended gate may be at least as important as minimizing the total cost of delivering the containers. In what follows, we consequently assess if the decision made by considering a total cost minimization objective are similar to the one taken when maximizing the extended gate utilization. Thus, we take this two objectives into account.

3.2 Model formulations

All the notations used in the articles are summarized in Appendix A (available online as supplementary material). The hinterland supply chain under study consists of a single deep-sea terminal inside a port (considered as the origin) with $N \in \mathbb{N}^*$ destinations ($\mathbb{N}^*$ refers to the set of natural numbers). We consider a single period in the model as classical done in the hub location literature. A deterministic constant flow n_j must be shipped from origin to destination $j \in \{1,...,N\}$. Here, n_j is expressed in number of containers, and only one type of container is available (40-foot containers); thus, we assume that $n_j \in \mathbb{N}^*$ for all $j \in \{1,...,N\}$. At most one extended gate must be located among $M \in \mathbb{N}^*$ candidate locations. Each potential extended gate location is referred to as location $i \in \{1,...,M\}$. In addition, we refer to the origin (i.e., the deep-sea terminal) as location $i = 0$.

The truck transportation cost function considered is linear in the number of containers shipped and the cost of transporting one container from location $i \in \{0,...,M\}$ to destination $j \in \{1,...,N\}$ by truck is referred to as $Z^1_{i,j}$. This cost depends on the distance from i to j, on the rate proposed by the truck operators, on the empty container management practice developed by the destination and on the level of congestion around location i.

While delivery takes place through intermodal transportation, train transportation is used from the port to the extended gate. We define the cost of shipping one container by train from the port to the extended gate $i \in \{1,...,M\}$ as $Z^2_{0,i}(K)$, where $K \in \mathbb{R}^*_+$ is the total number of containers shipped by train ($\mathbb{R}^*_+$ is the set of strictly positive real numbers). Flow-dependent economies of scale are taken into account; consequently, we assume that $Z^2_{0,i}(K)$ is non-negative for all $K \in \mathbb{R}^*_+$ and non-increasing in K, for all $i \in \{1,...,M\}$. Empty container are usually stored in empty depots outside the deep-sea terminal. Thus, empty containers cannot be shipped back to the empty depot by using the same train. In addition, if an export match is found, the containers are usually shipped back by truck due to time consideration. We can consequently assume train is used solely for shipping container from the deep-sea terminal to the extended gate.

In addition to train and truck transportation costs, intermodal transportation implies additional container handling operations at the extended gate. We define the cost of transshipping one container at extended gate $i \in \{1,...,M\}$ as $Z^{1,2}_i(K)$. Note that the total number of containers transshipped at extended gate i is equal to the total number of containers shipped by train. The transshipment cost depends on the location considered to account for the difference in land and labor costs as well as for the difference in extended gate layout, equipment, and size. This cost also accounts for the difference in cost from transshipping to a truck or to a train at the origin, if any. In addition, we consider that $Z_0^{1,2}(K) = 0$, for all $K \in \mathbb{R}_+^*$. The transshipment cost per container depends on the total amount of containers transshipped at extended gate i. We further assume that $Z_i^{1,2}(K)$ is non-negative for all $K \in \mathbb{R}_+^*$ and non-increasing in K, for all $i \in \{1,...,M\}$.

The total cost of shipping one container from the port to destination $j \in \{1,...,N\}$ by intermodal transportation through extended gate $i \in \{1,...,M\}$ is equal to $Z_{0,i}^2(K) + Z_i^{1,2}(K) + Z_{i,j}^1$. To simplify the notation, we define $Z_{0,i}^3(K)$ as follow:

$$Z_{0,i}^3(K) = Z_{0,i}^2(K) + Z_i^{1,2}(K) \text{, for all } i \in \{0,...,M\}\text{, for all } K \in \mathbb{R}_+^*. \quad (1)$$

Note that $Z_{0,0}^3(K) = 0$ for all $K \in \mathbb{R}_+^*$. By applying the properties defined for $Z_{0,i}^2(K)$ and $Z_i^{1,2}(K)$, we can state that $Z_{0,i}^3(K)$ is non-negative for all $K \in \mathbb{R}_+^*$ and non-increasing in K, for all $i \in \{1,...,M\}$. As is usually the case in the hub location literature, we further define $0 \le X_{i,j} \le 1$ as the proportion of flow from the port to destination j being routed through extended gate i. Here, $X_{0,j} = 1$ indicates that the entire flow from the port to destination j is delivered by direct shipment, while $X_{i,j} = 1$, where $i \in \{1,...,M\}$, indicates that the entire flow from the port to destination j is delivered by intermodal transportation using extended gate i.

The first objective considered consists in minimizing the total transportation and transshipment cost. This objective function may be expressed as follows:

$$MIN \sum_{i=0}^{M} \sum_{j=1}^{N} n_j \left(Z_{0,i}^3(K) + Z_{i,j}^1 \right) X_{i,j}\ , \quad (2)$$

The following set of constraints must be considered:

$$\sum_{i=1}^{M} y_i \le 1, \quad (3)$$

$$y_i \in \{0;1\}, \forall i \in \{1,...,M\}, \quad (4)$$

$$X_{i,j} \le y_i, \forall j \in \{1,...,N\}, \forall i \in \{1,...,M\}, \quad (5)$$

$$\sum_{i=0}^{M} X_{i,j} = 1, \forall j \in \{1,...,N\}, \quad (6)$$

$$X_{i,j} \geq 0, \forall i \in \{0,...,M\}, \forall j \in \{1,...,N\}, \text{ and} \tag{7}$$

$$K = \sum_{i=1}^{M} \sum_{j=1}^{N} n_j X_{i,j} \ . \tag{8}$$

y_i are binary variables equal to 1 if extended gate i is open and 0 otherwise. Constraint (3) ensures that, at most, one extended gate can be opened. Constraints (5) imply that flow can be routed only through an open extended gate. Constraints (6) ensure that the total amount of flow is shipped from origin to destinations. Constraints (7) ensure that the proportions of flow routed are non-negative. Finally, constraint (8) is used to account for the number of containers routed by intermodal transportation.

Objective function (2) with constraints (3) - (8) is the classical hub location problem with flow-dependent economies of scale. This corresponds to the single actor view of the problem, i.e., when all the destinations align their decisions in order to minimize the total transportation and transshipment cost (SO allocation) and when the terminal operator also decides to locate the extended gate in order to minimize the total cost. This problem serves as a benchmark for assessing the impact of having multiple actors involved.

We consider several variations of this problem. First, we consider that the destinations act independently and seek to minimize their own transportation and transshipment cost. Second, we consider another objective for locating the extended gate by considering that the terminal operator seeks to maximize the terminal utilization. This second objective function may be expressed as follows:

$$MAX \ K \ . \tag{9}$$

The different variations are all solved by backward induction. First, we focus on the allocation decisions made by the destinations (i.e., the followers of the game) and we assume that the extended gate $i \in \{1,...,M\}$ is open. Second, the optimal extended gate location for the two objective functions considered is obtained by enumeration. Section 4 focuses on the first step of the backward induction procedure, i.e., on the allocation problem. After stating some preliminary results in Subsection 4.1, Subsection 4.2 focuses on identifying all the existing user equilibria for the shippers' allocation game. The SO allocation is then identified in Subsection 4.3. The full procedure is illustrated via an example in Section 5.

4. The shippers' allocation problem

In this section, we aim at solving the shippers' allocation problem (i.e., the problem of deciding how to allocate the demand flow between direct shipment and intermodal transportation for each destinations). We assume that an extended gate $i \in \{1,...,M\}$ is open and we focus on the routing decisions made by the destinations. The destinations indeed need to determine which share of their demand flow is routed via the extended gate i (i.e., determine $X_{i,j}$), knowing that the remaining share of their demand flow will be routed by direct shipment from the port ($X_{0,j} = 1 - X_{i,j}$). After stating a necessary condition for obtaining a user equilibrium and a system optimal allocation that enables simplifying the analysis, we identify the user equilibria first and then the system optimal allocation.

