Integrated Maintenance and Spare Part Optimization for Moving Assets

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We consider an integrated maintenance and spare part optimization problem for a single critical component of a moving asset for which the degradation level is observable. Degradation is modelled as a function of the current operating mode, mostly dictated by where the moving asset physically is. The spare part is stocked at the home base where the moving asset eventually stops by. Alternatively, the spare part can be stocked on board of the moving asset to prevent costly expedited deliveries. The costs associated with spare part deliveries and part replacements depend on the operating mode. Our objective is to minimize the total expected discounted cost of spare part deliveries, part replacements, and inventory holding over an infinite planning horizon. We formulate the problem as a Markov decision process and characterize the structure of the optimal policy, which is shown to be a bi-threshold policy in each operating mode. Our numerical experiments show that the cost savings obtained by the integrated optimization of spare part inventory and part replacement decisions are significant. We also demonstrate the value of the integrated approach in a case study from the maritime sector.

Keywords: maintenance, replacement, spare parts, moving assets, inventory, maritime, condition-based maintenance

1. Introduction

Maintenance and spare parts holding costs are known to be a significant part of the overall operating costs for many moving assets such as maritime assets (e.g., ships, vessels, submarines), aircrafts, commercial vehicles (e.g., trucks, buses, trains), and military equipment (e.g., frigates, strike-fighter). For maritime assets, maintenance activities can contribute in the range of 25-35% to the operating costs (Turan et al., 2009). For commercial airlines, maintenance accounts for around 10% of airlines' total costs (Lam, 1995). The total spare parts inventory in the aviation industry is estimated to be 45 billion US dollars in 1995 (Flint, 1995). In addition to the significant cost associated with maintenance and spare parts inventory, unexpected downtimes can lead to a significant loss of revenues and can affect health, safety, and environment. For instance, daily shipping operations of tankers yield as much as $20,500 depending on the vessel size (Unctad, 2013). Downtime of the Navy ships can seriously affect national safety. For such assets, maintenance and spare parts optimization is essential to decrease overall operating costs and to increase asset availability.

A component is called critical when the consequences associated with its failure are significant. In practice, such components are usually maintained by the so-called repair-by-replacement strategy, i.e., the component is removed from the asset and replaced by a new or as good as new spare part. This reduces maintenance execution time and the resulting downtime (van Houtum and Kranenburg, 2015). In this setting, the availability of spare parts is crucial since emergency shipments and the

downtime while waiting for the spare parts may be very costly. Traditionally, maintenance and spare parts inventory decisions are made separately. However, a separate optimization may lead to poor solutions since it ignores the interdependencies between the two decisions. In general, the integrated optimization provides remarkable improvement in terms of cost and availability compared to separate optimization (Kabir and Al-Olayan, 1996; van Horenbeek et al., 2013).

For many technical systems, it is possible to measure parameters of critical components that reveal the actual degradation behavior (van Houtum and Kranenburg, 2015). This enables the prevention of failures by means of condition-based maintenance (CBM), i.e., by executing a preventive replacement at the moment that a certain degradation threshold is exceeded. In practice, measurements can be performed via periodic inspections or advanced sensor technologies. Physical degradation models investigate the quantitative relation between degradation, usage, and environment (see, e.g., Tinga, 2010; Tinga and Janssen, 2013). In this paper, we consider a critical component of a moving asset for which the degradation level is observable and can be modelled as a function of the current operating mode. Operating modes represent where the moving asset physically is, how the component is used, and under which environmental conditions it operates. For many moving assets, the sequence and the duration of operating modes are more realistically modelled as random variables (see, e.g., Alam and Al-Saggaf, 1986; Çekyay and Özekici, 2015).

Under the common repair-by-replacement strategy, the spare part must be on board of the moving asset when replacing the component. In practice, the spare part is usually available at a fixed location in the home base, where the moving asset eventually stops by. Depending on the application, the home base can be a harbor, hangar, or garage. Typically, the cost associated with delivering a spare part is negligible when the moving asset is already in the home base. However, the cost of a spare part delivery might be significant when the asset is operating at a remote location. Such a delivery is made by means of an aircraft, helicopter, boat, dedicated vehicle, etc. To prevent costly deliveries, the spare part can be stocked on board of the moving asset. A holding cost is incurred if the spare part is kept on board. This holding cost might be relatively high due to several factors including the spare part's unit-value, the risk of obsolescence, and limited space availability on board.

This work is motivated by our collaboration with Fugro Marine Services (FMS). Fugro is the world's largest integrator of geotechnical, survey, subsea and geosciences services. A major part of Fugro's turnover relates to the research of the seafloor, i.e., collecting and interpreting data related to the earth's seafloor. For these operations, the so-called survey vessels are employed. The availability of these vessels is of high importance since the activities performed generate high revenue. FMS is Fugro's in-house vessel managing company. FMS manages maintenance related activities for survey vessels among others. We consider a critical component of one of these vessels in the case study presented in Section 6. FMS has treated the component under consideration as reliable based on the

original equipment manufacturer (OEM) specifications. The replacement of this component was planned to be performed on a time-based schedule and it was decided to not to hold the spare part on board. Nevertheless, the component has failed before its preventive replacement time while the vessel was operating at a remote location. The expedited delivery of the spare part has amounted to €10,000 approximately. In addition, the downtime cost due to one-day delay in operations, which is significantly varying over time, was within the range of €20,000 – €50,000 at that time. This situation could have been avoided if the spare part of this relatively cheap component was held on board of the vessel. On the other hand, it is neither realistic nor economical to always hold spare parts on board. First of all, the available space on board is usually very limited. Some of the critical components are very expensive resulting in significant holding costs. This shows that there is a need for a structured approach to make the decisions on "when to put a spare part on board" and "when to replace the component" just-in-time. Such an approach should take into account the randomness in the sequence and the duration of operating modes as well as the relation among operating modes, component degradation, and cost parameters. To the best of our knowledge, this problem is not yet studied in the literature.

We formulate the above-presented problem as a Markov decision process model in which the objective is to minimize the total expected discounted cost. Given that many moving assets have very long service times; we develop an infinite horizon model. We characterize the structure of the optimal policy, which is found to be a bi-threshold policy in each operating mode. Moreover, we provide insights regarding the performance of the optimal policy compared to four benchmark policies. In these benchmark policies, replacement decisions are optimized and spare part inventory decisions are pre-defined as: never keep a spare part on board or always keep a spare part on board. Our numerical experiments show that, compared to the benchmark policies considered, the cost savings obtained by the optimal policy can be significant.

The paper is organized as follows. Section 2 presents the related literature. Section 3 formulates the problem as a Markov decision process model. Section 4 characterizes the structure of the optimal integrated policy. Section 5 provides numerical experiments and several insights regarding the performance of the optimal policy compared to four benchmark policies. Section 6 presents a case study for demonstration purposes. Section 7 draws some conclusions and suggests potential future research directions.

2. Literature Review

Our paper relates to three main research streams, namely, CBM optimization for components subject to Markovian degradation, maintenance optimization of mission-based systems, and integrated maintenance and spare parts inventory optimization.

A large body of the CBM literature models degradation as a Markov process (Elwany et al., 2011). The earliest and basic replacement models for components subject to Markovian degradation can be found in Derman (1970), Kolesar (1966), and Kawai et al. (2002). For these models, it is shown that under some monotonicity assumptions the optimal replacement policy minimizing expected total discounted cost in an infinite time horizon is a threshold-type policy. This means that there exists a threshold with respect to the degradation level above which the optimal decision is to replace the component and below which the optimal decision is to do nothing. Over the last decades, the structure of optimal replacement policies have been investigated in different problem settings (see, e.g., Makis and Jiang, 2003; Elwany et al., 2011; Kurt and Kharoufeh, 2010). Çekyay et al. (2011) reviewed CBM models with Markovian degradation for which threshold-type policies are optimal. They showed that these polices are not necessarily optimal in a variety of cases and the optimal policy may have a rather complex structure.

