

# Summary

## *Cooperative Resource Pooling Games*

*with applications to the railway sector*

The Netherlands has one of the most intensively used railway networks of the world. Every day, more than one million passengers are transported on a compact, capacitated railway network of 6830 kilometers length. A highly reliable and available railway network is needed in order to guarantee such kind of service. However, there are disruptions too, which may affect availability of the railway network negatively. In that perspective, it is important that disruptions are repaired as soon as possible. This requires bringing the right service engineers, the right equipment, and the right spare parts to the disruption as quickly as possible. The current administrator of the Dutch railway network, i.e., ProRail, regulates this by outsourcing maintenance to several competitive contractors. Via performance based contracts, these contractors are held responsible for specific regions of the railway network. In particular, the contractors each hold an individual set of maintenance resources, dedicated for the execution of the maintenance in their own region. Looking from a national perspective, these dedicated maintenance resources can be used more efficiently. For instance, contractors can set up an arrangement in which a group of common maintenance resources are pooled and, as a consequence, can obtain some interesting cost savings. Although such cost savings can be considerably large, establishing a pooling arrangement between several independent, self-interested contractors is not easy! How can we, for instance, make sure that no individual contractor, nor any subgroup of contractors has reasons to split off from the collaboration? Such requirements are crucial for a sustainable cooperation and in that perspective, the construction of an allocation or rule that allocates the cost savings in such a way that no (group of) contractors want(s) to split off is a necessity.

In this monograph, we focus on this aspect for several types of situations in which service providers can pool their resources. These situations are all inspired by the Dutch railway sector. We address the cost savings allocation problem by making use of concepts of cooperative game theory. In particular, we formulate four resource pooling situations and for each of them we define an associated cooperative game. We formulate these

resource pooling games in terms of costs, (additional) profit, or cost savings directly. In any case, we are interested in how to allocate the total amount obtained under full collaboration. In particular, we are interested in the existence of allocations that makes no individual service provider, nor any subgroup of service providers worse off, or in terms of cooperative game theory, we are interested in core non-emptiness of the associated game. In addition, we focus on several allocation rules and appealing fairness properties they might satisfy.

Next, we describe the various resource pooling situations and associated cooperative games and subsequently highlight the most important results.

In the first resource pooling situation, we study an environment with several service providers who pool their critical, low-utilization resources with unavailability to increase joint profit. As an example, one can think of contractors who each own a single tamping machine. Such tamping machines are critical as tamping is required immediately, low-utilized as they are used a few times per year only, and sometimes unavailable, because they are subject to failures and repair leadtimes are long. For the associated availability game, we show that the core is non-empty in general. In addition, we show an even stronger result, namely the existence of an allocation of the joint profit for every possible coalition such that each player's payoff increases as the coalition to which the player belongs to grows larger. Moreover, we introduce four allocation rules and investigate them on several fairness properties. We investigate whether the allocations resulting from those allocation rules are increasing in the availability and in the profit function. Furthermore, we investigate whether the allocations resulting from those allocation rules are the same for players who are similar in the underlying setting or equivalent in terms of the associated availability game. Finally, we also investigate whether the allocations resulting from those allocation rules are members of the core.

In the second resource pooling situation, we consider an environment with several service providers, each keeping a single spare part in stock to protect against downtime of their technical systems. The costs related to the downtime of these technical systems are assumed to be different per service provider. As an example, one can think of contractors who each keep spare parts in stock for a specific railway segment with different penalty costs specified in their performance based contracts. We assume that service providers are able to reduce joint downtime costs by pooling the spare parts according to the one-by-one critical level policy. Under such a policy, players are added one-by-one to the group of players that are allowed to satisfy demand for an increasing number of spare parts in the on-hand stock. We refer to this as one-by-one pooling. For

the associated one-by-one pooling game, we show an interesting relationship with Böhm-Bahwerk horse market games. As a consequence, we can show that our game has a non-empty core. In addition, we present a class of allocation rules for which the resulting allocations are core members. Last, we study a simple and intuitive allocation rule within this class of allocation rules that satisfies interesting fairness properties.

In the third resource pooling situation, we investigate an environment that has quite some similarities with the second resource pooling situation. However, this time, we assume that service providers can collaborate by pooling their spare parts according to an optimal pooling strategy, to which we refer to as stratified pooling. The assumption of optimal pooling makes the mathematical analysis quite challenging. Still, we are able to show that the core of the associated stratified pooling game is non-empty. To this end, we use that the underlying resource pooling situation can be described by a Markov decision process and the optimal spare parts pooling strategy as a stationary decision rule in this Markov decision process. In particular, we use this modelling technique to prove core non-emptiness of stratified pooling games.

In the last resource pooling situation, we study an environment with several service providers who each may or may not own a single resource to cover their region completely. The service providers can increase total covering by pooling their resources. As an example, these service providers can represent (a part of) a region of a contractor with or without a repair van. Covering a region (of a service provider) implies that one can avoid penalty costs set by performance based contracts. For the associated maximal covering location game, we show that the core may be empty. This implies that there exist situations for which the allocation of the joint profit can always be improved upon by at least one coalition. Although collaboration is not always beneficial, we provide several sufficient conditions that ensure core non-emptiness. These conditions are in terms of the number of players, the type of graph, the number of resources, and an underlying integer linear program. Finally, for each condition, we provide an example showing that when the condition is not satisfied, core non-emptiness is not guaranteed.