4.1 Preliminary result

We start by formally defining a user equilibrium (UE). As Fisk (1984) points out, Wardrop's first principle of user equilibrium is related to the Nash equilibrium principle in non-cooperative game theory. Indeed, Haurie and Marcotte (1985) show that a Nash equilibrium in an atomic congestion game converges to a Wardrop equilibrium when the number of players increases. In the setting we consider, each destination (i.e., each shipper) is considered a player of the game. Each destination $j \in \{1,...,N\}$ aims at minimizing its total cost function expressed as follows:

$$Z_{i,j}(X_{i,j}, K) = n_j\left((1 - X_{i,j})Z^1_{0,j} + X_{i,j}(Z^3_{0,i}(K) + Z^1_{i,j})\right). \qquad (10)$$

A Nash equilibrium is obtained when neither player can unilaterally reduce its transportation cost by changing its decision. Nash (1950) proves that there is at least one Nash equilibrium under general assumptions by applying a fixed point theorem. However, this equilibrium may be obtained by requiring at least one player to choose a probability distribution over the set of potential actions to protect against other players' reactions. This type of strategy is a mixed strategy, as opposed to a pure strategy. A pure strategies Nash equilibrium is such that each player chooses an action for sure. This type of equilibrium may not always exist (see e.g., rock-paper-scissors or matching pennies games).

We can notice that even though the cost function (10) does not depend on the other players' decisions directly, it does so via the number of containers that will be shipped through the extended gate. This feature is consistent with atomic congestion games as defined by Rosenthal (1973), except that in our setting, the more intermodal transportation is used, the better. Rosenthal (1973) proves that there exists at least a pure strategies Nash equilibrium for atomic congestion games. We may expect that a pure strategies Nash equilibrium exists as well in our setting and we focus only on pure strategies Nash equilibria in what follows. In order to fit to shippers' practices, we additionally assume that the extended gate is used by a destination only if this leads to a strict decrease in cost compared to direct shipment. We consequently define a user equilibrium (UE) as a pure strategy Nash equilibrium such that the players choosing to use the extended gate are strictly better off in doing so, compared to using direct shipment only. We show in the next section that a UE always exists for the shippers' allocation game, and that several UE may exist in some situations.

We define a system optimum (SO) allocation as an allocation minimizing the total cost with the minimum volume shipped via the extended gate. We show in subsection 4.3 that this definition ensures the uniqueness of the SO allocation, due to the structural properties of the problem.

$X_{i,j}$ satisfies the single routing condition if and only if $X_{i,j} \in \{0;1\}$, i.e., if the entire flow of destination j is routed either by direct shipment or by intermodal transportation via extended gate i. Theorem 1 states that the single routing condition is satisfied for each destination under any UE (if any) and under any SO allocation (if any).

Theorem 1: *Any UE and any SO allocation satisfies the single routing condition for all* $j \in \{1,...,N\}$.

All proofs appear in Appendix B (available online as supplementary material). We recall here that we exclude urgent deliveries from the model as the unplanned share of the demand will necessarily be shipped directly from the port and consequently this does not influence the decisions made and the results obtained. Taking urgent deliveries into account, a destination may be delivered both by direct shipment (for urgent deliveries) and by intermodal transportation (for non-urgent deliveries) in practice. Theorem 1 states that the single routing condition holds for the non-urgent share of the demand at destination, i.e., the quantity considered in the model. Theorem 1 also implies that a SO allocation exists for all $i \in \{1,...,M\}$, as $\{0;1\}^N$ is a finite set.

4.2 Identifying the user equilibria of the game

In this subsection, we prove that a UE always exists and that several user equilibria may exist for some instances of the shippers' allocation game. Then, we identify key structural properties that enable identifying the existing equilibria. Finally, we define the effort that the terminal operator needs to exert to obtain a given UE by characterizing the minimum number of destinations that need to be convinced to get the base volume necessary for becoming competitive and we show how to compute this effort.

We start by turning the cost function for each destination as expressed in Formula (10) into a profit function by computing the cost savings of a strategy compared to using only direct shipment:

$$P_{i,j}(X_{i,j},K) = X_{i,j} n_j \left(Z^1_{0,j} - Z^1_{i,j} - Z^3_{0,i}(K)\right). \qquad (11)$$

By applying Theorem 1, $X_{i,j} \in \{0;1\}$ under any UE. Consequently, under any UE, destination j will use the extended gate if and only if $P_{i,j}(1,K) > 0$ (by definition of a UE, we set $X_{i,j} = 0$ if $P_{i,j}(1,K) = 0$). As, $P_{i,j}(1,K)$ depends on K (the total number of container shipped by train), we define the minimum volume required to make intermodal transportation via extended gate i economically viable for destination j as $K^{i,j}$ (we recall that we consider i as fixed in this section). $K^{i,j}$ is the smallest integer value of K such that $P_{i,j}(1,K) > 0$. If $K^{i,j}$ exists, then it is unique. If $K^{i,j}$ does not exist, then $X_{i,j} = 0$ independently of the decisions of the other destinations and we set $K^{i,j} = \infty$.

Theorem 2 proves that at least one UE exists for the shippers' allocation game and Theorem 3 states that the UE may not always be unique.

Theorem 2: *The shippers' allocation game possesses at least one user equilibrium.*

Theorem 3: *The shippers' allocation game may possess several user equilibria.*

As we show in the proof (see Appendix B in the supplementary material), Theorem 3 holds even when considering only two destinations. This occurs when neither of the destinations has enough volume to individually make use of the extended gate and when the combination of both destinations' volumes makes the use of the extended gate profitable for each. This example can be viewed as a stag hunt game (Skyrms, 2001) for which two pure strategy Nash equilibria exist. In what follows, we develop structural properties of the shippers' allocation game. These properties enable us to identify all the existing user equilibria for the game.

$K^{i,j}$ also enables us to define the conditions to reach a UE. Let L_i be the number of user equilibria of the shippers' allocation game associated to extended gate i ($L_i \geq 1$ by applying Theorem 2). For all $l \in \{1,...,L_i\}$, let $U_{i,l}$ be the set of destinations using the extended gate (such that $X_{i,j} = 1$) under equilibrium l ($U_{i,l}$ may be empty). Then, $U_{i,l}$ satisfies the following conditions:

$$\max_{j \in U_{i,l}}(K^{i,j}) \leq \sum_{j \in U_{i,l}} n_j \, , \tag{12}$$

$$K^{i,t} > \sum_{j \in U_{i,l}} n_j + n_t \, , \ \forall t \in \{1,...,N\} \setminus \mathrm{U_{i,l}} \, . \tag{13}$$

Condition (12) implies that the volume generated by the destinations included into $U_{i,l}$ is large enough to have all these destinations using the extended gate. Conditions (13) imply that the remaining destinations are not able to individually join the set of destinations using the extended gate. The shippers' allocation game highlights nice structural properties, stated in Theorem 4 and Corollaries 1, 2 and 3.