There is a vast literature on systems performing missions which are composed of different phases or stages. In the literature, these systems are called phased-mission systems or mission-based systems. These systems are analogous to those that are subject to different operating modes and mission environments. Many papers focused on reliability and availability analysis for such systems (see, e.g., Esary and Ziehms, 1975; Kim and Park, 1994; Mura and Bondavalli, 1999; Kharoufeh et al., 2010). To the best of our knowledge, maintenance and replacement models for mission-based systems and for systems subject to randomly varying environments are limited. Waldmann (1983) investigated the structure of an optimal replacement policy for a system subject to stochastic degradation in a random environment. The author provided sufficient conditions to establish the optimality of threshold-type policies. Özekici (1995) modelled the environment by a semi-Markov process and used the intrinsic aging concept for degradation. The intrinsic age represents the cumulative hazard accumulated in time during the operation of the component in the randomly varying environment. Under increasing failure rate distribution functions in all environments and reasonable cost structures, the author showed that the threshold-type intrinsic age replacement and repair policies are optimal. In a similar setting, Çekyay and Özekici (2015) and Zhang et al. (2013) provided extensions to deal with the multi-component case. Ulukus et al. (2012) modelled the environment as a finite state Markov process. They assumed that the system accumulates degradation at a linear rate whose value depends on the environment. They showed that there exists an optimal threshold-type replacement policy for each environment. Considering similar degradation dynamics, Flory et al. (2015) extended the work of Ulukus et al. (2012) to partially observable environments. Çekyay and Özekici (2012) defined both the mission process and the degradation process as finite state Markov processes where the generator of the degradation process depends on the phases of the mission. They discussed optimal repair and replacement problems and characterized the optimal policies under some monotonicity assumptions. In this paper, our assumptions regarding mission and degradation processes are similar. The mission

process (operating modes) is described as a finite state Markov process. The degradation process of the component is defined by another finite state Markov process and is modulated by the mission process. Differently from the existing papers in this research stream, we investigate an integrated problem by taking the spare part inventory decision into account.

In the last decades, several integrated maintenance and spare parts inventory optimization models have been developed. Many papers in this research stream rely on the assumption that components' life-time/reliability distributions are given. Armstrong and Atkins (1996) considered a single component subject to age-based replacement. They investigated the optimal combination of replacement and ordering time that minimizes the total replacement and inventory costs. Armstrong and Atkins (1998) provided several extensions to the previous work, generalizing the cost terms and the replenishment time of spare parts. Kabir and Al-Olayan (1996) proposed a policy for integrated optimization of age-based replacement and spare parts ordering. They considered a number of identical components. Their simulation model reported a remarkable improvement on the total cost compared to separate optimization. To the best of our knowledge, only a few articles are available for components subject to CBM. Wang et al. (2008) considered a number of identical components and developed a condition-based replacement and spare parts ordering policy. They modelled the degradation of components as discrete-time Markov chains. They used Monte Carlo simulation to evaluate the performance of the proposed policy. Wang et al. (2009) combined Monte Carlo simulation and a genetic algorithm to determine the optimal parameters of the condition-based replacement and spare parts ordering policy. Elwany and Gebraeel (2008) enabled the use of sensor information and proposed a method which dynamically updates replacement and ordering decisions based on the physical condition of the equipment. They highlighted the advantages of using the proposed methodology compared to that of Armstrong and Atkins (1996). We refer the reader to van Horenbeek et al. (2013) for a complete review of this research stream. Unlike our work, existing research does not investigate integrated maintenance and inventory optimization problem from a moving asset perspective. To the best of our knowledge, none of these models incorporates the relation among operating modes, component degradation, and cost parameters.

3. Markov decision process formulation

In this section, we present the Markov decision process formulation of our integrated maintenance and spare part optimization problem.

Let I be a non-empty set representing different operating modes of the moving asset. Operating modes dictate where the moving asset physically is, how the component is used, and under which environmental conditions it operates. For example, operating modes can be defined as "wait in the home base", "transit to location l", and "perform mission m in environment e", etc. We assume that the operation process evolves as a continuous-time Markov chain (CTMC), $X \equiv \{X_t, t \geq 0\}$ on discrete

state space I. Therefore, the duration of operating mode $i \in I$ is assumed to be exponentially distributed with rate μ_i. The probability that the system jumps from operating mode $i \in I$ to operating mode $k \in I$ is denoted by $Q(i,k)$. Generally, $Q(i,i) = 0$ for all $i \in I$, but technically one can allow $Q(i,i)$ to be larger than zero. As further demonstrated in Section 6, the operation process of a real-life system can be well-represented by such a Markov process.

The degradation of the component also evolves as a CTMC, $Y \equiv \{Y_t, t \geq 0\}$ on a discrete state space $J \equiv \{0, 1, ..., F\}$, representing degradation levels from perfect (0) to failure state (F), where $F > 1$. Transitions in degradation levels are from j to $j+1$ for $j \in \hat{J}$, where $\hat{J} \equiv J \setminus \{F\}$. The degradation process is modulated by the operation process, i.e., transition rate λ_{ij} for the transition from j to $j+1$ depends on the current operating mode $i \in I$. We note that the number of degradation levels and operating modes can be arbitrarily large. Therefore, this constitutes a fairly general degradation process.

We assume that at most one spare part can be stocked on board of the moving asset. This assumption is not unrealistic considering limited space availability on board. Moreover, the component lifetime is usually much higher than the time between two successive home base visits, i.e., in practice, it is not likely to observe two or more failures during one mission. We denote the number of spare parts on board by $u \in U$, where $U \equiv \{0,1\}$.

We define the state of the Markov decision process as $(i, j, u) \in I \times J \times U$, a realization of the joint process (X, Y, Z) where X denotes the current operating mode, Y is the current level of degradation, and Z is the number of spare parts on board. We restrict our attention to a model in which decisions are only made at transition instants. A decision maker observes the system at each decision epoch (i.e., each time instant at which a transition occurs) and makes decision $a \in \{0,1,2\}$ which can be one of the followings:

0: Do nothing
1: Deliver a spare part
2: Replace the part

The set of possible actions $A(i, j, u)$ in state $(i, j, u) \in I \times J \times U$ is defined as:

$$A(i, j, u) = \begin{cases} \{0,1\} & \text{if } i \in I, j \in \hat{J}, u = 0 \\ \{0,2\} & \text{if } i \in I, j \in \hat{J}, u = 1 \\ \{1\} & \text{if } i \in I, j = F, u = 0 \\ \{2\} & \text{if } i \in I, j = F, u = 1 \end{cases}$$

At a decision epoch, action 0 (do nothing) can be chosen if the component has not failed. Action 1 (spare part delivery) is possible if there is no spare part on board. Action 2 (part replacement) can be performed only if the spare part is already on board.

We assume that a part replacement or a spare part delivery can be scheduled to occur immediately and instantaneously. Therefore, action 1 brings the system from state $(i, j, 0)$ to state $(i, j, 1)$ immediately. Similarly, action 2 brings the system from state $(i, j, 1)$ to state $(i, 0, 0)$ immediately. After each transition, a new action can be taken. That is, a spare part delivery can be immediately followed by a part replacement and the other way around. For example, if the component has failed when the spare part was not on board, a spare part delivery and a part replacement should take place consecutively, bringing the system from state $(i, F, 0)$ to state $(i, F, 1)$ and then from state $(i, F, 1)$ to state $(i, 0, 0)$ instantaneously. In this case, both actions take place at the same time instant.

The assumption of instantaneous replacements and deliveries is reasonable when the duration of these actions is much shorter than the mean component lifetime. In practice, even short downtimes can lead to significant costs. In this model, the downtime is translated into cost parameters. We distinguish a part replacement cost and a spare part delivery cost. At the time that a preventive replacement occurs in operating mode $i \in I$, the replacement cost $c_i^{\mathrm{pr}} > 0$ is charged. If the system is found to be failed, then a corrective replacement must be performed at a corrective replacement cost c_i^{cr}, where $c_i^{\mathrm{cr}} \geq c_i^{\mathrm{pr}}$ for $i \in I$. The replacement cost parameters c_i^{pr} and c_i^{cr} include the cost of labour and the cost of downtime due to preventive and corrective replacement actions, respectively.

For a spare part delivery that takes place when the system is up in operating mode $i \in I$, a preventive spare part delivery cost $c_i^{\mathrm{pd}} \geq 0$ is incurred. The cost of a preventive delivery includes the price of the spare part and the transportation cost. The transportation cost depends on the location of the moving asset, i.e., on the operating mode. We assume that a spare part is always available at the home base. By definition, the moving asset eventually stops by the home base. When the moving asset is at the home base, the spare part can be directly put on board with a negligible transportation cost (regular delivery). If the moving asset is at a remote location at the moment a spare part is requested, the delivery should be made by means of an aircraft, helicopter, boat, dedicated vehicle, etc. (expedited delivery). We translate the difference between regular and expedited deliveries into cost terms, by differentiating the spare part delivery cost with respect to operating modes. In principle, preventive deliveries do not cause downtime if the component has not failed. If the component has failed, additional costs may be incurred due to the cost of an emergency shipment and the cost of downtime while waiting for the spare part. To represent these additional costs, we introduce a corrective spare part delivery cost c_i^{cd}, where $c_i^{\mathrm{cd}} \geq c_i^{\mathrm{pd}}$ for $i \in I$. Moreover, holding the spare part on board of the moving asset incurs a holding cost $h > 0$ per time unit.

