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Solving Routing Problems by Exploiting the Dual of a master LP Formulation

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Abstract

This paper introduces a duality analysis of a master Linear Programming (MLP) formulation of the Vehicle Routing Problem with Time Windows (VRPTW). The considered MLP model is the slightly modified version of the relaxation of the Dantzig-Wolfe decomposition by expressing a VRPTW solution as a non-negative convex combination of constructed routes. The MLP model is basically the so-called reformulation of the VRPTW used in many Branch-and-Price (BP) algorithms. Our dual analysis shows that a pricing competition occurs in the dual model and the dual values of decision variables can guide us in making certain decision like customer grouping and introducing a new vehicle to an existing (incomplete) solution. By using our dual interpretation, we propose a heuristic algorithm that greedily constructs a routing plan by iteratively solving the MLP model as a central optimization mechanism. The objects to select in the MLP model are routes that are constructed by using a Dynamic Programming (DP) based method. We keep total number of routes bounded by a constant number, hence the size of the MLP model is fixed. A complete routing plan, i.e. an integer solution to the MLP model, is obtained by making the aforementioned decisions. We provide further details of the algorithm and show its efficiency by means of a computational study.

Keywords: Vehicle Routing with Time Windows, Linear Programming, Primal-Dual Method, LP Duality, Dynamic Programming.

1 Introduction

The Vehicle Routing Problem with Time Windows (VRPTW) is one of the basic benchmark problems in optimization. It is a generalization of the Vehicle Routing Problem (VRP) that was firstly introduced by Dantzig et al. (1959). The NP-hardness of the VRPTW is rooted from the Traveling Salesman Problem (TSP). Many exact solution methods to the VRPTW use a reformulation

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with set packing structure and employ Column Generation (CG) method in a Branch-and-Bound search, for example (Desrochers et al. 1992). Although a remarkable progress is obtained in the size of solved instances, exact algorithms suffer from long running times in solving real-life instances. The main issue is that the pricing sub-problem is NP-hard and requires either long time or large amount of memory to solve optimally. Some researchers worked on developing heuristics based on Branch-and-Bound methods whose termination criterion is either time or solution quality or a mix of both.

The basic idea of our approach is to express the customer visits as decision variables, so not enforcing the requirement that every customer should be by exactly one vehicle. Initially, we have an incomplete VRPTW solution when solved the MLP model, and then we greedily assign certain customers to used vehicles and reach a complete routing solution. The decision of assigning customers to used vehicles is based on the evidence obtained from the solutions of the MLP model and its dual. Relaxing the requirement of visiting customers exactly once is not a new idea, for example (Kohl and Madsen 1997) and (Kallehauge et al. 2006) worked on exact algorithms based on Lagrangian relaxation in which the relaxed constraints of the VRPTW are the aforementioned ones.

In this work, the proposed heuristic approach makes use of the information given by LP-duality in order to find good-quality solutions to the VRPTW. Our approach has important similarities to exact algorithms. It uses the master LP model with set packing structure that is used by many Branch-and-Price algorithms. Our algorithm does not backtrack during its course, and it makes greedy decisions in this sense. As customers are assigned to used vehicles, fixed-size path sets are revised in order to update the information obtained from dual solutions.

Our contribution. The contribution of this paper is two-fold. Firstly, we provide a dual interpretation of the MLP formulation of the VRPTW. Our dual interpretation shows that a competition happens among customers in the dual model, and the values of dual decision variables provide us some information about the customer competition. As the second contribution of this paper, we propose a heuristic approach to solve the VRPTW that makes use of the information provided by our dual interpretation. Our computational experimentation shows that our heuristic has short running time and it is promising to find good quality solutions.

The paper is organized as follows. Related work in the literature is mentioned in Section 2. Basic concepts and necessary notation is introduced in Section 3. The dual analysis by using primal-dual method of a master LP model of the VRPTW is provided in Section 4. Section 5 presents our proposed heuristic algorithm to the VRPTW by firstly outlining its important properties in an overview. Computational results are reported in Section 6. Finally, conclusions and possible research directions are discussed in Section 7.

2 Related Work

There is an extensive literature for the VRPTW that contains a wide range of algorithms like priority rule based simple heuristics, (adaptive) large neighborhood search, and exact algorithms. A good summary of the literature of the VRPTW till 1990s can be found in the review of Desrochers et al. (1988). In this survey, the authors mention that the literature lacks (at that time) an exact approach to the VRPTW, and few years later Desrochers et al. (1992) proposed one of the first exact solution methods to the VRPTW. Authors' method solves a reformulation of the VRPTW with a set packing structure and employs the CG technique in order to do bounding in a Branch-and-Bound search. The corresponding sub-problem amounts to finding the shortest path in a modified network with time windows and capacity constraints, and it is solved by using the Dynamic Programming method. The largest instances size that is solved optimally was with 14 customers till that time, and the results of Desrochers et al. (1992) showed that a high ratio of 25-customer instances are solved optimally within 10 minutes by using the computation power of 1990s. Some years later, Fisher et al. (1997) proposed two optimization algorithms to the VRPTW, namely a Lagrangian Relaxation/Variable splitting approach and a K-tree approach. In the former, two sub-problems (a semi-assignment problem and shortest path problem with time windows and capacity constraints) are solved. The authors report that 100-customer benchmark instances with clustered and randomly located customers to optimality with varying solution times are between 10 and 70 minutes. The conclusion is that both optimization algorithms perform best especially on the instances with clustered customers.

Customer grouping or fixing decision variables in a VRPTW formulation is a general trick researchers used, for example Cacchiani et al. (2014) propose a heuristic approach to the Periodic Vehicle Routing Problem (PVRP). The proposed algorithm solves a master LP model by fixing binary variables to 1 whose solution values are 1, and fixing the value of the variable that has the highest fractional value. After the fixing, the authors find new columns by taking into account the changed dual values of the master LP model. In another paper, Huang and Hsu (2011) introduce binary variables to allow not visiting/outsourcing some customers, and minimize weighted sum of these variable in the objective. The authors propose a Lagrangian heuristic to the Vehicle Routing Problems with the Private Fleet and the Common Carrier (VRPPC).