Theorem 4: *Let $a, b \in \{1,...,N\}$ be two distinct destinations such that $K^{i,b} \geq K^{i,a}$, then:*

$$b \in U_{i,l} \Rightarrow a \in U_{i,l} \text{, for all } l \in \{1,...,L_i\}.$$

Corollary 1: *Consider an instance of the shippers' allocation game with $L_i \geq 2$ user equilibria. Then, it is possible to arrange the $L_i \geq 2$ user equilibria such that:*

$$U_{i,l} \subset U_{i,l+1}, \forall l \in \{1,...,L_i - 1\}.$$

Corollary 2: *Consider an instance of the N shippers' allocation game with* L_i *user equilibria. Then:*

$$L_i \leq \left\lfloor \frac{N}{2} \right\rfloor + 1 \cdot$$

Corollary 3: *Consider an instance of the shippers' allocation game with* $L_i \geq 2$ *user equilibria. Then, for all* $l \in \{1,...,L_i - 1\}$, $U_{i,l+1}$ *performs strictly better than* $U_{i,l}$ *in terms of extended gate utilization and in terms of total cost.*

Theorem 4 enables simplifying the search for the user equilibria by arranging the destinations by increasing order of $K^{i,j}$. We assume w.l.o.g. for the remaining of this subsection that the destinations are arranged such that $K^{i,1} \leq ... \leq K^{i,M}$. Conditions to reach a UE with the first k ($1 \leq k < N$) destinations are expressed as follows:

$$K^{i,k} \leq \sum_{j \in \{1,...,k\}} n_j \,, \tag{14}$$

$$K^{i,t} > \sum_{j \in \{1,...,k\}} n_j + n_t \,, \ \forall t \in \{k+1,...,N\}. \tag{15}$$

The conditions to reach a UE with no destination using the extended gate are:

$$K^{i,t} > n_t \,, \ \forall t \in \{1,...,N\}, \tag{16}$$

and the condition to reach a UE with all the destination using the extended gate is:

$$K^{i,N} \leq \sum_{j \in \{1,...,N\}} n_j \,. \tag{17}$$

We apply these conditions in the example presented in Table 1, with 10 destinations. The UE identified are $U_{i,2} = \{1;2;3;4;5\}$ by applying Conditions (14) and (15), as well as $U_{i,1} = \varnothing$ by applying Condition (16). Note that $\{1;2\}$ satisfies Condition (14). However, $K^5 < \sum_{j \in \{1,2\}} n_j + n_5$ (i.e.,21<16+6), which contradicts Condition (15).

destination j	n_j	$K^{i,j}$	$\sum n_j$
1	7	9	7
2	9	14	16
3	2	19	18
4	2	20	20
5	6	21	26
6	4	31	30
7	5	50	35
8	15	62	50
9	12	∞	62
10	6	∞	68

Table 1: An example of the 10 shippers' allocation game

Corollary 2 states that the maximum number of UE for a given instance of the *N* shippers' allocation game grows linearly in *N*. As the conditions (14) – (17) can be checked in polynomial time in *N*, we can identify all the UE for the *N* shippers' allocation game in polynomial time of *N*. This ensures practical application.

Our results show that several user equilibria may exist for the shippers' allocation game. We also show how to identify the existing set of UE. To our knowledge, the only models considering UE in an intermodal hub location problem are proposed by Yamada et al. (2009) and by Meng and Wang (2011). In their analysis, the total transportation cost is minimized under constraints stating that a destination may not be routed via a path that increases its individual transportation cost compared to direct shipment. Thus, these two articles focus on identifying the UE that minimizes the total cost (i.e., U_{i,L_i} by applying Corollary 3). Our results question this choice of focusing solely on the UE minimizing the total cost as we prove that several equilibria may exist. Corollary 3 shows that the terminal operator would favor U_{i,L_i} as this equilibrium performs better for the two objectives of interest for the terminal operator. However, we cannot ensure that U_{i,L_i} is the equilibrium that would be found in practice as soon as $L_i > 1$.

In what follows, we consequently propose to identify the minimal number of destinations who have to use the extended gate to ensure that the total cost given by objective function (2) and the extended gate utilization given by objective function (9) are at least as good as the ones obtained under $U_{i,l}$, $l \in \{1,...,L_i\}$. Indeed, the terminal operator often needs to convince some shippers to use the extended gate in order to obtain the base volume necessary for becoming competitive. Let $Card(U)$ be the cardinality of a set U. We propose and use the following definition of a set compatible with $U_{i,l}$ to identify the set of shippers who are able to generate this base volume.

Definition 1: *Let* $U_{i,l}$, $l \in \{1,...,L_i\}$ *be a user equilibrium. A set U is called compatible with* $U_{i,l}$ *if this one satisfies the following conditions:*

- $U \subseteq U_{i,l}$,
- $\forall k \in U$, $K^{i,k} \leq \sum_{j \in U} n_j$,
- *If* $U_{i,l} \setminus U \neq \varnothing$, *there exists a permutation of the elements of* $U_{i,l} \setminus U$ *denoted as* $(1,...,Card(U_{i,l}) - Card(U))$ *such that for all* $k \in (1,...,Card(U_{i,l}) - Card(U))$, $K^{i,k} \leq \sum_{j \in U} n_j + \sum_{j=1}^{k} n_k$.

We can first notice that if all the destinations of a set U compatible with $U_{i,l}$ use the extended gate, the volume generated ensures that each of these destinations are better off as compared to using direct shipment. In addition, the destinations of $U_{i,l} \backslash U$ (if any) can iteratively join the extended gate, by following the sequence of the permutation $(1,..., Card(U_{i,l}) - Card(U))$. Consequently, if the terminal operator manages to convince all the destinations included in a set U compatible with $U_{i,l}$ to use the extended gate, the total cost given by objective function (2) and the extended gate utilization given by objective function (9) will be at least as good as the ones obtained under $U_{i,l}$. Identifying a set compatible with $U_{i,l}$ in practice is difficult as this implies identifying the sequence that enables destinations of $U_{i,l} \backslash U$ to iteratively join the extended gate. Theorem 5 enables simplifying the search for the sets compatible with $U_{i,l}$.

Definition 2: *Let* $U_{i,l}$, $l \in \{1,..., L_i\}$ *be a user equilibrium. Let* $U_{i,l}^{\min}$ *be the set of all sets* U *satisfying the following conditions:*

- $U \subseteq U_{i,l}$,
- $\forall k \in U$, $K^{i,k} \leq \sum_{j \in U} n_j$,
- *if* $l > 1$, $U \not\subset U_{i,l-1}$.

Theorem 5: *Let* $U_{i,l}$, $l \in \{1,..., L_i\}$ *be a user equilibrium. Let* $U_{i,l}^{comp}$ *be the set of all sets compatible with* $U_{i,l}$ *and let* $U_{i,l}^{\min}$ *be the set of sets defined in Definition 2:*

Then, $U_{i,l}^{\min} = U_{i,l}^{comp}$

Theorem 5 enables easily verifying if a given set is compatible with $U_{i,l}$. We further define the minimal number of destinations who need to be convinced to obtain performances as least as good as the ones obtained under $U_{i,l}$. First, we can notice that $U_{i,l}^{comp}$ is non-empty as $U_{i,l}$ is compatible with $U_{i,l}$. In addition, $U_{i,l}^{comp}$ is a finite set as there is a finite number of subsets of a finite set ($U_{i,l}$ is a finite set). We can consequently define $Eff(U_{i,l}) = \min_{U \in U_{i,l}^{comp}} (Card(U))$. $Eff(U_{i,l})$ corresponds to the effort that the terminal operator needs to exert to ensure obtaining the performances of $U_{i,l}$. In practice, we do not necessarily need to identify all the sets compatible with $U_{i,l}$ to compute

$Eff(U_{i,l})$. Indeed, if we identify a set U compatible with $U_{i,l}$ such that there is no set compatible with $U_{i,l}$ with cardinality less or equal to $Card(U)-1$, then $Eff(U_{i,l}) = Card(U)$.

In the example proposed above with 10 destinations, we obtain that $Eff(U_{i,1}) = 0$ (as $U_{i,1} = \varnothing$) and $Eff(U_{i,2}) = 2$ (as $\{1;2\}$ is compatible with $U_{i,2}$, and there is no singleton satisfying $K^{i,j} \le n_j$ as $U_{i,1} = \varnothing$). We compute $Eff(U_{i,l})$ is Section 5 and we further analyze the impacts of having more than one user equilibrium and derive several insights.