We apply uniformization to convert the continuous-time Markov decision problem into an equivalent discrete-time Markov decision problem. The uniformization is applied to the subset of states in which action 0 is possible. We add fictitious transitions from state $(i,j,u) \in I \times \hat{J} \times U$ to itself, ensuring that the total rate out of a state is equal for all states $(i,j,u) \in I \times \hat{J} \times U$. The latter refers to the so-called uniformization rate (see Alagoz and Ayvaci, 2010). We select a positive uniformization rate v such that $v \geq \max\{v_{ij} \mid i \in I, j \in \hat{J}\}$ where $v_{ij} = \mu_i + \lambda_{ij}$. In our Markov decision model, how long the system stays in a state depends on the action taken. Under uniformization, if action 0 is taken, the next transition (and decision epoch) is after an exponentially distributed time with mean $1/v$ in each state $(i,j,u) \in I \times \hat{J} \times U$. In this case, one of the following transitions can occur at the subsequent transition instant: (1) the system can jump from degradation level $j \in \hat{J}$ to $j+1$ with probability λ_{ij}/v, (2) the system can jump from operating mode $i \in I$ to operating mode $k \in I$, $k \neq i$ with probability $\mu_i Q(i,k)/v$, or (3) the system can occupy the same state after transition with probability $1 - v_{ij}/v$. For illustration purposes, we depict an exemplary process in Appendix I.

We assume a continuous discount rate $\alpha > 0$ so that any cost incurred at some future time t is discounted by a factor $e^{-\alpha t}$. Hence, the discount factor in the uniformized process can be defined as:

$$\beta = \int_0^\infty v e^{-(v+\alpha)\tau} d\tau = \frac{v}{v+\alpha}$$

The discounted holding cost of keeping a spare part on board during t time units is:

$$\int_0^t h e^{-\alpha \tau} d\tau = \frac{h}{\alpha}(1 - e^{-\alpha t})$$

If there is a spare part on board and action 0 is taken, then the expected discounted holding cost between two decision epochs is:

$$\int_0^\infty \frac{h}{\alpha}(1 - e^{-\alpha t}) v e^{-vt} dt = \frac{\beta}{v} h$$

The total expected discounted cost between two decision epochs $r(i,j,u,a)$ if action a is taken in state (i,j,u) is equal to:

$$r(i,j,u,a) = H(i,j,u,a) + C(i,j,u,a)$$

where $H(i,j,u,a)$ is the continuous cost and $C(i,j,u,a)$ is the lump sum cost of action $a \in A(i,j,u)$ in state (i,j,u):

$$H(i,j,u,a) = \begin{cases} \dfrac{\beta}{v}h & \text{if } i \in I, j \in \hat{J}, u = 1, a = 0 \\ 0 & \text{Otherwise} \end{cases}$$

$$C(i,j,u,a) = \begin{cases} c_i^{\text{pd}} & \text{if } i \in I, j \in \hat{J}, u = 0, a = 1 \\ c_i^{\text{cd}} & \text{if } i \in I, j = F, u = 0, a = 1 \\ c_i^{\text{pr}} & \text{if } i \in I, j \in \hat{J}, u = 1, a = 2 \\ c_i^{\text{cr}} & \text{if } i \in I, j = F, u = 1, a = 2 \\ 0 & \text{Otherwise} \end{cases}$$

Let $V(i,j,u)$ be the value function representing the minimum expected total discounted cost using the optimal policy if the operating mode is i, the degradation level is j, and the number of spare parts on board is u. The optimal replacement problem can be formulated by the following dynamic programming equation:

$$V(i,j,u) = \min_{a \in A(i,j,u)} \{r(i,j,u,a) + \Gamma_a V(i,j,u)\} \tag{1}$$

where operators Γ_0, Γ_1, Γ_2 refer to the actions of doing nothing, a spare part delivery, and a part replacement, respectively:

$$\Gamma_0 V(i,j,u) = \frac{\beta}{v}\left[\sum_{k \in I, k \neq i}\mu_i Q(i,k)V(k,j,u) + \lambda_{ij}V(i,j+1,u) + (v - v_{ij})V(i,j,u)\right] \text{ for } i \in I, j \in \hat{J}, u \in U \tag{2}$$

$$\Gamma_1 V(i,j,0) = V(i,j,1) \qquad \text{for } i \in I, j \in J \tag{3}$$

$$\Gamma_2 V(i,j,1) = V(i,0,0) \qquad \text{for } i \in I, j \in J \tag{4}$$

Equation (2) models possible future random events following the decision of doing nothing. It consists of three types of transitions. The first one is the transition in operating modes, bringing the system from state (i,j,u) to (k,j,u) where $i,k \in I, i \neq k$. The second one represents the component's degradation, bringing the system from state (i,j,u) to $(i,j+1,u)$ where $j \in \hat{J}$. The third one represents the transitions from state (i,j,u) to itself which is a consequence of the uniformization. Equation (3) corresponds to the decision of delivering a spare part, in which case the system jumps immediately from state $(i,j,0)$ to state $(i,j,1)$. Finally, Equation (4) represents the replacement decision which resets the number of spare parts on board and the degradation level to 0. Under (3) and (4), at each decision epoch, a part replacement can immediately follow a spare part delivery and the other way around.

We note that the state space of the model is discrete, the action space $A(i,j,u) \in \{0,1,2\}$ is finite for each $(i,j,u) \in I \times J \times U$. The model satisfies the conditions of Theorem 6.2.10 in Puterman (1994), which establishes the existence of an optimal deterministic stationary policy and the convergence of

the value iteration algorithm to the optimal value. A value iteration algorithm that is designed to deal with immediate and instantaneous actions is given in Appendix II.

4. Optimal Policy

In this section we analyze the structure of the optimal policy. We start by stating three theorems that establish monotonicity results related to the component's degradation level. From this, we derive the structure of the optimal policy, which is an operating-mode-dependent bi-threshold policy. The proofs of the theorems are given in Appendixes III - V.

Theorem 1: $V(i,j,u)$ is non-decreasing in $j \in J$ for each $i \in I$ and $u \in U$.

This theorem states the following. Consider two components in operating mode $i \in I$ while $u \in U$ spare parts are on board. If the degradation level of the first component is higher than the second one, then the minimum expected total cost of the first component cannot be less than the minimum expected total cost of the second component.

Theorem 2: $V(i,j,u)$ is submodular in $(j,u) \in J \times U$, i.e.,

$$V(i,j,1) - V(i,j,0) - V(i,j-1,1) + V(i,j-1,0) \leq 0 \text{ for each } i \in I \text{ and } j \in J \setminus \{0\}.$$

In Theorem 2, we consider the difference between the two cases, (i) there is a spare part on board and (ii) there is no spare part on board. We show that in a certain operating mode, when component's degradation increases, the increase in the minimum expected total cost in case (i) cannot be greater than that of case (ii). In other words, if the spare part is not on board, the cost increase resulting from component's degradation is higher compared to the case where the spare part is on board.

Theorem 3: In each $i \in I$, there exists an optimal part replacement threshold $\psi(i) \leq F$ and an optimal spare part delivery threshold $\xi(i) \leq F$ such that:

$$a*(i,j,u) = \begin{cases} 0 & \text{if } j < \xi(i), u = 0 \\ 1 & \text{if } j \geq \xi(i), u = 0 \\ 0 & \text{if } j < \psi(i), u = 1 \\ 2 & \text{if } j \geq \psi(i), u = 1 \end{cases}$$

This theorem shows that the optimal policy is an operating-mode-dependent bi-threshold policy on the degradation level.

Remark: The spare part delivery threshold can be greater than or equal to the part replacement threshold, which may seem counterintuitive in sense that one would expect a spare part delivery to precede a part replacement in all operating modes. This is because, whether a replacement decision is made or not depends on the number of spare parts on board. If there is no spare part on board at a certain degradation level and operating mode, the additional cost of delivering a spare part may not

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pay off the risk of a failure. However, if there is already a spare part on board for the same degradation level and operating mode, it may be optimal to replace the component.

5. Numerical experiments

In this section, we execute numerical experiments in order to test the performance of the optimal policy and to assess the value of integrated spare part and maintenance optimization. We compare the optimal policy to four benchmark policies in which replacement decisions are not integrated with spare part optimization. These policies simplify spare part inventory decisions by "never" or "always" keeping the spare part stock on board. Replacement decisions are optimized in all benchmark policies.

