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Branch-Cut-and-Price for the Vehicle Routing Problem with Multiple Time Windows

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The Vehicle Routing Problem with Multiple Time Windows (VRPMTW) extends the classical VRP by allowing each customer to be served within one of several non-overlapping time windows. Because different depot departure times lead to different feasible time window selections (resulting in a time window pattern) and waiting times along a route, both the departure time and the time window pattern must be determined jointly. This added scheduling dimension substantially increases problem complexity, and existing solution methods are only heuristic. We propose the first exact algorithm for the VRPMTW using a branch-cut-and-price framework. The master problem is a set covering model, and the pricing problem is an elementary shortest path problem in which both the optimal depot departure time and the route's time-window pattern must be determined. We develop two algorithms for the pricing problem: (i) a labeling algorithm based on a graph expansion that duplicates each customer by its time windows, and (ii) a tailored labeling algorithm on the original graph that explicitly handles multiple windows. The latter handles a piecewise linear, typically discontinuous, ready-time function, which complicates label extension and dominance. To overcome this, we develop dominance rules tailored to the structure of the ready time function, including a novel partial dominance rule that prunes inferior departure time sub-intervals when full dominance does not apply. Computational results show that our method solves instances with up to 100 customers and 10-time windows per customer, thereby establishing exact benchmarks for evaluating existing and future heuristic approaches.