4.3 System optimum allocation and price of stability

The price of stability associated to the extended gate *i* for the *N* shippers' allocation game with $L_i \ge 1$ user equilibria is defined as the ratio between the total cost obtained under U_{i,L_i} and the minimum cost that could be obtained. This concept has firstly been proposed by Anshelevich et al. (2008). This is an optimistic evaluation of the consequences of having several actors interacting in the *N* shippers' allocation game. Indeed, U_{i,L_i} is the user equilibrium that performs the best in terms of total cost according to Corollary 3. The price of stability associated to the extended gate *i* is referred to as PoS_i (note that $PoS_i \ge 1$ for all $i \in \{1,...,M\}$). The price of stability is the optimistic version of the price of anarchy introduced by Koutsoupias and Papadimitriou (1999). We propose to use this concept of price of stability to measure the value of collaboration in what follows.

The computation of the price of stability involves identifying a SO allocation, as defined in subsection 4.1. Under a SO allocation, some destinations may decide to use the extended gate even if this choice increases their individual cost, as soon as the volume added for train transportation and transshipment operations provides a stronger cost reduction for the other players already using the extended gate. Let S_i be the set of all players using the extended gate under a SO allocation (S_i may be empty). Theorem 6 states the key structural property of S_i.

Theorem 6: *Assume that there exist two distinct destinations* $a,b \in \{1,...,N\}$ *such that* $Z^1_{0,a} - Z^1_{i,a} \ge Z^1_{0,b} - Z^1_{i,b}$, *then:*

$$b \in S_i \Rightarrow a \in S_i .$$

Theorem 6 implies that ranking and relabeling the destinations $j \in \{1,...,N\}$ by decreasing order of $Z^1_{0,j} - Z^1_{i,j}$ helps in finding the SO allocation. We assume w.l.o.g. for the remaining of this subsection that the destinations are arranged such that $Z^1_{0,1} - Z^1_{i,1} \ge ... \ge Z^1_{0,N} - Z^1_{i,N}$. The function obtained by comparing the total cost incurred when the first k destinations use the extended gate and the total cost incurred when all the destinations use direct truck transportation can be expressed as follows:

$$\sum_{j=1}^{k} n_j (Z_{0,j}^1 - Z_{i,j}^1) - \sum_{j=1}^{k} n_j Z_{0,i}^3 \left(\sum_{j=1}^{k} n_j \right). \tag{18}$$

This profit function can be evaluated for all $0 \le k \le N$ and S_i is obtained by selecting the highest value obtained with the minimum number of destinations using intermodal transportation. Accordingly, Theorem 6 implies the uniqueness of the SO allocation as defined in Subsection 4.1. Note that the profit function as evaluated by Formula (18) may not always be a monotonic function of k. The following Lemma shows that ordering the destinations by decreasing order of $Z_{0,j}^1 - Z_{i,j}^1$ implies ordering the destinations by increasing order of $K^{i,j}$.

Lemma 1: *Assume that the destinations are ordered such that* $Z_{0,j}^1 - Z_{i,j}^1 \ge Z_{0,j+1}^1 - Z_{i,j+1}^1$ *for all* $j \in \{1,..., N-1\}$*, then:*

$$K^{i,j} \le K^{i,j+1}, \textit{ for all } j \in \{1,..., N-1\}.$$

Finally, Lemma 2 enables comparing the set of destinations using the extended gate under the SO allocation and the set of destinations using the extended gate under U_{i,L_i}.

Lemma 2: *Let* S_i *be the set of destinations using extended gate i under the system optimum allocation and let* U_{i,L_i} *be the set of destinations using the extended gate under the user equilibrium performing the best in terms of total cost, then:*

$$U_{i,L_i} \subseteq S_i.$$

Lemma 2 implies that $PoS_i > 1$ if and only if $U_{i,L_1} \ne S_i$. Lemma 2 also implies that the SO allocation performs at least as good at U_{i,L_i} in terms of extended gate utilization.

5. Example and insights

This section is based on an example representing features of the hinterland network in the Netherlands. The main objective here is to explore the implications of having multiple actors involved in such a supply chain. We apply the modeling developed in Section 3, and the theoretical results of Section 4 enable us to quickly solve the problem. In this example, $N = 25$ and $M = 10$. More information about the data used for this example may be found in Appendix C (available online as supplementary material).

We determine $U_{i,l}$ and $Eff(U_{i,l})$, for all $i \in \{1,...,M\}$ and for all $l \in \{1,...,L_i\}$. This allows us to compute the total cost for all $U_{i,l}$ as well as the extended gate utilization by dividing the actual number of containers shipped to the extended gate by the total amount of containers (i.e., $K / \sum_{j=1}^{N} n_j$). The results appear in Table 2. Note that in case the extended gate is not used, the total transportation cost is €11 274.

	Total Cost (€)	*Extended Gate Utilization (%)*	$Eff(U_{i,l})$
U1,1	11 274	0%	0
U1,2	10 410	30%	5
U1,3	9 565	43%	10
U2,1	11 274	0%	0
U2,2	9 802	48%	12
U3,1	11 274	0%	0
U4,1	11 274	0%	0
U4,2	11 070	22%	6
U5,1	11 274	0%	0
U5,2	10 653	24%	6
U6,1	11 274	0%	0
U6,2	10 676	35%	10
U7,1	11 274	0%	0
U7,2	11 197	28%	6
U7,3	10 908	57%	9
U8,1	11 274	0%	0
U8,2	9 084	52%	6
U9,1	11 274	0%	0
U10,1	11 274	0%	0

Table 2: The user equilibria for an example with 25 shippers and 10 extended gate locations

We can notice that several equilibria exist for most of the potential extended locations considered. Out of 10 potential locations, 7 possess several UE. In addition, a user equilibrium involves having no destination using the extended gate for all the potential locations considered.

Insight 1: *Several user equilibria often exist in practice. In most of the cases, one of these equilibria involves not using the extended gate.*

Insight 1 is consistent with the practical findings of the maritime economics and transport geography literature. Indeed, Van Der Horst and De Langen (2008) mention that introducing a new hinterland service requires a base volume for becoming competitive. This base volume may often not be met by a single shipper. In case the base volume is not met, the user equilibrium will consist of having no destination using the extended gate. Accordingly, we evaluate the minimal number of destinations that the terminal operator needs to convince to ensure that at least the level of performance provided under $U_{i,l}$ is obtained by computing $Eff(U_{i,l})$. None of the user equilibria with at least one destination using the extended gate may be obtained without convincing at least 5 destinations for the presented example, as shown in Table 2.

The different user equilibria may be evaluated by the terminal operator based on the total cost (to be minimized), the extended gate utilization (to be maximized) and the convincing effort measured by $Eff(U_{i,l})$ (to be minimized). We apply multiobjective optimization to identify the set of efficient user equilibria. A user equilibrium $U_{i,l}$ is said to be efficient if none of the other equilibria perform at least as good at $U_{i,l}$ for two of the objectives considered and strictly better than $U_{i,l}$ for the remaining objective. Note that user equilibria obtained for different locations of the extended gate can be compared by the terminal operator as this one decides on where to locate the extended gate. The literature on combinatorial multiobjective optimization is well developed and enables implementing reliable and fast methods to identify the efficient solutions (see e.g., Ehrgott, 2005; Phelps and Köksalan, 2003). For the example of interest here, the efficient user equilibria may be obtained by direct comparison as the number of user equilibria is limited. The results appear in Table 3. We additionally define $\delta_{0,i}$ as the distance from the deep-sea terminal to the extended gate (expressed in km).

i	*Total Cost (€)*	*Extended Gate Utilization (%)*	$Eff(U_{i,l})$	$\delta_{0,i}$ *(km)*
N/A	11 274	0%	0	N/A
1	10 410	30%	5	197
8	9 084	52%	6	182
7	10 908	57%	9	31

Table 3: The efficient user equilibria

Based on the information provided in Table 3, the terminal operator can decide on a targeted total cost and extended gate utilization, select the best extended gate location accordingly and convince the destinations in accordance with $Eff(U_{i,l})$ (note that the destinations who need to be convinced appear in $\arg\min_{U \in U_{i,l}^{comp}}(Card(U))$). Table 3 highlights that the convincing effort that the terminal operator is willing to make influences the optimal extended gate location.