In the first benchmark policy, the spare part stock is never kept on board. That is, a spare part delivery is always followed by a part replacement. In addition, preventive deliveries are not allowed when the moving asset is outside the home base. As a consequence, preventive part replacements can only occur in the home base. We refer to this policy as never spare part policy (NP). The second policy is similar to NP, with the only difference of allowing preventive deliveries when the moving asset is outside the home base. We call this policy as never spare part policy with preventive deliveries (NPP). In NPP, the spare part delivery threshold becomes equal to the part replacement threshold in each operating mode. In the third policy, the spare part is put on board whenever the asset is in the home base (if it is not already on board). We refer to this policy as always spare part policy (AP). In AP, preventive deliveries in other operating modes than the home base are not allowed. That is, the spare part delivery threshold is zero in the home base and equals to the failure threshold in all other operating modes. Finally, we relax AP to allow preventive deliveries outside the home base and call this policy as always spare part policy with preventive deliveries (APP). The setup of the experiments is described in Section 5.1 and the results are discussed in Section 5.2.

5.1. Setup

We consider a critical component with 4 operating states $I \equiv \{0,1,2,3\}$ where 0 represents the home base of the moving asset, 1 and 3 intermediate states, and 2 the mission state. Intermediate states may correspond to transit states from home base to the mission site or vice versa. The operating mode transition probability matrix is defined as in Table 1, representing a deterministic cyclic sequence of operating modes.

Table 1: Operating mode transition probability matrix

$Q(i,k)$	0	1	2	3
0	0.00	1.00	0.00	0.00
1	0.00	0.00	1.00	0.00
2	0.00	0.00	0.00	1.00
3	1.00	0.00	0.00	0.00

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Three alternatives are considered regarding the operating mode transition rates (see Table 2). Alternative 1 fits to a commercial setting similar to the real-life case that we present in Section 6. In that case, missions and home base visits have short durations. Alternative 2 reflects our observations from the defense industry (Navy ships) where mission durations and home base visits are long. To enrich our comparative analysis, we also consider Alternative 3, in which mission durations are very long compared to home base visits.

Table 2: Operating mode transition rates

Operating mode transition rate (per year): μ_i	0	1	2	3
High	180	720	90	720
Low	6	26	6	26
Low in mission	180	52	6	52

Depending on the component, degradation can be uniform in different operating modes, high during missions or high during home base visits. For example, the power on/off switching has much influence on degradation for some electronic components, in which case degradation rates may be higher in the home base where the systems are often switched on/off. On the other hand, if the degradation is linearly related to operating time and the component mostly operates during missions, the degradation would be higher in missions (cf. the case study in Section 6). In these numerical experiments, we consider 10 degradation levels $J \equiv \{0,1,..,9\}$ and different degradation profiles for each operating mode $i \in I$ as given in Table 3.

Table 3: Degradation rates

Degradation rates (per year): λ_{ij}	i \ j	0	1	2	3	4	5	6	7	8	9
Uniform	0	2	2	2	2	4	6	8	10	12	-
	1	2	2	2	2	4	6	8	10	12	-
	2	2	2	2	2	4	6	8	10	12	-
	3	2	2	2	2	4	6	8	10	12	-
High in mission	0	2	2	2	2	4	6	8	10	12	-
	1	4	4	4	4	8	12	16	20	24	-
	2	6	6	6	6	12	18	24	30	36	-
	3	4	4	4	4	8	12	16	20	24	-
High in home base	0	6	6	6	6	12	18	24	30	36	-
	1	4	4	4	4	8	12	16	20	24	-
	2	2	2	2	2	4	6	8	10	12	-
	3	4	4	4	4	8	12	16	20	24	-

We take the same preventive replacement costs $c_i^{\mathrm{pr}} = €1,000$ for all operating modes $i \in I$. The cost parameters considered for corrective replacement c_i^{cr}, transportation c_i^{tr}, and the spare part's price c^{sp} are given in Table 4. We note that the preventive spare part delivery cost consists of the cost of transportation and the spare part's price, i.e., $c_i^{\mathrm{pd}} = c_i^{\mathrm{tr}} + c^{\mathrm{sp}}$ in operating mode $i \in I$. Additional spare part delivery cost in case of failure c_i^{ad} is the difference between preventive and corrective deliveries, reflecting a possible downtime cost or the additional cost of an emergency shipment, i.e.,

$c_i^{\mathrm{cd}} = c_i^{\mathrm{pd}} + c_i^{\mathrm{ad}}$ in operating mode $i \in I$. Three alternatives are considered for the holding cost rate h^{ra}, 10% (low), 25% (medium), and 50% (high). The holding cost per time unit is calculated by multiplying the holding cost rate with the spare part's price, i.e., $h = h^{\mathrm{ra}} c^{\mathrm{sp}}$. The annual discount rate of money is taken as 2%, resulting in the continuous rate $\alpha = -\ln(0.98)$. We note that the cost ranges considered are based on our observations from real-life cases that consist of both commercial (survey vessels) and defence equipment (the Navy ships) in the maritime sector. Overall, we obtain $3^6 . 2 = 1458$ instances with the combination of different alternatives considered.

Table 4: Cost parameters in thousand €

Cost parameters (in thousand €)	Alternatives	Operating mode i			
		0	1	2	3
Transportation cost (c_i^{tr})	Low	0	0.1	0.2	0.1
	Medium	0	1	2	1
	High	0	10	20	10
Additional delivery cost (c_i^{ad})	Yes	0	25	50	25
	No	-			
Corrective replacement cost (c_i^{cr})	Low	1.5			
	Medium	5			
	High	50			
Spare part's price (c^{sp})	Low	0.05			
	Medium	5			
	High	50			

5.1. Results

We assess the value of the integrated approach by comparing its performance with the four benchmark policies in terms of average and maximum cost increase. The cost increase is calculated by $CS = (C - C^*)/C^*$, where C is the expected total discounted cost under the benchmark policy and C^* is the optimal cost obtained by our integrated maintenance and spare part optimization model. In addition, we calculate the percentage of instances for which the expected total discounted cost obtained under the benchmark policy coincides with the optimal cost (denoted by OC %).

Our numerical experiments show that the integrated approach has significant value. As shown in Table 5, the average cost increases under the benchmark policies NP, NPP, AP, and APP are 78%, 20%, 30%, and 27% respectively. The benchmark policies are not very likely to give the optimal cost. The percentage of instances for which the optimal cost coincides with the cost obtained by NP, NPP, AP, or APP is 5%, 22%, 2%, or 12%. Not surprisingly, NP and AP are outperformed by NPP and APP, respectively. The average cost performance of NPP is found to be better than that of APP. In addition, "never spare part policies" (NP and NPP) are more likely to give the optimal cost than "always spare part policies" (AP and APP) for most of the cases. Nevertheless, the maximum cost increases in "never spare part policies" are significantly higher compared to "always spare part

policies", meaning that the latter is usually more robust. In what follows, we summarize our observations regarding the performance of benchmark policies in different parameter settings.

Observation 1: AP and APP are relatively well-performing (badly-performing) when the spare part is cheap (expensive) and the holding cost rate is low (high). On the other hand, NPP performs the best (worst) when the price of the spare part is high (low) and the transportation costs are low (high).

Observation 2: When the spare part is never stocked on board, relying on preventive replacements in the home base deteriorates the cost performance significantly. The cost performance of NP is significantly worse compared to NPP when the transportation costs are low, corrective replacement and corrective delivery costs are high, degradation is fast during missions, and the moving asset is mostly in mission. Moreover, since preventive deliveries are favorable when transportation costs are low, the cost performance of NP is non-monotonous with respect to transportation costs.

Observation 3: When the spare part is put on board whenever the moving asset is at the home base, the cost saving resulting from preventive deliveries performed in other operating modes than the home base is usually very limited. The difference between AP and APP is relatively high when the degradation is fast during missions and the spare part's price is low.

Observation 4: The cost performance of all benchmark policies is very sensitive to the structure of the degradation rates. The performance of "never spare part policies" deteriorates when component's degradation is fast during missions. On the other hand, "always spare part policies" perform relatively badly when degradation is fast in the home base. For uniform degradation rates that we consider, the component's life-time is longer compared to other alternatives. This is shown to cause a very poor performance in "always spare part policies".

Observation 5: Contrary to "always spare part policies", the performance of "never spare part policies" is very sensitive to the changes in operating mode transition rates. This is because, when the spare part is not stocked on board, the total cost of spare part deliveries is highly dependent on the operation process. In particular, NPP performs very well when the operating mode transition rates are high, i.e., when the home base visits are frequent. These cases greatly improve the average performance of NPP.

Observation 6: The cost performance of all benchmark policies is almost insensitive with respect to corrective replacement and corrective delivery costs. This is because, the trade-off between preventive and corrective costs is already taken into account while optimizing replacement decisions.