In the paper of Günlük et al. (2006) multi-depot VRPTW is studied. The authors propose so-called Fix-Price Heuristic that works in a similar manner of our heuristic algorithm. In their follow-on fixing procedure, the columns with solution values not smaller than 0.95 are fixed to 1. Then all columns are updated in order respect the fixing decisions. Next, the LP model is solved, and fixing decisions are made as long as variables with convenient solution values are found. When no variable with desired solution value is found, the threshold value is decreased to 0.85. The procedure is terminated if no variables having solution values greater than or equal to the reduced threshold value. Besides this fixing procedures, the proposed heuristic approach of Günlük et al. (2006) has other components to solve the studied problem efficiently.

One of the recent works on the VRPTW is conducted by Nagata and Bräysy (2009). The authors propose a sophisticated approach for reducing the number of routes, and it is based on the ejection pool that is combined with a concept reminiscent of the Guided Local Search. The benchmark instances described by Gehring and Homberger (2001) are used in experimentation of the proposed approach. By limiting the solution time to multiples of 10 minutes (maximum 5 hours), the authors were able to find new best known solutions for several instance sizes between 400 and 1000 customers. To the best of our knowledge, Nagata and Bräysy (2009) have currently the best solutions for the instances of large size in the literature. We refer to surveys by Bräysy and Gendreau (2005a), and Bräysy and Gendreau (2005b) for more recent exact heuristic algorithms for the VRPTW.

3 Preliminaries

This section briefly describes the VRPTW, and defines several concepts that are necessary for a formal description of our heuristic method.

3.1 Problem description

An instance of the VRPTW consists of a set $N = \{0, 1, \ldots, n\}$ of locations on a plane, where 0 is the depot and others are customer locations, a set V of homogenous vehicles of capacity $Q \in \mathbb{Z}_+$. Every customer $i \in N \setminus \{0\}$, also denoted by N', requires a service of length $sv_i \in \mathbb{Q}$ time units for a demand of amount $q_i \in \mathbb{Q}$. The service at customer i can only start in time interval $[e_i, l_i]$ where $e_i, l_i \in \mathbb{Z}_+$ are called earliest time (or release date), and latest arrival time (or due date) respectively. Hence, arriving earlier than e_i requires waiting till e_i, but later than l_i implies violation of the feasibility. The planning horizon of the problem is defined by the time window of the depot and it is denoted by $[e_0, l_0]$. The distance between customers i and i' is denoted by $d_{i,i'}$, and is equal to the Euclidean distance of the arc $[i, i'] \in N \times N$ on a plane where customer locations are specified as $x-$ and $y-$coordinates. We assume that a unit distance is traveled in a unit time, i.e. the distance of an arc is equal to the travel time on it.

> Problem: Vehicle Routing Problem with Time Windows (VRPTW)
>
> Instance and feasibility:
>
> Set N of customers with demands and time windows and the depot, set V of homogenous vehicles with capacities.
>
> A feasible route of a vehicle is a sequence of visited customers such that total demand does not exceed the vehicle capacity, and that every visited customer is serviced within its time window, and depot departure and depot arrival stay within the planning horizon. A feasible routing solution is a set of feasible routes such that every customer is serviced exactly by one vehicle, and the number of used vehicles does not exceed the number of available vehicles in the depot.
>
> Question: Does there exist an routing plan with number of vehicles less than k and for k vehicles with a smaller total travel distance less than D?

3.2 Preprocessing

Given an instance of the VRPTW, we conduct preprocessing steps. Firstly, we adapt the time windows as follows

$$e_i = \max\{e_i, e_0 + d_{0,i}\}, l_i = \min\{l_i, l_0 - d_{i,0}\}, \quad c \in N \tag{1}$$

Having found adapted time windows, incoming and outgoing arcs around customers are ranked with respect to their adapted lengths. An adapted length of an arc is a the sum of its own distance and a ratio of the minimum waiting time occuring due to using that arc. It is found as

$$w_{[i,i']} = d_{i,i'} + \alpha\left(\max\{e_{i'} - (e_i + s_i + d_{i,i'}), 0\}\right) \tag{2}$$

where $\alpha \in (0,1)$, and we use $\alpha = 0.1$ in our implementation. Incoming and outgoing arc lists of customers are non-decreasingly ordered and the indices of arcs in these lists become their ranking. Clearly, an arc has a tail (head) ranking, i.e. the index of it in the outgoing (incoming) arc list of its tail (head). Let $tr_{[i,i']}$ ($hr_{[i,i']}$) denote the tail (head) ranking of arc $[i,i']$.

Definition 1. *(Incompatible customers) Two customers that cannot be served in a feasible route due to their conflicting time windows or total demand exceeding vehicle capacity are called incompatible.*

3.2.1 Simple lower bounds on the number of used vehicles

Let L_{Veh} denote the lower bound on the number of vehicles in all feasible routing solutions for a given VRPTW instance.

Using total demand. Trivially, We can find minimum number of vehicles to serve all customers by the total customer demand as

$$L_{Veh} \geq L_P = \left\lceil \frac{\sum_{i \in N'} q_i}{Q} \right\rceil \tag{3}$$

Using incompatible customers. A customer set in which every pair of customers is incompatible also gives us a lower bound on the number of vehicles in a feasible routing solution. The maximum cardinality of aforementioned customer set can be found by solving the maximum independent set problem in customer network. Unfortunately, this problem is NP-Hard in strong sense. Therefore, we settle to a heuristic for finding a maximal independent set. In this heuristic, an independent set of customer is constructed greedily. Having chosen a customer to add to the independent set, all customers in the network that are connected by an arc to the chosen customer are deleted. This continues until no customer is left to chose. The decision of selecting the first customer to start the independent set can be made by checking several criteria like number of incoming and outgoing arcs and service demand.