Insight 2: *The optimal extended gate location depends on the effort that the terminal operator is willing to make.*

Insight 2 implies that the terminal operator needs to assess the convincing effort and the outcome of this effort (if the destinations will or will not be convinced) in order to choose the optimal extended gate location. The question of how to convince destinations to use the extended gate in practice is of great interest. For example, the destinations may agree on guaranteed minimum volumes shipped via the extended gate before implementing this one. Such practice is currently used in the Netherlands, where 11 shippers in the region of Westland have signed an agreement to transport 10 000–15 000 containers per year by barge from the port of Rotterdam to the container terminal of Hoek of Holland (project Fresh Corridor 7). In this example, the project is not led by a terminal operator but we may expect such experience to be relevant for extended gate projects as well. We refer to the maritime economics literature focusing on hinterland supply chains for related discussions (see, e.g., Van Der Horst and De Langen, 2008).

Table 3 also shows that the total cost and extended gate utilization may be conflicting objectives. Indeed, minimizing the total cost and maximizing the extended gate utilization do not lead to the same location decisions for the example studied here.

Insight 3: *The classical objective of minimizing the total cost cannot be used as a proxy for maximizing the extended gate utilization as these two objectives may be conflicting.*

Insight 3 is somehow counterintuitive as we might expect that lowering the total cost would encourage more destinations to use the extended gate. However, the distance from the port to terminals helps understanding the dynamic. This information is presented in Table 3. Note that the distances are quite low for the example considered, as the example represents features of the hinterland in the Netherlands. Higher distances are expected if the results are applied to another area. We can notice that the extended gate utilization is decreasing in the distance from the port. Indeed, the more far away the extended gate is from the port, the more destinations are located closer to the port than to the extended gate. If destination j is located closer to the port than to extended gate i, we may expect that $Z^1_{0,j} < Z^1_{i,j}$, leading to $K^{i,j} = \infty$. Thus this destination would not use the extended gate (independently of the volume shipped to the extended gate). In addition, we can expect that the train transport cost is increasing in $\delta_{0,i}$, thus the minimal volume required to have the destinations using the extended gate tends to be increasing in the extended gate distance from the port. As a result, the extended gate utilization is expected to be higher when this one is located closer to the port. On the other hand, midrange locations perform better in terms of total cost as extended gate utilization is still reasonable and the distance from the port to the extended gate enables efficient use of intermodal transportation.

Insight 4: *Extended gates close to the port perform better in terms of utilization and extended gate at midrange locations perform better in terms of total cost.*

As Roso et al. (2009) point out, inland terminals may be located at different distances from the port when fulfilling different functions. Our results suggest to assess the comparative advantages and drawbacks of inland terminal locations in terms of total cost and utilization. We can also expect extended gates to be located closer to the port compared to other types of inland terminals as we highlight in Section 3 that utilization is of primary importance for terminal operators.

In order to fully explore the implications of having multiple actors involved in intermodal hinterland supply chains, we determine S_i and PoS_i for all $i \in \{1,...,M\}$. We recall here that the price of stability PoS_i is the ratio between the cost obtain under U_{i,L_i} and the cost obtain under S_i. This one enables us to evaluate the value of collaboration. The results appear in Table 4.

	Total Cost (€)	*Extended Gate Utilization (%)*	PoS_i
S1	9 272	54%	1,03
S2	9 579	61%	1,02
S3	11 186	41%	1,01
S4	10 580	74%	1,05
S5	9 897	52%	1,08
S6	9 936	57%	1,07
S7	10 846	70%	1,01
S8	8 946	63%	1,02
S9	10 112	67%	1,11
S10	10 309	57%	1,09

Table 4: The System Optimum allocations

The results of Table 4 enable us to draw additional insights. First, we can notice from Table 4 that the price of stability is relatively low for all the locations of the example. On the other hand, collaboration is compulsory for getting the extended gate used for locations 3, 9 and 10. This means that collaboration is critical for implementing the extended gate in some cases. The results obtained for location 3 are particularly surprising. For this location, there is a unique UE consisting in not using the extended gate (see Table 2). On the other hand the SO allocation lead to an extended gate utilization of 41%. We might expect the value of collaboration to be quite high (compared to other locations) in such situation. However, the value of collaboration is very low for location 3.

Insight 5: *Collaboration may be critical for successfully implementing an extended gate even if the value of collaboration is generally quite low.*

Finally, our results can help explain why some intermodal transportation projects are predicted to be effective in theory while being very difficult to turn into profitable projects in practice. This statement is in accordance with Rodrigue et al. (2010), who report that "both public and private actors have a tendency to overestimate the benefits and traffic potential and underestimate the costs and externalities of inland port projects" (p. 528). Most of the models used for estimating the benefits and traffic potential of inland terminal projects neglect coordination issues. The setting corresponds to the SO allocation of the proposed model. Lemma 2 implies that the SO allocation performs better than any UE in terms of total cost and extended gate utilization in most of practical situations. This enables understanding why public and private actors tends to overestimate the benefits and traffic potential in practice. The impacts of this overestimation can be substantial. For locations 3, 9 and 10, the SO allocation lead to a solution that may seem appealing, while the only existing UE has no destinations using the extended gate.

Insight 6: *The overestimation in extended gate utilization resulting from ignoring that multiple actors interact can lead to misleadingly consider an extended gate project as profitable.*

Our results lead us to conclude that taking the multiple actors feature of intermodal hinterland networks into account is of primary importance, especially because no single actor usually fulfills the role of supply chain leader in hinterland supply chains (Bontekoning et al., 2004).

6. Conclusions

In this article, we analyze some implications of having multiple actors involved in intermodal hinterland supply chains by focusing on a new problem referred to as the extended gate location problem. In this problem, the terminal operator first decides on where to locate the extended gate. Then the shippers decide if they want to take their containers at the deep-sea terminal or at the extended gate. We formulate the shippers' allocation problem as a non-cooperative game and we derive key structural properties of the game. This allows us to identify all the existing equilibria for the game. In addition, we show how to compute the minimum number of shippers who need to be convinced to reach a given equilibrium and we measure the value of collaboration by comparing our results to the ones obtained for the shippers' allocation leading to the optimal cost. We apply the results to an example based on the features of the hinterland network in the Netherlands and provide related insights.

Our main findings are as follows. First, we show that several equilibria often exist in practice. In most of the cases, the terminal operator needs to convince some shippers to use the extended gate in order to obtain the base volume necessary for becoming competitive. Second, we show that the optimal location depends on the convincing effort the terminal operator is willing to make. Third, we highlight that the minimization of the total cost and the maximization of the extended gate utilization may be conflicting objectives and we propose to apply multiobjective optimization to identify the set of efficient solutions. Finally, we highlight that collaboration may be critical for successfully implementing an extended gate even if the value of collaboration may also be quite low. Overall, we prove that the multiple actors feature of intermodal hinterland networks is critical and needs to be accounted for. We focus on the extended gate location problem in this article but our results may serve as a basis for appropriately taking multiple actors into account in other hinterland hub location problems. The results may also generalize to the entire container supply chain, but further research is necessary for this end.

A natural extension of the article would consist in studying the problem of opening multiple extended gates. This problem is an adaptation of the p-hub median problem known to be NP hard. We expect some of the structural properties demonstrated for the single extended gate location problem to hold when considering multiple extended gates. If our expectations materialize, the methods developed for solving the single actor p-hub median problem could then be adapted. A second way of extending this article would consist in studying how to provide incentives to the shippers, such that they act in accordance with the system optimum solution. This may be done by proposing rebates to some shippers in case they use the extended gate. We refer to Maillé and Stier-Moses (2009) for an illustration on how rebates may be used in the traffic assignment literature. The shippers may also seek for cooperation. In this case, the shippers' allocation problem may be viewed as a cooperative game and the key question consists in identifying stable cost allocations (see e.g., Özener and Ergun, 2008). We hope that our results help pave the way for further research from the operations management and transportation science community on container transportation systems.