Table 5: Cost increases under the benchmark policies compared to the optimal policy

Parameter	Alternative	Never spare part policies						Always spare part policies					
		NP			NPP			AP			APP		
		Avr.(%)	Max.(%)	OC (%)	Avr.(%)	Max.(%)	OC (%)	Avr.(%)	Max.(%)	OC (%)	Avr.(%)	Max.(%)	OC (%)
Degradation rates	Uniform	40%	199%	7%	13%	108%	31%	41%	140%	2%	41%	140%	6%
	High in mission	154%	1057%	2%	33%	264%	14%	23%	74%	2%	14%	62%	23%
	High in home base	41%	200%	4%	14%	108%	22%	26%	105%	2%	25%	105%	7%
Operating mode transition rates	High	9%	34%	8%	5%	32%	22%	31%	140%	0%	31%	140%	0%
	Low	108%	950%	3%	25%	243%	29%	29%	134%	3%	24%	134%	17%
	Low in mission	119%	1057%	2%	29%	264%	15%	30%	132%	3%	25%	132%	19%
Corrective replacement cost	Low	56%	613%	10%	19%	264%	18%	29%	140%	4%	27%	140%	12%
	Medium	62%	645%	2%	20%	264%	25%	29%	137%	2%	27%	137%	12%
	High	117%	1057%	1%	20%	264%	24%	32%	137%	0%	27%	137%	12%
Transportation cost	Low	78%	1057%	7%	3%	14%	45%	35%	140%	0%	32%	140%	5%
	Medium	76%	1034%	4%	13%	73%	16%	31%	136%	0%	28%	136%	7%
	High	82%	963%	3%	44%	264%	5%	23%	129%	6%	21%	129%	24%
Add. exp. cost in case of failure	No	47%	617%	8%	19%	264%	25%	29%	140%	4%	27%	140%	12%
	Yes	109%	1057%	1%	20%	264%	19%	31%	137%	0%	27%	137%	12%
Spare part's price	Low	170%	1057%	0%	48%	264%	0%	8%	74%	5%	1%	6%	33%
	Medium	51%	283%	3%	10%	85%	20%	35%	115%	1%	33%	115%	4%
	High	15%	73%	10%	2%	18%	47%	47%	140%	0%	47%	140%	0%
Holding cost rate	Low	81%	1057%	3%	22%	264%	15%	13%	74%	2%	10%	32%	15%
	Medium	78%	1051%	3%	20%	263%	19%	27%	74%	2%	24%	73%	14%
	High	75%	1041%	7%	18%	260%	33%	50%	140%	1%	47%	140%	8%
Overall Avr. / Max.		78%	1057%	5%	20%	264%	22%	30%	140%	2%	27%	140%	12%

6. Case Study: Cooling fan of a survey vessel

In this section, real-life data acquired from Fugro Marine Services (FMS) are utilized for demonstrating our integrated maintenance and spare part optimization model. As briefly introduced in Section 1, a major part of Fugro's business activity relates to the research of the seafloor, for which survey vessels are being employed. These vessels use *diesel electric propulsion (DEP) systems* which produce thrust and create movement. The DEP system consists of several subsystems. The *centrifugal cooling fan* of the *frequency convertor* is part of the DEP system. The frequency converter generates heat as a consequence of its process and needs to be cooled down by a cooling fan. Since the failure of the cooling fan can lead to a fire, the DEP system needs to be stopped if the cooling fan has failed, which causes downtime. The cooling fan can be easily replaced in every location and its spare part stock can be kept on shore or on board of the vessel.

The daily reports of survey vessels contain information about the duration of different states such as staying in the harbor, sailing to a location, performing operations, or waiting for suitable weather conditions. In case of unsuitable weather conditions, the vessel sails to a location near the shore and waits there for a while. If weather conditions get better, the vessel sails back to operation site. Otherwise, the vessel returns back to the harbor. Based on these daily reports, we have categorized the operating modes as follows: "harbor", "transit-to-mission", "mission", "transit-to-harbor", and "weather". We have analyzed a data set of three years regarding the sequence of these operating modes and their duration. As for the appropriateness of the Markovian model, we have also tested whether the durations of operation modes are well-represented by exponential distributions. For all operating modes except transit-to-mission state, the exponential distribution assumption is not rejected by the Kolmogorov-Smirnov test at the 95% confidence level. The corresponding transition rates and the transition probability matrix are given in Table 6. Based on the failures experienced by FMS, the failure of the ball bearing is found to be the main reason of the cooling fan's failure. According to the OEM, the service life of the bearing system mainly depends on the thermal load on the bearing, i.e., the thermal fatigue. The OEM provides the L_{10} fatigue life of the bearing which is the time at which 10% of the bearings can be expected to have failed. The OEM states that the L_{10} fatigue life is approximately 40,000 operating hours at an ambient temperature of 40 °C. Based on this information, FMS has treated cooling fans as reliable and held no spare parts on board. The current policy employed by FMS corresponds to a time-based block replacement policy. The replacement is done during dry-dock maintenance which is mandatory every 5 years. Corrective maintenance is done if the cooling fan has failed before dry-dock maintenance.

Over the last 3 years, FMS has observed significantly short service lives. The earliest failures have occurred within the range of 2,000 – 5,000 operating hours. In the pursuit of representing the expected

failure times of the ball bearing better, we have used the fatigue life prediction model presented in Harris and Barnsby (2001):

$$L_{na} = a_1 a_2 a_3 L_{10} \tag{5}$$

where L_{na} is the adjusted bearing fatigue life and a_1, a_2, and a_3 are life adjustment factors for the selected reliability level, material, and operating conditions, respectively. Since the vessel under consideration always operates at the same geographical region, environmental conditions are assumed to be similar in different operating modes. After applying (5) (based on the qualitative information obtained from FMS), the adjusted fatigue life of the ball bearing is found to be 4,300 operating hours. The latter explains the current observations of FMS much better.

Table 6: Transition rates and the transition probability matrix

Transition Matrix: $Q(i,l)$	Harbor	Transit-to-mission	Mission	Transit-to-harbor	Weather	Transition/year: μ_i
Harbor	0.00	1.00	0.00	0.00	0.00	151
Transit-to-mission	0.00	0.00	1.00	0.00	0.00	1039
Mission	0.00	0.19	0.00	0.46	0.35	102
Transit-to-harbor	1.00	0.00	0.00	0.00	0.00	924
Weather	0.00	0.61	0.00	0.39	0.00	194

The degradation process is modelled based on our service life estimation in (5) and considering how the cooling fan is used in different operating modes. We note that the cooling fan is being in use only when the bow thruster is operating. This is required to make the vessel more maneuverable in mission, harbor, and weather states. Using the expert knowledge, we have assessed the operating time of the cooling fan in different operating modes. Assuming that the degradation is linearly correlated with the operating time, the cooling fan is expected to degrade 17 times faster during missions than during harbor or weather states. In transit states, the degradation of the cooling fan is assumed to be negligible since it is never utilized. Based on this information, we build an Erlangian degradation process for each operating mode where $F = 10$ and the rates out of each state to the next degradation state are 0.41, 0.00, 7.13, and 0.41 for harbor, transit, mission, and weather states respectively. By doing this, we divide the lifetime of the component in 10 equal parts. There is small amount of data to statistically support the degradation process that we propose. Nevertheless, this degradation process is found reflective considering what FMS has observed so far.

The cost parameters differ between operating modes and are as listed in Table 7. The preventive replacement costs mainly consist of the cost of labor. The preventive delivery costs include the spare part's price which is €3,600 and the cost of transportation which is €10,000 (applied if the vessel is in transit or in mission). During weather state the vessel is waiting near the shore which results in lower transportation costs. During harbor state the cost of transportation is negligible. The corrective replacement and delivery costs can be significantly high during mission and transit-to-mission states

due to the resulting downtime. A corrective replacement is estimated to take 4 hours and an expedited delivery lasts almost 1 day. The downtime cost associated with the delay in operations is an intangible parameter and is considerably varying over time. In this case study, we take the downtime cost as €30,000 per day, not to mean that it is actually what FMS incurs or bases its decisions on. We refer the reader to Section 5 for the impact of the downtime cost on the solution performance (this is reflected in corrective costs). In this case study, holding cost rate is taken as 25% per year and the continuous rate is set to $\alpha = -\ln(0.98)$.

Table 7: Cost parameters

Operating mode	Preventive replacement cost c_i^{pr}	Corrective replacement cost c_i^{cr}	Preventive delivery cost c_i^{pd}	Corrective delivery cost c_i^{cd}
Harbor	€ 100	€ 400	€ 3,600	€ 3,600
Transit-to-mission	€ 100	€ 5,400	€ 13,600	€ 43,600
Mission	€ 100	€ 5,400	€ 13,600	€ 43,600
Transit-to-harbor	€ 100	€ 400	€ 13,600	€ 13,600
Weather	€ 100	€ 400	€ 8,600	€ 8,600

We compare the optimal policy with the current policy of FMS as well as the benchmark policies, NP, NPP, AP, and APP. We observe NP and AP give the same solution as NPP and APP, respectively. This is because, preventive delivery costs are high in non-harbor states and the frequency of harbor visits is very high compared to the frequency of failures. This makes preventive deliveries unfavorable in non-harbor states.