Let IND denote the set of maximal independent set found by using the heuristic described above. Then we define L_{Veh} as follows

$$L_{Veh} = \max\{L_P, |IND|\} \tag{4}$$

3.3 Route centers, routes and route sets

We attach a used vehicle initially to a certain customer that is called the "route center" of that vehicle. Let RC denote the set of route center, and in the initialization of our algorithm the customers in a maximal independent set in the customer network are assigned as route centers

$$RC = IND \tag{5}$$

If we have $L_{Veh} > |IND|$, then the set of route centers should be extended to reach a feasible route solution. Extending the set of route centers may also be necessary in case $L_{Veh} = |IND|$, since we initialize the algorithm with the minimum number of vehicles (or router centers). During the course of the algorithm, we construct a fixed-size route set for every route center. In the following subsection we describe routes and explain our DP based route construction method.

In our solution approach routes are the building blocks, since the master LP model that we solve iteratively during the course of our algorithm selects routes to find a routing plan. The sequence $r = (r(1), r(2), \ldots, r(|r|))$ of customers is called a "route". Routes are simple, i.e. visiting every customer at most once, and visit exactly one route center. The route center can be in any place of the sequence of visited customers. We do not explicitly show depot in the expression of a route.

Definition 2. *(Transition quality of a route) Let r denote a route visiting its route center in k^{th} position. Then the transition quality of r is the sum of the head ranking of arcs before the route center and the tail ranking of arc after the route center. It is given by*

$$tq_r = \sum_{i=1}^{k-1} hr_{[r(i),r(i+1)]} + \sum_{j=k}^{|r|-1} tr_{[r(j),r(j+1)]} \tag{6}$$

Note that the transition quality of a route is a quality measure with respect to the route center, and it is flat since there may be a high number of route having the same transition quality value.

Route sets. Let the set $\mathcal{R}_{rc}$ denote all routes visiting the route center $rc \in RC$, and let the set of routes in $\mathcal{R}_{rc}$ of length l are denoted by $\mathcal{R}_{rc,l}$. It is easy to see that $\mathcal{R}_{rc,1} = \{(rc)\}$, and $|\mathcal{R}_{rc,2}| \leq 2|N|$. However route sets $\mathcal{R}_{rc,l}$ for $l \geq 3$ may have huge size in general. Hence in order to keep our algorithm to halt in polynomial time we require that $|\mathcal{R}'_{rc,l}| \leq L_l$ for $l \geq 3$ where L_l is a constant number. So $\mathcal{R}'_{rc}$ becomes a fixed-size route set of the route center rc.

4 Master LP model and its dual analysis

Our master LP model allows us to start with a partial feasible solution that serves a subset of customers initially. It contains customer assignment variables to decide which customers are to be served by available vehicles introduced so far. The objective has primary goal " maximizing" the number of selected customers and secondary "minimizing" the total distance traveled in the selected paths. A big coefficient is used to have the hierarchy in two aforementioned goals. Note that our master LP finds a routing plan for a given number of vehicles, that is $|RC|$. Next, we give the formulation of our master LP model. Table 1 explains the parameters and decision variables. The formulation of the master IP model is given in (29)-(10).

Table 1: Sets, parameters, and variables

Sets	
N'	set customer locations, $N' = N \setminus \{0\}$
$\mathcal{R}$	set of all constructed routes,
Parameters	
M	the objective coefficient of primary goal variables
c_r	cost (traveled distance) of route $r \in \mathcal{R}$
δ_r^i	indicates if route $r \in \mathcal{R}$ visits customer $i \in N'$
Decision Variables	
x_r	selection variable of path $r \in \mathcal{R}$
y_i	selection variable of customer $i \in N'$,

$$(SP) \qquad \text{Max} \qquad M\left(\sum_{i\in N'} y_i\right) - \sum_{r\in\mathcal{R}} c_r x_r \tag{7}$$

subject to:

$$\sum_{r\in\mathcal{R}} \delta_r^i x_r - y_i = 0, \qquad i \in N' \tag{8}$$

$$y_i \in \{0,1\}, \quad i \in N' \tag{9}$$

$$x_r \in \{0,1\}, \quad r \in \mathcal{R} \tag{10}$$

Constraints (8) couple selections of a customer and the routes visiting that customer. Note that in standard IP reformulations in the literature, for example Desrochers et al. (1992), all y_c variables are fixed to the value 1 as the right hand side of (8). Next, we relax all binary variables in (9)-(10) and we obtain the master LP model as

$$(P) \qquad \text{Max} \qquad M\left(\sum_{i\in N'} y_i\right) - \sum_{r\in\mathcal{R}} c_r x_r \tag{11}$$

subject to:

$$\sum_{r\in\mathcal{R}} \delta_r^i x_r - y_i = 0, \quad i \in N' \tag{12}$$

$$y_i \leq 1, \quad i \in N' \tag{13}$$

$$y_i \geq 0, \quad i \in N' \tag{14}$$

$$x_r \geq 0, \quad r \in \mathcal{R} \tag{15}$$

In the following section we use the primal-dual method in order to analyze how an optimal solution in the dual model is obtained which will enable us to interpret the values of dual variables in optimal solutions.

4.1 Dual analysis via primal-dual method

In this section, we show how the dual of the master LP model in (11)-(15) is solved optimally by incorporating the primal-dual method. After the explanations, numerical examples will also be given in the end of this section. The primal-dual method was proposed by Dantzig et al. (1956), and it has been used to design approximation algorithms for many problems in graph theory. In mathematical programming, it is known that many ideas of the exact algorithms to a number of network design problems are implicit in the primal-dual algorithms. Interested reader is referred to Goemans and Williamson (1996) for an extensive analysis of the primal-dual method in network design problems.