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Appendix A

Notation	metric	description
N	$N \in \mathbb{N}^*$	*number of destinations*
M	$M \in \mathbb{N}^*$	*number of candidate locations for the extended gate*
n_j	$n_j \in \mathbb{N}^*$	*number of containers requiring shipment to destination j*
j	$j \in \{1;\ldots;N\}$	*index for destinations*
i	$i \in \{1;\ldots;M\}$	*index for extended gates (i=0 is used for the deep-sea port)*
$\delta_{0,i}$	*km*	*distance from the port to extended gate i*
$Z^1_{i,j}$	*€/container*	*truck transportation cost from i to destination j*
K	$K \in \mathbb{R}^*_+$	*number of container shipped by train*
$Z^2_{0,i}(K)$	*€/container*	*train transportation cost from the port to extended gate i*
$Z^{1,2}_i(K)$	*€/container*	*transshipment cost at the extended gate i*
$Z^3_{0,i}(K)$	*€/container*	*train + transshipment costs from the port to extended gate i*
$X_{i,j}$	$0 \le X_{i,j} \le 1$	*portion of the demand at destination j shipped via i*
X_i	$X_i = (X_{i,j})_{j \in M}$	*strategy vector*
y_i	*binary*	*y_i=1 if extended gate i is opened*
$P_{i,j}(X_{i,j},K)$	$P_{i,i}(X_{i,i},K) \in \mathbb{R}$	*cost savings of a strategy compared to using only direct shipment*
L_i		*number of UE associated to the allocation game with extended gate i*
$U_{i,l}$	$U_{i,l} \subset \{1;\ldots;N\}$	*set of destinations using the extended gate under equilibrium l*
$K^{i,j}$	$K_{i,j} \in \mathbb{N}^*$	*minimum volume for intermodal transportation for destination j*
$U^{comp}_{i,l}$	$U^{comp}_{i,l} \subset P(U_{i,l})$	*Set of sets compatible with user equilibrium l*
$Eff(U_{i,l})$	$Eff(U_{i,l}) \in \mathbb{N}^*$	*number of destinations that need to be convince to reach the equilibrium*
PoS_i	$PoS_i \in \mathbb{R}_+$	*price of stability for location i*
S_i	$S_i \subset \{1;\ldots;N\}$	*set of destinations using extended gate i under the SO allocation*

Appendix B

Proof of Theorem 1

UE:

Let $X_i = (X_{i,j})_{j\in N}$ be a user equilibrium of the allocation game (if any) and let $K = \sum_{j\in N} n_j X_{i,j}$. By contradiction, assume that that $1 > X_{i,j} > 0$ for a given $j \in \{1,...,N\}$. This implies that $Z_{i,j}(X_{i,j}, K) < n_j Z^1_{0,j}$ with $Z_{i,j}(X_{i,j}, K)$ defined in Formula (10). Thus $n_j(Z^1_{i,j} + Z^3_{0,i}(K)) < n_j Z^1_{0,j}$. Because $Z^3_{0,i}$ is non-increasing in K, we obtain that $n_j(Z^1_{i,j} + Z^3_{0,i}(K + (1 - X_{i,j})n_j)) < n_j Z^1_{0,j}$. This contradicts the fact that X_i is a user equilibrium as choosing $X_{i,j} = 1$ enables reducing $Z_{i,j}(X_{i,j}, K)$.

SO allocation:

Let $X_i = (X_{i,j})_{j\in M}$ be a system optimum allocation (if any). By contradiction, assume that that $1 > X_{i,j} > 0$ for a given $j \in \{1,...,N\}$. Using the results obtained for UE, we can conclude that choosing $X_{i,j} = 1$ will reduce the transportation cost for destination *j* without increasing the costs for the other destinations (as this implies increasing K, and $Z^3_{0,i}$ is non-increasing in K). We consequently end up with a contradiction. This concludes the proof.

Proof of Theorem 2

We can construct a UE as follows: We begin by setting $X_{i,j} = 0 \ \ \forall j \in \{1;...;N\}$. If none of the players can increase their profit by individually using the extended gate, then considering direct shipment for all the players is a UE. Otherwise, there is a destination $k \in \{1;...;N\}$ such that $Z_{i,k}(X_{i,k}, K | \ X_{i,k} = 1) < Z_{i,k}(X_{i,k}, K | \ X_{i,k} = 0)$. Let $X_{i,k} = 1$. If none of the remaining destinations can increase their profit by individually joining the extended gate, then considering direct shipment for all the players except for player *k* is a UE. Otherwise, the same procedure can be repeated as a destination included in the set of players using the extended gate at one step will never have any incentive to change its decision to direct shipment (because $Z^3_{0,i}$ is non-increasing in *K*). Because $\{1;...;N\}$ is a finite set, the proposed procedure necessarily converges; thus, a UE always exists for the shippers' allocation game. This concludes the proof.

Proof of Theorem 3

Consider an instance of the 2 shippers' allocation game with the following characteristics: $n_1 = 2$, $n_2 = 2$, $K^{i,1} = 4$ and $K^{i,2} = 4$. We can notice that $P_{i,1}(X_{i,1} = 1 \ ; X_{i,2} = 0) < 0$ and $P_{i,2}(X_{i,1} = 0 \ ; X_{i,2} = 1) < 0$. We conclude that $X_i = (0;0)$ is a user equilibrium allocation for the game. Moreover, $P_{i,1}(X_{i,1} = 1; X_{i,2} = 1) > 0$ and $P_{i,2}(X_{i,1} = 1; X_{i,2} = 1) > 0$ as $K^{i,1} = K^{i,2} \geq n_1 + n_2$, thus, $X_i = (1;1)$ is a second user equilibrium allocation for the game. This concludes the proof. We refer to Appendix C and Table 2 for a more detailed example with several UE.

Proof of Theorem 4

Assume that there exist two destinations $a, b \in \{1,...,N\}$ such that $a \neq b$ and $K^{i,b} \geq K^{i,a}$. Let $U_{i,l}$ be a user equilibrium of the shippers' allocation game ($l \in \{1,...,L_i\}$) and let $K_l = \sum_{j\in U_{i,l}} n_j$. By definition of $K^{i,j}$, $b \in U_{i,l} <=> K^{i,b} \leq K_l$ and $a \in U_{i,l} <=> K^{i,a} \leq K_l$. Then:

$b \in U_{i,l}$ => $K^{i,b} \leq K_l$ => $K^{i,a} \leq K_l$ => $a \in U_{i,l}$. This concludes the proof.

Proof of Corollary 1

Consider an instance of the shippers' allocation game with $L_i \geq 2$ user equilibria. Let $U_{i,l1}$ and $U_{i,l2}$ be two distinct user equilibria. If $U_{i,l1} = \varnothing$ then $U_{i,l1} \subset U_{i,l2}$ and if $U_{i,l2} = \varnothing$ then $U_{i,l2} \subset U_{i,l1}$. Let $a \in \{1;...;N\}$ such that $a \in U_{i,l1}$ and $K^{i,a} = \max_{j \in U_{i,l1}}(K^{i,j})$ and let $b \in \{1;...;N\}$ such that $b \in U_{i,l2}$ and $K^{i,b} = \max_{j \in U_{i,l2}}(K^{i,j})$. If $K^{i,a} \geq K^{i,b}$ then all the destinations included in $U_{i,l1}$ are included in $U_{i,l2}$ by applying Theorem 4 so $U_{i,l1} \subset U_{i,l2}$. Else, $K^{i,a} < K^{i,b}$ then all destinations included in $U_{i,l2}$ are included in $U_{i,l1}$ by applying Theorem 4 so $U_{i,l2} \subset U_{i,l1}$. Consequently, we can order the L_i user equilibria by descending order of $\max_{j \in U_{i,l}}(K^{i,j})$. This concludes the proof.