Figure 1 demonstrates the thresholds values obtained by the optimal policy and the benchmark policies. We observe that "never spare part policies" encourage early replacements in the harbor to deal with the risk of corrective deliveries/replacements in non-harbor states. On the other hand, "always spare part policies" postpone the replacement in the operating modes in which the corrective replacement costs are relatively low (harbor, transit-to-harbor, and weather states).
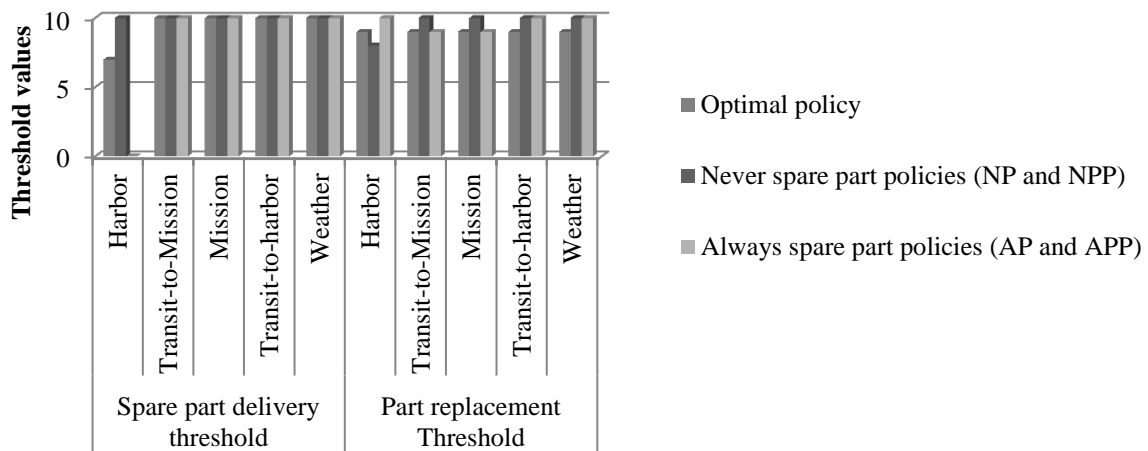

Figure 1: Threshold values obtained for different policies

Table 8 summarizes the performance of different policies in terms of cost. We note that the total cost obtained by the current policy is coherent considering what FMS has faced for the last few years. Assuming that the degradation model that we have built reflects the degradation of the cooling fan, the current policy performs significantly badly compared to the optimal policy, with a cost increase of 745%. This is because the initial service life was overestimated and the current policy excludes the relation between operating modes and degradation.

Table 8: Cost performance of different policies

	Expected total discounted cost over an infinite horizon	Cost increase compared to the optimal policy
Current policy	€ 804,933	745%
Never spare part policies (NP and NPP)	€ 105,784	11%
Always spare part policies (AP and APP)	€ 131,736	38%
Optimal policy	€ 95,290	

We observe that "never spare part policies" outperform "always spare part policies". This is because harbor visits are very frequent, offering many opportunities for cheap preventive replacements in the harbor. Nevertheless, compared to the optimal policy, both simple policies are shown to be very costly. The total cost increases of about 11% and 38% are observed under "never spare part policies" and "always spare part policies", respectively. Even under very frequent harbor visits, it is not optimal to "never" keep the spare part stock on board. The optimal policy provides a solution in which the component is stocked on board when the risk of failure is high, providing a trade-off between spare part delivery, part replacement and holding costs.

7. Conclusion

In this paper, we have considered an integrated maintenance and spare part optimization problem for a single critical component of a moving asset. We have proposed a Markov decision model and analyzed the optimal integrated policy. We have showed that the optimal policy is an operating-mode-dependent bi-threshold policy. Numerical experiments have been provided to assess the value of the optimal policy compared to four benchmark policies. In these benchmark policies, replacement decisions are optimized but not integrated with spare part inventory decisions. These policies simplify spare part inventory decisions by "never" or "always" keeping the spare part stock on board. Through a comparative analysis, we have showed that our integrated approach has significant value. Simplifying spare part inventory decisions result in an average cost increase of at least 20.0%. The benchmark policies are unlikely to be optimal except some specific conditions such as very high or very low spare part's price or transportation costs.

The problem studied in this paper was motivated by a real-life problem faced by Fugro Marine Services. From our collaboration with the maritime sector, we observe that failure-related historical data is usually limited. This makes the characterization of the degradation process challenging. In modelling the degradation and failure behaviors of the components, physical models are known to be less data demanding than data-driven models. Moreover, the difference associated with operating modes can be incorporated into physical models, using the quantitative relation between usage, environment, and degradation. Nevertheless, considerable effort is generally needed to develop such models.

In this paper, we assume that spare parts are always available at the home base. In practice, the home base is a central warehouse in which the spare parts stock is limited and pooled to serve several moving assets. If spare parts are directly assigned to moving assets and stocked on board, the benefit of inventory pooling at the central warehouse vanishes. In particular, this happens if lateral transshipments from one asset to another are not allowed. In our model, this effect can be taken into account implicitly, by inflating the holding cost rate. However, this approach is approximate. An exact approach would be to analyze this problem from multi-echelon spare parts inventory perspective, considering a number of moving assets served by a central warehouse (see, e.g., Muckstadt, 2005; van Houtum and Kranenburg, 2015).

Furthermore, in this paper, the consequences associated with system failure are translated into cost parameters. In practice, estimating intangible corrective cost terms is not straightforward, in particular, when failures affect people's health, safety, and environment. If modelling the trade-off between preventive and corrective costs is not obvious, the focus should lie on availability and reliability measures. The relation between system design and availability-reliability requirements needs to be well-investigated in systems' design phase.

We note that although our motivation comes from a maritime application, the model and the results presented in this paper may be applied to other moving assets such as aircrafts, commercial vehicles, and military equipment. The rapid development of advanced sensor technologies is making components' condition monitoring more affordable and feasible. The condition information provided by sensors can be especially useful for critical systems operating in environments that vary randomly over time. This is because the changes in environmental conditions and system usage can be related to the rate at which degradation accumulates (Ulukus et al., 2012). The model that we propose is greatly valuable considering the trend of shifting from time-based maintenance to condition-based maintenance. Moreover, repair-by-replacement is a common practice for high-value capital assets in general (Driessen et al., 2014). For moving assets, our model can be of value when there is an option of stocking the spare part on board. Depending on the application to be performed, the model can be

easily tailored to adjust to the number of operating modes, their duration/sequence, associated costs, and degradation parameters.

Acknowledgement

Appendix I.   Uniformization Example

For illustration purposes, we depict an exemplary process in Figure A.1. We consider two operating modes $I \equiv \{0,1\}$ and three degradation levels $J \equiv \{0,1,2\}$. We assume that the system operates under the following policy. In operating mode $i = 0$, a spare is delivered when degradation level j is greater than 1. In operating mode $i = 1$, spare part delivery is performed when the component has failed. In both operating modes, the component is replaced only correctively. In the uniformized process, when action 0 is taken under the given policy, the probability that the system jumps from degradation level $j \in \hat{J}$ to $j+1$ is $p_{ij} = \lambda_{ij}/v$ and the probability that the system jumps from operating mode $i \in I$ to $k \neq i$ is $q_{ik} = \mu_i Q(i,k)/v$. In Figure A.1, transitions from a state to itself are not depicted for ease of exposition (the corresponding probabilities are $q_{ii} = 1 - v_{ij}/v$). Immediate transitions under the given policy are shown by dotted arcs.
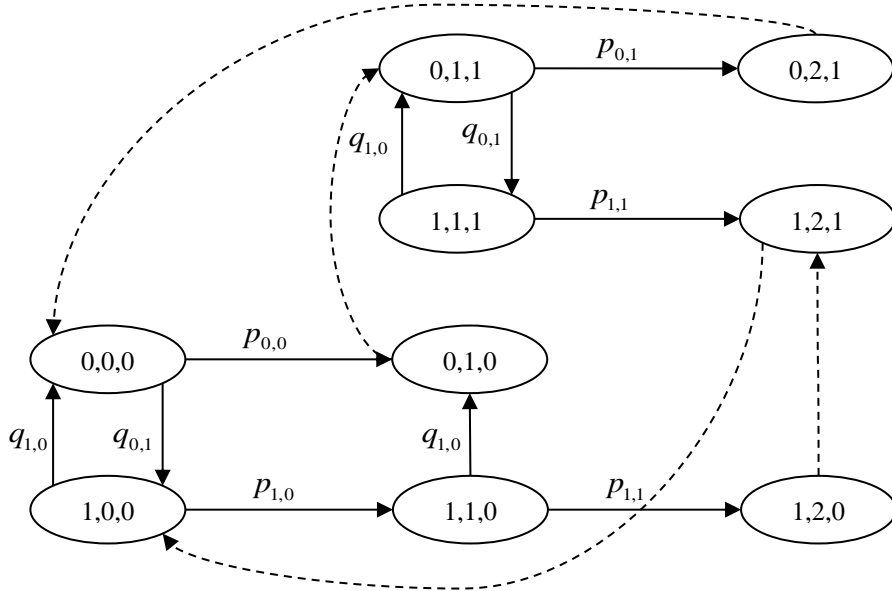