To start our analysis, we give the dual of our master LP model by letting λ_i,γ_i, κ_i be the dual variables corresponding to the constraints (12)-(14).

$$(D) \qquad \text{Min} \qquad \sum_{i \in N'} \gamma_i \tag{16}$$

subject to:

$$(\gamma_i + \kappa_i) - \lambda_i \geq M, \quad i \in N' \tag{17}$$

$$\sum_{i \in C_r} \lambda_i \geq -c_r, \quad r \in \mathcal{R} \tag{18}$$

$$\gamma_i \geq 0, \quad i \in N' \tag{19}$$

$$\kappa_i \leq 0, \quad i \in N' \tag{20}$$

Complementary slackness condition. By the Complementary Slackness (CS) theorem, given primal and dual feasible solutions $(y, x; \lambda, \gamma)$ are optimal if and only if the following equalities are satisfied

$$x_r \left(\sum_{i \in C_r} \lambda_i + c_r \right) = 0, \quad P \in \mathcal{P} \tag{21}$$

$$y_i \Big(\gamma_i + \kappa_i - \lambda_i - M) = 0, \quad c \in N' \tag{22}$$

$$(1 - y_i)\gamma_i = 0, \quad c \in N' \tag{23}$$

$$(y_i)\kappa_i = 0, \quad c \in N' \tag{24}$$

For the detailed analysis of the CS conditions and an extensive analysis of the linear optimization, we refer to the the book of (Bertsimas and Tsitsiklis 1997). In primal-dual method, a given dual feasible solution is improved towards the optimal solution by using "restricted primal" model which minimizes the violations from CS conditions. The basic idea is that the satisfaction of CS conditions is greedily increased till full satisfaction is reached. In order give the formal definition of the restricted primal model, we need to define several sets related to CS conditions as

$$K = \{r \in \mathcal{R} \mid \sum_{i \in C_r} \lambda_i + c_r = 0\} \tag{25}$$

$$J = \{i \in N' \mid \gamma_i + \kappa_i - \lambda_i = M\} \tag{26}$$

$$I = \{i \in N' \mid \gamma_i = 0\} \tag{27}$$

$$L = \{i \in N' \mid \kappa_i = 0\} \tag{28}$$

The set K is said to contain all routes in price balance in the dual solution. Note that only route in K can have positive x_r values by CS condition (21). Similarly, only customers in J can have positive selection values. Finally, we define the slack variable s_i for the customers not in the set I to quantify the violation from the CS condition. The violation of CS condition (22) due to customers not in set J is simply the value of y_i.

Restricted primal model For a given a dual feasible solution (λ, γ) with sets K, J, I; we can formulate a restricted primal problem that minimizes the violation of CS conditions as

$$(RP) \qquad \text{Min} \qquad z_{RP} = \sum_{i \notin I} s_i + \sum_{r \notin K} x_r + \sum_{i \notin (J \cup L)} y_i \tag{29}$$

subject to:

$$\sum_{r \in \mathcal{R}} \delta_r^i x_r - y_i = 0, \quad i \in N' \tag{30}$$

$$y_i \leq 1, \quad i \in I \tag{31}$$

$$y_i \geq 0, \quad i \in N' \tag{32}$$

$$y_i + s_i = 1, \quad i \notin I \tag{33}$$

$$x_r \geq 0, \quad r \in \mathcal{R} \tag{34}$$

$$s_i \geq 0, \quad i \notin I \tag{35}$$

By constraints (31), customers in I can have any y value, and the violation of CS condition (23) of those not in I amounts to the value of slack variable s in constraints (33).

Case $z^*_{RP} = 0$*:* Master LP is solved to optimality, i.e. all CS conditions are satisfied.

Case $z^*_{RP} > 0$*:* Dual feasible solution $(\lambda, \gamma, \kappa)$ is improved to another dual feasible solution $(\lambda'', \gamma'', \kappa'')$ with smaller objective value. To explain how this improvement is achieved, we first need to consider the dual of the (RP) model.

$$(DRP) \qquad \text{Max} \sum_{i \in N'} \gamma'_i \tag{36}$$

subject to:

$$\gamma'_i + \kappa'_i - \lambda'_i \leq 1, \quad i \notin (J \cup L) \tag{37}$$

$$\gamma'_i + \kappa'_i - \lambda'_i \leq 0, \quad i \in (J \cup L) \tag{38}$$

$$\sum_{i \in C_P} \lambda'_i \leq 1, \quad r \notin K \tag{39}$$

$$\sum_{i \in C_P} \lambda'_i \leq 0, \quad r \in K \tag{40}$$

$$\gamma'_i \leq 1, \quad i \notin I \tag{41}$$

$$\kappa'_i \geq 0, \quad i \in I \tag{42}$$

$$\gamma'_i \leq 0, \quad i \in I \tag{43}$$

For the sake of simplicity in the notation, decision variables γ' are used in (DRP) model. They are different from γ dual variables in the dual of the MLP model. In case $z^*_{RP} > 0$, we have $\sum_{i\in N'} \gamma'_i = z^*_{RP} > 0$ by strong duality. The improved dual solution is found as

$$(\lambda'', \gamma'', \kappa'') = (\lambda, \gamma, \kappa) - \Delta(\lambda', \gamma', \kappa') \tag{44}$$

where $\Delta > 0$ is called dual improvement step value. Then we have

$$\sum_{i\in N'} \gamma''_i = \sum_{i\in N'} \gamma_i - \Delta \sum_{i\in N'} \gamma'_i < \sum_{i\in N'} \gamma_i \tag{45}$$