Proof of Corollary 2

Consider an instance of the shippers' allocation game with $L_i \geq 2$ user equilibria. By applying Corollary 1, we can deduce that $Card(U_{i,l+1}) > Card(U_{i,l}), \forall l \in \{1;...;L_i - 1\}$. We aim at proving that $\forall l \in \{1,...,L_i - 1\}$, $Card(U_{l+1}) \geq Card(U_l) + 2$. By contradiction, assume that there is $l \in \{1,...,L_i - 1\}$, such that $Card(U_{i,l+1}) = Card(U_{i,l}) + 1$. Then, a single player is added to the set of players using the extended gate by applying Theorem 4. This player's profit is strictly greater than zero when using the extended given the other players' decisions; thus, U_{l+1} is not a user equilibrium. We conclude that $Card(U_{l+1}) \geq Card(U_l) + 2$. This leads to $L_i \leq \left\lfloor \frac{N}{2} \right\rfloor + 1$. This concludes the proof.

Proof of Corollary 3

Consider an instance of the shippers' allocation game with $L_i \geq 2$ user equilibria. Let $K_l = \sum_{j \in U_{i,l}} n_j$ for all $l \in \{1,...,L_i\}$. By applying Corollary 1, we can deduce that $K_l < K_{l+1}$ for all $l \in \{1,..., L_i - 1\}$. Thus, $U_{i,l+1}$ performs strictly better than $U_{i,l}$ in terms of extended gate utilization.

The total cost for all destinations $j \in \{1;...;N\} \backslash U_{i,l+1}$ does not differ for $U_{i,l}$ and $U_{i,l+1}$. The total cost for all destinations $j \in U_{i,l}$ is less or equal for $U_{i,l+1}$ as compared to $U_{i,l}$ (Because $Z^3_{0,i}$ is non-increasing in K). The total cost for all destinations $j \in U_{i,l+1} \setminus \mathrm{U_{i,l}}$ is strictly less for $U_{i,l+1}$ as compared to $U_{i,l}$ (as destination decide to use the extended gate only if this strictly decrease its cost compared to direct shipment). As $U_{i,l+1} \setminus \mathrm{U_{i,l}}$ is non-empty, we conclude that $U_{i,l+1}$ performs strictly better than $U_{i,l}$ in terms of total cost. This concludes the proof.

Proof of Theorem 5

Let $l \in \{1;...;L_i\}$ and $U_{i,l}$ be a UE of the shippers' allocation game.

$U^{\min}_{i,l}$ is non empty as $U_{i,l} \in U^{\min}_{i,l}$. Let $U \in U^{\min}_{i,l}$. Then, $U \subseteq U_{i,l}$ and $\forall k \in U$, $K^{i,k} \leq \sum_{j \in U} n_j$.

If $U_{i,l} \backslash U = \varnothing$, then $U = U_{i,l}$ and we conclude that if $l > 1$, $U \not\subset U_{i,l-1}$ by applying Corollary 1.

If $U_{i,l} \setminus U \neq \emptyset$, consider U' with $U \subseteq U' \subset U_{i,l}$.

Let $k = \min\left(k' \in \left(1,...,Card(U_{i,l}) - Card(U)\right) \middle| k' \notin U'\right)$. Such a k exists as $U' \subset U_{i,l}$. Then $K^{i,k} \leq \sum_{j \in U} n_j + \sum_{j=1}^{k} n_j \leq \sum_{j \in U'} n_j + n_k$ as $j \in U'$ for all $j \in \{1;...;k-1\}$. Hence U' is not a UE. We conclude that $U \in U_{i,l}^{comp}$.

$U_{i,l}^{comp}$ is non empty as $U_{i,l} \in U_{i,l}^{comp}$. Let $U \in U_{i,l}^{comp}$. So $U \subseteq U_{i,l}$. $K^{i,k} \leq \sum_{j \in U} n_j$ for all $k \in U$. If $l > 1$, $U \not\subset U_{i,l-1}$. We will construct sequence $\left(1,...,Card(U_{i,l}) - Card(U)\right)$ such that for all $j \in \left(1,...,Card(U_{i,l}) - Card(U)\right)$: $U_j = U \cup \left(1,..., j\right) \in U_{i,l}^{comp}$.

Let $U_0 = U$. Obviously, $U_0 \in U_{i,l}^{comp}$. Let $j \in \{1;...;Card(U_{i,l}) - Card(U) - 1\}$ and assume that $U_j \in U_{i,l}^{comp}$. Hence, $K^{i,k} \leq \sum_{j \in U_j} n_j$ for all $k \in U_j$, $U_j \not\subset U_{i,l-1}$ and $U_j \subset U_{i,l}$. As U_j is not a UE, there exists $j+1 \in N \setminus U_j$ such that: $K^{i,j+1} \leq \sum_{j \in U_j} n_j + n_{j+1}$. Then, $K^{i,j+1} \leq \sum_{j \in U_{i,l}} n_j + n_{j+1}$ and thus $j+1 \in U_{i,l}$ as otherwise, this would contradicts that $U_{i,l}$ is a UE. We conclude that $U_{j+1} \in U_{i,l}^{comp}$ and by construction, we conclude that $U \in U_{i,l}^{\min}$. This concludes the proof.

Proof of Theorem 6

Let $b \in S_i$, and let $a \in N$ such that $Z_{0,a}^1 - Z_{i,a}^1 \geq Z_{0,b}^1 - Z_{i,b}^1$. As $b \in S_i$, the last container of destination b increases the profit when being shipped via the extended gate, i.e.:

$$\sum_{j \in S_i} n_j Z_{0,i}^3\left(\sum_{j \in S_i} n_j\right) - (Z_{0,b}^1 - Z_{i,b}^1) - \left(\sum_{j \in S_i} n_j - 1\right) Z_{0,i}^3\left(\sum_{j \in S_i} n_j - 1\right) < 0 \Rightarrow Z_{0,b}^1 - Z_{i,b}^1 > \sum_{j \in S_i} n_j Z_{0,i}^3\left(\sum_{j \in S_i} n_j\right) - \left(\sum_{j \in S_i} n_j - 1\right) Z_{0,i}^3\left(\sum_{j \in S_i} n_j - 1\right).$$

As $Z_{0,a}^1 - Z_{i,a}^1 \geq Z_{0,b}^1 - Z_{i,b}^1$, we obtain that:

$Z_{0,a}^1 - Z_{i,a}^1 > \sum_{j \subset S_i} n_j Z_{0,i}^3\left(\sum_{j \subset S_i} n_j\right) - \left(\sum_{j \subset S_i} n_j - 1\right) Z_{0,i}^3\left(\sum_{j \subset S_i} n_j - 1\right)$ meaning that at least one container of destination a enables increasing the profit when being shipped via the extended gate. By applying Theorem 1, we conclude that $a \in S_i$. This concludes the proof.

Proof of Lemma 1

Let $a, b \in \{1,..., N\}$ such that $Z_{0,a}^1 - Z_{i,a}^1 \geq Z_{0,b}^1 - Z_{i,b}^1$. If $K^{i,a} = 1$ then $K^{i,a} \leq K^{i,b}$. Else, $P_{i,a}(1, K^{i,a} - 1) < 0$ by definition of $K^{i,a}$.

$P_{i,a}(1, K^{i,a} - 1) < 0 \Rightarrow n_a\left(Z_{0,a}^1 - Z_{i,a}^1 - Z_{0,i}^3(K^{i,a} - 1)\right) < 0 \Rightarrow Z_{0,a}^1 - Z_{i,a}^1 - Z_{0,i}^3(K^{i,a} - 1) < 0 \Rightarrow$
$Z_{0,b}^1 - Z_{i,b}^1 - Z_{0,i}^3(K^{i,a} - 1) < 0 \Rightarrow P_{i,b}(1; K^{i,a} - 1) < 0 \Rightarrow K^{i,b} > K^{i,a} - 1 \Rightarrow K^{i,b} \geq K^{i,a}$. This concludes the proof.