Figure A.1: Markov chain illustrating the uniformized process under a given policy

Appendix II.   Value iteration algorithm

Let $V_n(i,j,u)$ be the expected total discounted cost over a time horizon of n periods, where each period has an expected length $1/v$. Without loss of generality, we select $V_0(i,j,u)$ as follows:

$$V_0(i,j,u) = \begin{cases} 0 & \text{if } j \in \hat{J}, i \in I, u \in U \\ c_i^{\text{cd}} + c_i^{\text{cr}} & \text{if } j = F, i \in I, u = 0 \\ c_i^{\text{cr}} & \text{if } j = F, i \in I, u = 1 \end{cases} \tag{A.1}$$

We introduce an additional notation σ_a to represent the instantaneous part replacements and spare part deliveries:

$$\sigma_a = \begin{cases} 0 & \text{for } a = 0 \\ 1 & \text{for } a = 1,2 \end{cases}$$

For $(i,j,u) \in I \times J \times U$ and $n \in \mathbb{N}_0$, $V_{n+1}(i,j,u)$ can be computed by:

$$V_{n+1}(i,j,u) = \min_{a \in A(i,j,u)} \{ r(i,j,u,a) + \Gamma_a V_{n+\sigma_a}(i,j,u) \} \tag{A.2}$$

where $\Gamma_a V_{n+\sigma_a}(i,j,u)$ is defined as follows for $a = 0$, $a = 1$, and $a = 2$, respectively:

$$\Gamma_0 V_n(i,j,u) = \frac{\beta}{v} \left[\sum_{k \in I, k \neq i} \mu_i Q(i,k) V_n(k,j,u) + \lambda_{ij} V_n(i,j+1,u) + (v - v_{ij}) V_n(i,j,u) \right] \tag{A.3}$$

for $i \in I, j \in \hat{J}, u \in U$

$$\Gamma_1 V_{n+1}(i,j,0) = V_{n+1}(i,j,1) \qquad \text{for } i \in I, j \in J \tag{A.4}$$

$$\Gamma_2 V_{n+1}(i,j,1) = V_{n+1}(i,0,0) \qquad \text{for } i \in I, j \in J \tag{A.5}$$

Due to instantaneous part replacements and spare part deliveries, value iteration should substitute updated values of $V_{n+1}(i,j,u)$ at the $(n + 1)^{\text{th}}$ iteration. In state $(i,0,1)$, if the optimal decision is to replace the part, this decision will not be immediately followed by a spare part delivery. This is because, $V_{n+1}(i,0,1) = c_i^{\text{pr}} + c_i^{\text{pd}} + V_{n+1}(i,0,1)$ cannot hold since $c_i^{\text{pr}} > 0$ and $c_i^{\text{pd}} \geq 0$ for each $i \in I$. Therefore, $V_{n+1}(i,0,1)$ is equivalent to:

$$V_{n+1}(i,0,1) = \min\left\{ c_i^{\text{pr}} + \Gamma_0 V_n(i,0,0), \frac{\beta}{v} h + \Gamma_0 V_n(i,0,1) \right\} \text{ for } i \in I \tag{A.6}$$

Starting with (A.6), $V_{n+1}(i,j,u)$ can be evaluated in decreasing order of $u \in U$ and increasing order of $j \in J$ using (A.2). As such, (A.4) and (A.5) can be indirectly expressed in terms of $V_n(i,j,u)$ for each $(i,j,u) \in I \times J \times U$. This overcomes instantaneous transitions for all states. What follows

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presents the value iteration algorithm which is designed to solve (1). We refer to Puterman (1994, Theorem 6.3.1., p. 161) for further details about the convergence of such a value iteration algorithm to the optimal value of the infinite horizon problem.

Value iteration algorithm

Step 1: Set $n = 0$ and $\varepsilon = 0.0001$. Select $V_0(i,j,u)$ as (A.1).

Step 2: For each $(i,j,u) \in I \times J \times U$, compute $V_{n+1}(i,j,u)$ in decreasing order of $u \in U$ and increasing order of $j \in J$: If $(j,u) = (0,1)$ use (A.6). Otherwise, use (A.2).

Step 3: If for each $(i,j,u) \in I \times J \times U$, $|V_{n+1}(i,j,u) - V_n(i,j,u)| < \varepsilon(1-\beta)/2\beta$

Then, go to Step 4. Otherwise, increment n by 1 and return to Step 2.

Step 4: For each $(i,j,u) \in I \times J \times U$, choose the optimal stationary policy by:

$$a^*(i,j,u) = \underset{a \in A(i,j,u)}{\arg\min}\{r(i,j,u,a) + \Gamma_a V_{n+\sigma_a}(i,j,u)\} \text{ and stop.}$$

Appendix III. Proof of Theorem 1

In order to prove Theorem 1, we first introduce Lemmas $1 - 4$.

Lemma 1: For each $i \in I$ and $n \in \mathbb{N}_0$, if $\Gamma_0 V_n(i,j,1)$ is non-decreasing in $j \in \hat{J}$, then $V_{n+1}(i,j,1)$ is non-decreasing in $j \in \hat{J}$.

Proof of Lemma 1: From (A.2), $V_{n+1}(i,j,1)$ can be re-written as follows for each $i \in I$, $j \in \hat{J}$, and $n \in \mathbb{N}_0$:

$$V_{n+1}(i,j,1) = \min\left\{c_i^{\mathrm{pr}} + V_{n+1}(i,0,0), \frac{\beta}{v}h + \Gamma_0 V_n(i,j,1)\right\} \tag{A.7}$$

For each $i \in I$, the cost of preventive replacement $c_i^{\mathrm{pr}} + V_{n+1}(i,0,0)$ and the term $\frac{\beta}{v}h$ are constant and do not depend on $j \in \hat{J}$. Therefore, if $\Gamma_0 V_n(i,j,1)$ is non-decreasing in $j \in \hat{J}$, then (A.7) is also non-decreasing in $j \in \hat{J}$. $\qquad\qquad\square$

Lemma 2: For each $i \in I$ and $n \in \mathbb{N}_0$:

    (a) $V_n(i,j,1)$ is non-decreasing in $j \in J$

    (b) $\Gamma_0 V_n(i,j,1)$ is non-decreasing in $j \in \hat{J}$

Proof of Lemma 2: We first prove Lemma 2(a) by induction.

Basis: Lemma 2(a) holds for $n = 0$ by (A.1).

Induction Step: For each $i \in I$, assume that $V_n(i,j,1)$ is non-decreasing in $j \in J$ for a given $n > 0$ (induction hypothesis). From (A.3), we have the following for $j \in \hat{J} \setminus \{0\}$:

$$\Gamma_0 V_n(i,j,1) - \Gamma_0 V_n(i,j-1,1)$$

$$= \frac{\beta}{v} \left[\sum_{k \in S, k \neq i} \mu_i Q(i,k) V_n(k,j,1) + \lambda_{ij} V_n(i,j+1,1) + (v - v_{ij}) V_n(i,j,1) \right]$$

$$- \frac{\beta}{v} \left[\sum_{k \in S, k \neq i} \mu_i Q(i,k) V_n(k,j-1,1) + \lambda_{i,j-1} V_n(i,j,1) + (v - v_{i,j-1}) V_n(i,j-1,1) \right]$$

$$= \frac{\beta}{v} \left[\sum_{k \in S, k \neq i} \mu_i Q(i,k) \big(V_n(k,j,1) - V_n(k,j-1,1) \big) + \lambda_{ij} \big(V_n(i,j+1,1) - V_n(i,j,1) \big) \right. \tag{A.8}$$

$$\left. + (v - v_{i,j-1}) \big(V_n(i,j,1) - V_n(i,j-1,1) \big) \right]$$

The second equality follows from the definition of v_{ij} and $v_{i,j-1}$ which are equal to $\mu_i + \lambda_{ij}$ and $\mu_i + \lambda_{i,j-1}$, respectively. We note that $v \geq \max\{v_{ij} \mid i \in I, j \in J\}$ by definition. Therefore, (A.8) is greater than or equal to zero for each $i \in I$ and $j \in \hat{J} \setminus \{0\}$ under the induction hypothesis. Thus, $\Gamma_0 V_n(i,j,1)$ is non-decreasing in $j \in \hat{J}$ for each $i \in I$. Using Lemma 1, $V_{n+1}(i,j,1)$ is non-decreasing in $j \in \hat{J}$ for each $i \in I$.