Preserving dual feasibility gives us the maximum value of Δ. This is done by checking constraints (17)-(19) as

$$\gamma''_i + \kappa''_i - \lambda''_i \geq M \Rightarrow \Delta \leq \min_{i\notin(J\cup L), \gamma'_i+\kappa'_i-\lambda'_i>0} \left\{ \frac{\gamma_i + \kappa_i - \lambda_i - M}{\gamma'_i + \kappa'_i - \lambda'_i} \right\} \tag{46}$$

$$\sum_{r\in C_r} (\lambda_i - \Delta\lambda'_i) + c_r \geq 0 \Rightarrow \Delta \leq \min_{r\notin K, \sum_{i\in C_r} \lambda'_i>0} \left\{ \frac{\sum_{i\in C_r} \lambda_i + c_r}{\sum_{i\in C_r} \lambda'_i} \right\} \tag{47}$$

$$\gamma''_i = \gamma_i - \Delta\gamma'_i \geq 0 \Rightarrow \Delta \leq \min_{i\notin I, \gamma'_i>0} \left\{ \frac{\gamma_i}{\gamma'_i} \right\} \tag{48}$$

Sets J, K, I and L are updated according to solution values. This dual improvement procedure is repeated, by iteratively solving the (DRP) model, till we obtain objective value $\sum_{i\in N'} \gamma'_i = 0$ that implies the optimal dual solution is obtained.

Interpreting dual problem. In the dual model, γ (κ) variables represent positive (negative) budget. If one of γ and κ variables is non-zero, then the other must be zero by CS conditions (21)-(24) . Hence, $(\gamma + \kappa)$ is the "budget" of customers in the dual solution. Dual improvement step Δ is the "price" paid/collected in a dual improvement iteration. In dual iterations there are some price-collecting customers and some price-paying customers. The decision variable γ' (κ') in the (DRP) model tells us if a customer is a price collector (payer). The value of variable γ'_i (κ'_i) is the portion of the collected (paid) price for customer i.

A route constrains the total value of the budgets of visited customers. Dual constraints (18) enforce that the total budget of customers visited by route $r \in \mathcal{R}$ can have the smallest value $-c_r$, and routes with a minimum total budget are said to be in price balance. Once a route reaches to price balance, it is added to set K, and total budget should not change in the next dual iteration (constraints (40)). A route may leave price balance by increasing its total customer budget. In a dual iteration, constraints (39) enforce that routes not in price balance can have at most one dual improvement price drop in total

Table 2: Dual pricing rounds of the illustrative example

Instance Data						
$N' = \{i_1, i_2, i_3, i_4, i_5\}$, $RC = \{i_2, i_4\}$, $\mathcal{R}'_{i_2} = \{r_1, r_2, r_3\}, \mathcal{R}'_{i_4} = \{r_4\}$ $r_1 = (i_1, i_2, i_3), r_2 = (i_2, i_3), r_3 = (i_2, i_5), r_4 = (i_4, i_5)$ $c_{r_1} = 75$, $c_{r_2} = 50$, $c_{r_3} = 80$ $c_{r_4} = 60$						
Dual Pricing Solution						
Initial dual feasible solution: $\gamma_{i_k} = 1.5M, \kappa_{i_k} = 0, \lambda_{i_k} = 0.5M,\ i_k \in N'$						
Dual iteration	Payers	Collectors	Δ	K	I	$\sum \gamma'$
1	i_3, i_5	i_2	$M + 50$	$\emptyset$	$\emptyset$	$6.5M - 50$
2	i_2, i_4	i_5	$M + 50$	$\{r_2\}$	$\emptyset$	$5.5M - 100$
3	i_3, i_5	i_2	10	$\{r_2, r_4\}$	$\emptyset$	$5.5M - 110$
4	i_1, i_3, i_5	i_2, i_4	$0.5M - 60$	$\{r_2, r_4\}$	$\{i_3\}$	$5M - 50$
5	i_1, i_3	i_2	85	$\{r_1, r_2, r_4\}$	$\{i_3\}$	$5M - 135$
Final budgets: $\gamma_{i_1} = M - 25, \gamma_{i_2} = 2M - 50, \gamma_{i_3} = 0, \gamma_{i_4} = M - 110, \gamma_{i_5} = M + 50$						

budget. A *necessary condition* for a route to be selected in the optimal primal solution is to be in price balance. In light of our dual understanding, we have the following observation.

Observation 3. *A route center with a high γ value experiences high competition among customers to get serviced by one of its routes in the primal solution. Hence, the route center with smallest γ value in the dual solution is the most convenient for making customer grouping decisions.*

In the remainder of this section we give an illustrative example in order to show how dual pricing rounds occur, and present computational results of some initial master LP model due to Solomon benchmark instances.

An illustrative example Let us consider a simple problem instance that is given in Table 2. Clearly the only feasible, hence optimal, solution to the corresponding master LP model is $x^*_{r_1} = x^*_{r_4} = 1$, and $x^*_{r_2} = x^*_{r_3} = 0$ with objective value $5M - 135$.

Table 2 also the dual pricing solution after five dual iterations. Initially all customers have a budget of amount $\gamma = 1.5M$ which results in the dual objective value $7.5M$.

In the first iteration, customers i_5 and i_3 compete for being serviced in a route of route center i_2. In the second iteration, route center i_4 involves a competition with route center i_2 to service customer i_5 in its route. The competition in the first iteration repeated with a different price in third iteration. In the last two iterations, the prices are only for appearing in routes and price values tend to decrease.

Solomon benchmark instances. To solve the master LP model of the Solomon benchmark instances, the primal-dual algorithm is implemented in Java and *DRP* model is solved by using CPLEX 12.6.1. The optimal dual solutions found after dual improvement/pricing rounds are verified by the dual solution of the MLP model that has also found by using CPLEX 12.6.1. Numerical results and the number of dual rounds for corresponding instances are given in Table 3. We do not report the solution times, since our main concern here is

Table 3: Computational results of primal-dual method

Ins.	$\|RC\|^*$	z_D^*	#Rounds
C101	10	99171.06	58
C102	8	87245.36	131
C103	7	77336.33	68
C104	4	48652.58	27
C105	5	55488.42	29
C106	4	45130.29	31
C107	1	11853.27	69
C108	1	11853.27	60
C109	1	11867.77	39
R101	17	91559.96	328
R102	16	92059.64	222
R103	13	90859.44	278
R104	7	74808.94	164
R105	7	61133.50	76
R106	6	60989.03	65
R107	5	57724.92	87
R108	5	59987.25	70
R109	4	43079.24	21
R110	3	35690.76	20
R111	5	57181.50	61

* Initial set of route centers

to show the equality of the solutions values found by directly solving the MLP model and by solving its dual with pricing rounds.