Proof of Lemma 2

Any destination that is individually better when using the extended gate contributes to the global cost minimization because this decision has no negative effect on the other destinations. We conclude that $U_{i,L_i} \subseteq S_i$. This concludes the proof.

Appendix C

Data related to the example proposed in section 5

In this example, $N = 25$ and $M = 10$. The locations of the destinations and the possible extended gate are displayed in Figure 1C. These ones have been randomly generated. The crosses represent the destinations, the dots represent the possible extended gate location and the square represents the location of the port. The extended gate location numbers are also included in figure 1.

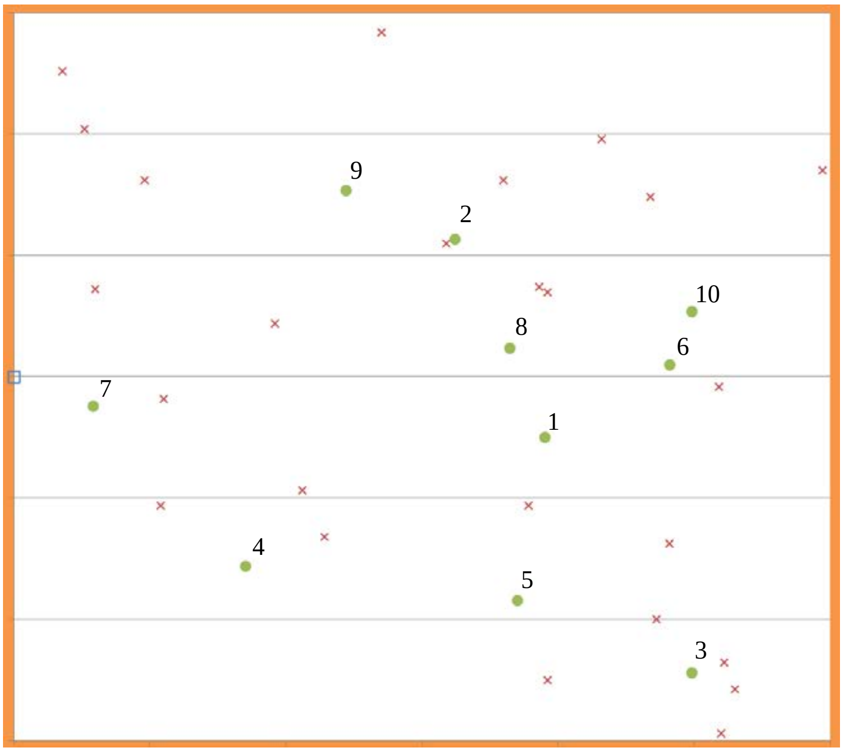

Figure 1C: Location of the destinations and possible extended gate location

The truck transportation costs are assumed to be linear in the distance traveled for each destination. The distance is calculated by considering the Euclidean norm. Table 1C provides the demand and location for each destination, as well as the truck transportation cost per container for direct shipment.

j	x_j	y_j	n_j	$Z^1_{0,j}$	j	x_j	y_j	n_j	$Z^1_{0,j}$
1	180	231	1	236	**14**	**18**	**276**	2	186
2	216	248	1	454	**15**	**297**	**235**	1	332
3	265	21	2	494	**16**	**48**	**231**	2	106
4	241	81	2	330	**17**	**259**	**146**	1	265
5	196	25	2	318	**18**	**260**	**3**	1	470
6	55	141	1	67	**19**	**26**	**252**	2	146
7	30	186	6	64	**20**	**114**	**84**	1	233
8	193	187	2	380	**21**	**196**	**185**	1	248
9	236	50	1	277	**22**	**54**	**97**	3	138
10	234	224	2	451	**23**	**96**	**172**	1	175
11	135	292	2	254	**24**	**189**	**97**	3	366
12	261	32	1	455	**25**	**159**	**205**	3	187
13	106	103	2	138					

Table 1C: Demand and direct shipment cost for each destination

Transloading costs at the extended gates are assumed to be affine in function of the number of container transloaded, i.e., that $KZ_i^{1,2}(K) = A_i + B_i K$. The parameters appear in Table 2C.

i	Ai	Bi
1	74	11
2	91	8
3	80	3
4	2	10
5	0	7
6	11	4
7	44	12
8	50	4
9	43	1
10	39	3

Table 2C: Parameters for the calculation of the transloading cost

The train transportation cost is linear in the distance traveled. The total train transportation cost per kilometer is piecewise linear in the number of container transported and is calculated as follows, $KZ_{0,i}^{2}(K) = a + bK + c.\min(d;K)$ with $a = 8$, $b = 0{,}1$, $c = 0{,}3$ and $d = 15$. The evaluation of $K^{i,j}$ for all $i \in \{1,\dots,M\}$ and for all $j \in \{1,\dots,N\}$ can then be performed by calculating $P_{i,j}(1;K)$ for all $K \leq \sum_{j \in N} n_j$. Table 3C provides some details of the calculations for extended gate 1 and destination 1.

We can deduce from Table 3C that $K^{1,1} = 33$.

K	20	21	22	23	24	25	26	27	28	29	30	31	32	**33**	34
$Z^1_{0,1}$-$Z^1_{1,1}$	107,8	107,8	107,8	107,8	107,8	107,8	107,8	107,8	107,8	107,8	107,8	107,8	107,8	**107,8**	107,8
$Z^3_{0,1}$(K)	157,5	151,5	146,0	141,0	136,4	132,2	128,3	124,6	121,3	118,2	115,3	112,5	110,0	**107,6**	105,3
$P_{1,1}$(1;K)	-49,7	-43,7	-38,2	-33,2	-28,6	-24,4	-20,5	-16,9	-13,5	-10,4	-7,5	-4,7	-2,2	**0,2**	2,5

Table 3C: Details for the calculation of $K^{1,1}$

Table 4C provides the value of $K^{i,j}$ for all $i \in \{1,\dots,M\}$ and for all $j \in \{1,\dots,N\}$.

j\i	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	33	12	∞	∞	∞	33	∞	18	12	31
2	14	6	∞	∞	46	12	19	8	6	11
3	9	20	7	8	7	12	11	11	32	16
4	11	21	16	14	11	15	17	13	28	19
5	17	∞	17	10	10	31	17	22	∞	46
6	∞	∞	∞	∞	∞	∞	22	∞	∞	∞
7	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
8	10	7	∞	27	25	12	12	6	8	13
9	17	36	17	16	13	23	24	20	∞	29
10	10	7	∞	39	25	9	16	7	7	8
11	∞	20	∞	∞	∞	∞	∞	44	11	∞
12	9	21	8	9	7	13	12	12	32	17
13	∞	∞	∞	20	∞	∞	20	∞	∞	∞
14	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
15	19	14	∞	∞	33	16	41	14	14	14
16	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
17	16	19	32	29	22	14	30	15	22	16
18	11	26	8	8	8	16	12	14	45	19
19	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
20	∞	∞	∞	8	37	∞	10	∞	∞	∞
21	18	13	∞	∞	42	20	25	11	15	21
22	∞	∞	∞	33	∞	∞	12	∞	∞	∞
23	∞	∞	∞	∞	∞	∞	17	∞	37	∞
24	8	18	22	9	9	16	9	9	24	21
25	44	14	∞	∞	∞	∞	∞	22	15	∞

Table 4C: Values of $K^{i,j}$

These data and the results provided in Section 4 enable us to identify all the existing UE as well as the SO allocation for all $i \in \{1,...,M\}$. The results appear in Section 5.