5 Heuristic Algorithm to solve the VRPTW

In this section we explain an heuristic algorithm to solve the VRPTW. Note that this heuristic is a special way of using the dual understanding of the MLP models. The heuristic algorithm utilizes the dual understanding that was the topic of the previous section. The algorithm has three phases; initialization, route center finding phase, and customer grouping phase. Before going into details, we shall explain basic properties of the route construction method, and provide an overview of the algorithm by means of its flowchart.

5.1 An overview of the approach

The heuristic algorithm starts constructing a routing plan by initializing the set of route centers which is done by finding a maximal independent set of (incompatible) customers. As shown in Figure 1, route sets of route centers are constructed by DP based route construction method. Then MLP model is solved by using CPLEX 12.6. Then primal and the dual solutions of the MLP are ready to proceed further.

The heuristic first checks if the number of route centers $|RC|$ is less than the lower bound on the number of vehicles L_{veh}. If this is true, then we can confidently find a new route center and extend the set $|RC|$. Next, a new route center evidence in the MLP solution is checked. If the evidence is found, finding a new route center procedure starts in which a fixed number of candidates are

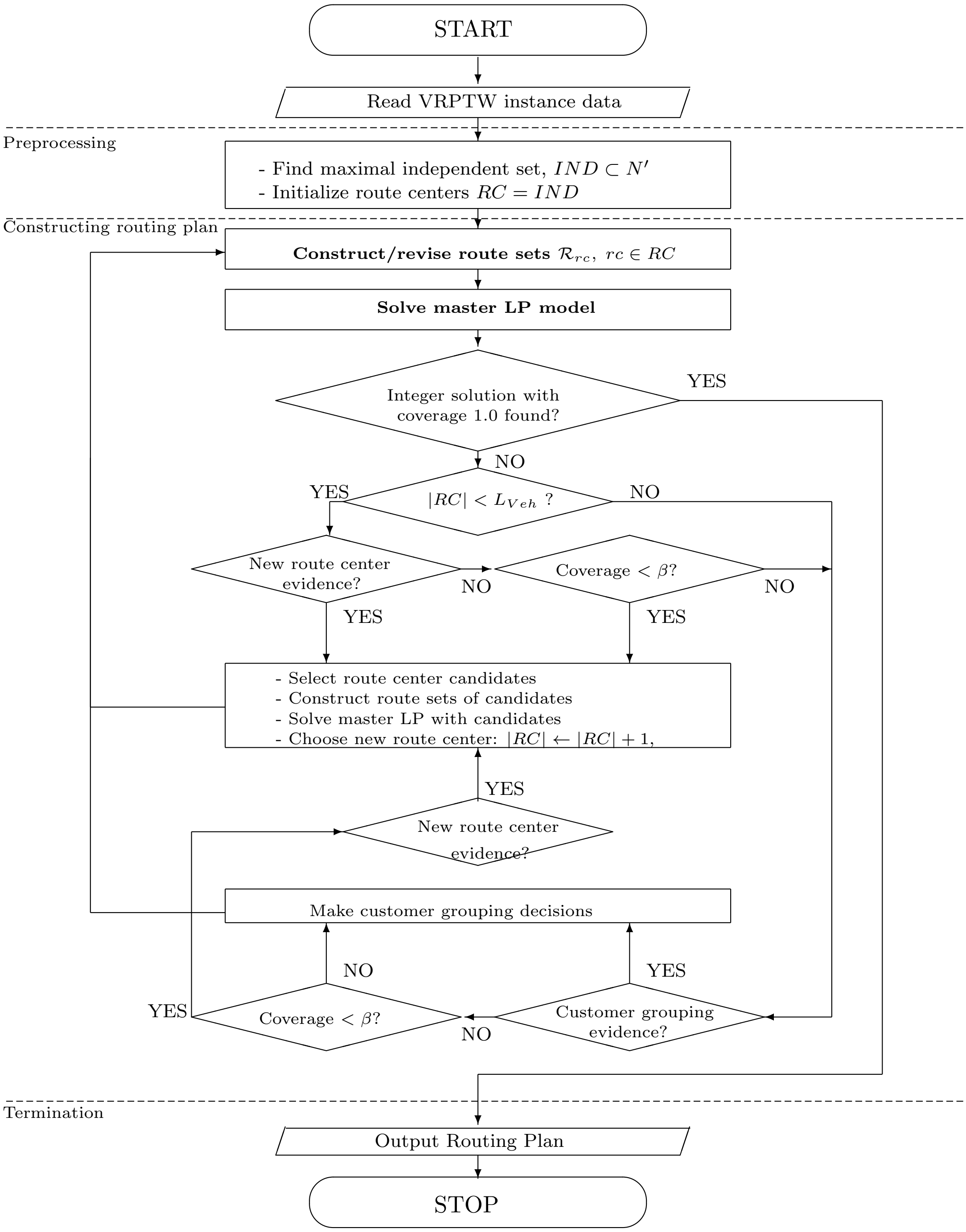

Figure 1: Flowchart of the heuristic algorithm to the VRPTW

determined whose route sets are constructed and MLP model with candidate routes are solved to determine the new route center. In case the evidence of a new route center is not clear, this could be a sign for the need to make some customer grouping decision in order to obtain a soon new route center evidence.

"Customer grouping" mainly helps the algorithm in reducing the problem size by attaching a customer to a route center. This is done gradually; first *linking* is decided requiring that a customer will always be in the routes of a certain route center. Second *siding* is decided which specifies if a customer will be visited before the linked route center or after. Finally, *fixing* results in making certain that an arc will be always used in all routes of a route center.

The dual solution of MLP model is used in both finding a new route center customer grouping. The subset of the route center candidates are selected from the customers that have highly negative γ values, and that are visited by the routes in dual price balance, in set K, with low transition quality. Moreover, customer grouping decisions are firstly made for route centers that have low γ values, which indicates that a weak competition happened in the dual solution of those ones by Observation 3.

An important property of MLP solution is the coverage which denotes the ratio of the selection of customers. If the coverage is low, then it strengthens the new route center evidence. Otherwise, we usually have strong customer grouping evidence. The parameter β Figure 1 is the threshold value for the coverage. Below that threshold value, we assume that the coverage cannot be increased to 1 by only making customer grouping decisions. In our implementation, we use 0.85 for β. Preliminary testing results of the heuristic algorithm showed that the results do not change significantly for β values from 0.5 to 0.90. Finally, an integer MLP solution with coverage value 1 gives us a complete routing plan, resulting in the termination of the algorithm.

5.2 Route construction.

Our route construction method has certain similarities to the one proposed by Kok et al. (2010). In the proposed DP heuristic, the number of states is bounded in every iteration. Moreover, the state expansions is also limited by a constant number. The authors tackle an extension of the VRPTW in which driving hour regulations are to be respected. In our route construction method, we also limit the number of routes of every length, and the number of routes that can be obtained by extending a given route is limited as well.

As explained in Section 3, given a route center $rc \in RC$, we have $\mathcal{R}_{rc,1} = \{(rc)\}$. The route set $\mathcal{R}'_{rc,l}$ for $l > 1$ is constructed by extending the routes in the set $\mathcal{R}'_{rc,l-1}$. Extending the routes is done by inserting convenient customers between to visits of existing routes. Feasible insertions are found by checking temporal constraints, and the vehicle capacity. We do not consider all possible insertion, but the ones that are efficient in terms of occurring detour cost. A constant number of such insertions are found in the preprocessing of the algorithm, and further insertion are checked when the necessity is seen during the course of the algorithm.

The route set $\mathcal{R}_{rc,2}$ is fully enumerated in order to be able to get dual solutions with as much as information possible. As mentioned before, route sets $\mathcal{R}_{rc,l}$ for $l \geq 3$ may have huge size in general. Hence we require that $|\mathcal{R}'_{rc,l}| \leq L_l$ for $l \geq 3$ where L_l is a constant number. So $\mathcal{R}'_{rc}$ becomes a fixed-size route set of the route center rc which results in a fixed size MLP model containing the selection variables of routes in sets $\mathcal{R}'_{rc,l}$, for all $l \geq 1$, and $rc \in RC$.

When finding the set $\mathcal{R}_{rc,l}$ by using insertions into routes in the set $\mathcal{R}_{rc,l-1}$, we use two criterion as quality measure: travel distance and transition quality.

5.3 Customer grouping

In Section 2, we mention several studies using variable fixing to reduce the problem size. To the best of our knowledge, the main criterion to choose which variables to fix is their primal values. In this work, we exploit the structure of the dual model, and observe that a binary decision variable with high primal selection value, i.e. close to 1, may not be convenient to be fixed.

The sample set of routes that is found by DP based route construction is usually very small compared to all possible routes for a given VRPTW instance. However, this sampling is strong in connections around route centers, since all routes are constructed by inserting further customers around route centers. Therefore, the heuristic algorithm checks customer grouping firstly for the customers visited before and after route centers. If such visits found that are strongly selected in primal solution and do not involve in high dual competition, then we say that a customer grouping evidence is obtained.

Definition 4. *(Central Path) The subsequence of customers in a route that contains only "fixed" arcs is called central path of that route.*

In fact, we check incoming to the central paths and outgoing arcs from the central paths are checked for customer grouping during the course of the algorithm. Once collected such arcs in a list, they are ordered with respect to their primal solution values. Ties are broken by checking the γ values of the corresponding central path customers.

So the arc to consider first for customer grouping becomes the one with highest primal selection value which is entering or leaving the central path with smallest γ value among all arcs in tie. The type of the grouping decision depends on the state of the adjacent customer to the central path. If it is not linked to a route center at all, we first link that customer to the route center of the central path. If it is already linked, then we decide if the customer should be visited before if the arc is entering into central path or after otherwise. The customer is said the be sided with respect to the central path. Finally, if the adjacent customer is already sided, then it is added to the central path.

5.4 Introducing a new vehicle

Note that the MLP model finds a routing solution for a given number of vehicles, that is the number of route centers $|RC|$. In general, optimal solutions of the VRPTW instances contain more vehicles than the minimum value L_{veh} that we find in the preprocessing phase. Another point is that the found maximal independent set of customers may even have smaller cardinality than L_{veh}. In this case, the algorithm extends the set of route centers RC more confidently till L_{veh} is reached. Then a route center evidence in the dual solution is expected to trigger new route center selection procedure. Fortunately, further customer grouping decisions lead to an evidence of new route center, if the available number of route centers is not sufficient to serve all customers.

The procedure of selecting a new route center starts with determining "route center candidates". Three properties of customers play important roles in finding good candidates. They are for customer $i \in N$; primal selection value y_i^*, dual variable γ_i^* value, and transition quality that is found by

$$\nu_i = \min_{rc \in RC} \{ \max_{r \in \mathcal{R}'_{rc} \cap K} \{tq_r\}\} \tag{49}$$

Note that the higher ν_i value of a customer, the worse connection quality of that customer to all route centers in the solution. We rank the customers in non-increasing order of y^* values, and select a fixed number of best ranked customers from the y^*-ordered list. Then we rank the in non-decreasing order of γ^* values, and select a fixed number of best ranked customers from the γ^*-ordered list. Similarly, last part of candidates are chosen by ordering all customers non-increasingly in transition quality ν_i values.

Let the set of route center candidates is denoted by $CAND$. Once candidate selection is completed, route sets of candidates are constructed by respecting all grouping decisions made previously. Then we include all selection variables of candidate routes in the MLP model by adding an extra constraint type enforcinf that as most one of them can be selected.

$$\sum_{r \in \mathcal{R}_c : c \in CAND} x_r \leq 1 \tag{50}$$

In the fractional solution, we declare the customer as the new route center that has the highest primal selection value in the candidate routes. Ties are broken by the number of candidate routes in K visiting a certain customer. Further ties are broken by γ^* values of customers. Once the new route center is found, all paths of candidates, including the ones of new route center, are removed and the path set of the new route center is constructed. Note that in most cases a new route center is usually chosen among candidates, however a customer that is not a candidate can be chosen a new route center in the solution of the modified MLP model hints so.

6 Computational Experimentation

We implemented the proposed heuristic algorithm in Java coding environment. During the course of the algorithm, all MLP models are solved by using CPLEX 12.6.1. The results that are presented in this section are obtained by using a personel computer with Intel i5 1.6 GHz Processor, and 8GB capacity of RAM.

100-customer instances of Solomon (1987) are used in our experimentation as benchmark instances. Table 4 gives preprocessing values of the instances. For example, C101 has initially the number of route centers equal to the lower bound on the number of vehicles. In instances C107-C109 and C204-C208 there is only one route center in the beginning. The high number of MLP iterations for these instances are due to runs of the model to find new route centers.

Table 5 shows the results found by our implementation. In a group of instances, the columns show the names of instances, solution properties of our heuristic; first number of vehicles and second traveled distance, and the percentage gap between the best known solution in travel distances. Finally, we

Table 4: Maximal Independent Sets,lower bound of number of vehicles

Instance	$\|IND\|$	L_{veh}	Instance	$\|IND\|$	L_{veh}
C101	10	10	C201	2	3
C102	8	10	C202	2	3
C103	7	10	C203	2	3
C104	4	10	C204	1	3
C105	5	10	C205	1	3
C106	4	10	C206	1	3
C107	1	10	C207	1	3
C108	1	10	C208	1	3
C109	1	10			

Table 5: 100-customer Solomon benchmark instances

	LP Heuristic		Time*	Iter		LP Heuristic		Time*	Iter
Ins.	V./D.	d(%)			Ins.	V./D.	d(%)		
C101	10/828.94	0.0	1.2	1	C201	3/591.56	0.0	2.0	3
C102	10/828.94	0.0	2.4	5	C202	3/591.56	0.0	1.9	3
C103	10/828.06	0.0	2.5	7	C203	3/591.17	0.0	2.8	4
C104	10/824.78	0.0	4.2	13	C204	3/630.43	6.7	2.7	5
C105	10/828.94	0.0	3.0	11	C205	3/589.72	0.1	2.4	5
C106	10/828.94	0.0	3.4	13	C206	3/592.21	0.6	2.4	5
C107	10/828.94	0.0	3.6	19	C207	3/589.27	0.1	2.8	5
C108	10/828.94	0.0	3.9	19	C208	3/591.89	0.6	2.3	5
C109	10/828.94	0.0	3.8	19					

*in seconds.

list number of MLP solving iterations for the instances in the last column of an instance.

The results of C10X and C20X instances are obtained in less than 5 seconds per instance. Our Heuristic has less than no gap between the best know solutions in the instance set C10X. The maximum gap becomes 6.7% for the instance set C20X in only one instance, and less than 1% in all other instances. We note that the solution of our algorithm is the first integer solution found, hence we do not have any improvement attempts on the constructed solution. Another important point to mention is that the solution times of our heuristic do not increase significantly as the route length in the solution of the instance increases. Especially, the instances with mixed customer time windows, e.g. C204, the number of feasible routes becomes quickly huge as the route length increases. Then route sets become important input for the MLP model in order to have high solution quality.

7 Conclusions and further directions

This paper has two main contributions. Firstly, it provides a dual analysis of a master LP formulation of the VRPTW by using primal-dual method. Secondly, a heuristic algorithm of the VRPTW is proposed by using the understanding from the dual analysis. In the dual analysis it is observed that the dual solution is the final state of a pricing competition among customers to appears in primal route of route centers. The most popular route center has high dual decision variable which means that the primal solution of that route center is not reliable to make customer grouping decisions. On the other hand, the existence of desperate customers in the dual solution, i.e. customers with highly negative dual variables, provides us an evidence for increasing the number of used vehicles. We underline the fact that all conclusions we present in this paper depend on the quality/size of the fixed-size route sets.

The proposed heuristic algorithm uses a master LP model of the VRPTW as a central optimization mechanism. It finds a complete routing solution by making customer grouping decisions by checking the evidence by not only using the primal selection values, but also the dual solution properties of the master LP model. To the best of our knowledge, this work is the first one incorporating the properties of the dual solution in solving routing problems. We believe that several issues in the proposed heuristic algorithm can be improved. For example, we make customer siding decision as soon as a linked customer is encountered. This can be done in different ways, for example a delay in number of iterations may be introduced in order to handle long-route VRPTW instances.

The master LP model under consideration of this paper can be used for several extensions of the VRPTW that involve real-life aspects like time-dependent travel times, stochastic travel times, and driving time regulations. These aspects of the problem will be considered in the route construction routine of the heuristic. In fact, a follow-up work of this paper has started that will focus on the VRPTW with stochastic travel times.

Finally, our dual analysis can be used to analyze other Operations Research problems that can be formulated as a master LP model. Then the dual interpretation will be useful in understanding the underlying processes resulting in the dual solution. This may give an opportunity to develop similar heuristic approachs for these problems.

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