It remains to prove that $V_{n+1}(i,F,1) - V_{n+1}(i,F-1,1) \geq 0$ for each $i \in I$. For $j = F$, we have:

$$V_{n+1}(i,F,1) = c_i^{\mathrm{cr}} + V_{n+1}(i,0,0) \geq c_i^{\mathrm{pr}} + V_{n+1}(i,0,0) \geq V_{n+1}(i,F-1,1) \quad \text{for all } i \in I, n \in \mathbb{N}_0 \tag{A.9}$$

The first inequality follows from the definition of preventive and corrective cost parameters. The second inequality holds by the definition of $V_{n+1}(i,F-1,1)$. Thus, for each $i \in I$, $V_{n+1}(i,j,1)$ is non-decreasing in $j \in J$, completing the induction and the proof of Lemma 1(a).

From Lemma 1(a), (A.8) is greater than or equal to zero for each $i \in I$, $j \in \hat{J} \setminus \{0\}$, and $n \in \mathbb{N}_0$, implying that Lemma 2(b) holds. □

We note that Lemma 2(b) is required to prove Theorem 2.

Lemma 3: For each $i \in I$ and $n \in \mathbb{N}_0$, if $\Gamma_0 V_n(i,j,0)$ is non-decreasing in $j \in \hat{J}$, then $V_{n+1}(i,j,0)$ is non-decreasing in $j \in \hat{J}$.

Proof of Lemma 3: From (A.2), $V_{n+1}(i,j,0)$ can be re-written as follows for each $i \in I$, $j \in \hat{J}$, and $n \in \mathbb{N}_0$:

$$V_{n+1}(i,j,0) = \min\left\{c_i^{\text{pd}} + V_{n+1}(i,j,1), \Gamma_0 V_n(i,j,0)\right\} \tag{A.10}$$

From Lemma 2(a), $V_{n+1}(i,j,1)$ is non-decreasing in $j \in J$ for each $i \in I$ and $n \in \mathbb{N}_0$. Lemma 3 follows from the definition of $V_{n+1}(i,j,0)$, which is the minimum between two non-decreasing terms in $j \in \hat{J}$. $\qquad\square$

Lemma 4: For each $i \in I$ and $n \in \mathbb{N}_0$:

(a) $V_n(i,j,0)$ is non-decreasing in $j \in J$

(b) $\Gamma_0 V_n(i,j,0)$ is non-decreasing in $j \in \hat{J}$

Proof of Lemma 4: The proof of Lemma 4 is along the same lines as the proof of Lemma 2, replacing, $u = 1$ by $u = 0$, Lemma 1 by Lemma 3, Lemma 2 by Lemma 4, and (A.9) by:

$$V_{n+1}(i,F,0) = c_i^{\text{cd}} + V_{n+1}(i,F,1) \geq c_i^{\text{pd}} + V_{n+1}(i,F,1) \geq V_{n+1}(i,F-1,0) \text{ for all } i \in I, \ n \in \mathbb{N}_0 \qquad\square$$

We note that Lemma 4(b) is required to prove Theorem 3.

Proof of Theorem 1: Theorem 1 follows from Lemma 2(a) and Lemma 4(a). Since they hold for all $n \in \mathbb{N}_0$, they also hold for the infinite horizon function $V(i,j,u)$ where n tends to infinity. $\qquad\square$

Appendix IV. Proof of Theorem 2

In order to prove Theorem 2, we first introduce Lemmas 5 – 7.

Lemma 5: For each $i \in I$, $j \in \hat{J} \setminus \{0\}$, and $n \in \mathbb{N}_0$, if action 2 is optimal in state $(i,j-1,1)$, then action 2 is also optimal in state $(i,j,1)$.

Proof of Lemma 5: Fix operating mode $i \in I$ and $n \in \mathbb{N}_0$. Suppose that $j \in \hat{J} \setminus \{0\}$.

From (A.7), the optimality of action 2 means that the cost associated with action 2 is less than or equal to the cost associated with action 0. Hence, if action 2 is optimal in state $(i,j-1,1)$, then:

$$c_i^{\text{pr}} + V_{n+1}(i,0,0) \leq \frac{\beta}{v}h + \Gamma_0 V_n(i,j-1,1)$$

From Lemma 2(b), we have:

$$c_i^{\text{pr}} + V_{n+1}(i,0,0) \leq \frac{\beta}{v}h + \Gamma_0 V_n(i,j,1)$$

Thus, action 2 is also optimal in state $(i,j,1)$, completing the proof. $\qquad\square$

Lemma 6: The following inequality holds for all $i \in I$, $j \in \hat{J} \setminus \{0\}$, and $n \in \mathbb{N}_0$:

$$V_{n+1}(i,j,1) - V_{n+1}(i,j-1,1) + V_{n+1}(i,j-1,0) \leq \Gamma_0 V_n(i,j,1) - \Gamma_0 V_n(i,j-1,1) + \Gamma_0 V_n(i,j-1,0) \qquad \text{(A.11)}$$

Proof of Lemma 6: Fix operating mode $i \in I$ and $n \in \mathbb{N}_0$. Suppose that $j \in \hat{J} \setminus \{0\}$.

From (A.10), we have $V_{n+1}(i,j-1,0) \leq \Gamma_0 V_n(i,j-1,0)$. Similarly, from (A.7), we have

$V_{n+1}(i,j,1) \leq \dfrac{\beta}{v} h + \Gamma_0 V_n(i,j,1)$. Therefore, if $V_{n+1}(i,j-1,1) = \dfrac{\beta}{v} h + \Gamma_0 V_n(i,j-1,1)$, then (A.11) holds.

Otherwise, if $V_{n+1}(i,j-1,1) = c_i^{\mathrm{pr}} + V(i,0,0)$, then $V_{n+1}(i,j,1) - V_{n+1}(i,j-1,1) = 0$ by Lemma 5. From Lemma 2(b), $\Gamma_0 V_n(i,j,1) - \Gamma_0 V_n(i,j-1,1) \geq 0$ and thus, (A.11) holds. □

Lemma 7: For each $i \in I$ and $n \in \mathbb{N}_0$, the following inequalities hold:

(a) $V_n(i,j,1) - V_n(i,j,0) - V_n(i,j-1,1) + V_n(i,j-1,0) \leq 0$ for $j \in J \setminus \{0\}$ $\qquad$ (A.12)

(b) $\Gamma_0 V_n(i,j,1) - \Gamma_0 V_n(i,j,0) - \Gamma_0 V_n(i,j-1,1) + \Gamma_0 V_n(i,j-1,0) \leq 0$ for $j \in J \setminus \{0\}$ $\qquad$ (A.13)

Proof of Lemma 7: We first prove Lemma 7(a) by induction.

Basis: Lemma 7(a) holds for $n=0$ under (A.1).

Induction Step: For each $i \in I$ and $j \in J \setminus \{0\}$, assume that Lemma 7(a) holds for a given $n > 0$ (induction hypothesis).

- *Case 1:* $j = F$.

By definition, $V_{n+1}(i,F,0) = c_i^{\mathrm{cd}} + V_{n+1}(i,F,1)$. Hence, for $n+1$, the left-hand side of (A.12) is:

$$-c_i^{\mathrm{cd}} - V_{n+1}(i,F-1,1) + V_{n+1}(i,F-1,0) \qquad \text{(A.14)}$$

Equation (A.14) is less than or equal to 0 since $V_{n+1}(i,F-1,0) \leq V_{n+1}(i,F-1,1) + c_i^{\mathrm{pd}}$ and $c_i^{\mathrm{pd}} \leq c_i^{\mathrm{cd}}$.

- *Case 2:* $j \in \hat{J} \setminus \{0\}$ and $V_{n+1}(i,j,0) = c_i^{\mathrm{pd}} + V_{n+1}(i,j,1)$.

For $n+1$, the left-hand side of (A.12) is:

$$-c_i^{\mathrm{pd}} - V_{n+1}(i,j-1,1) + V_{n+1}(i,j-1,0) \qquad \text{(A.15)}$$

From (A.10), (A.15) is less than or equal to 0.

- *Case 3:* $j \in \hat{J} \setminus \{0\}$ and $V_{n+1}(i,j,0) = \Gamma_0 V_n(i,j,0)$.

For $n+1$, the left-hand side of (A.12) becomes:

$$V_{n+1}(i,j,1) - \Gamma_0 V_n(i,j,0) - V_{n+1}(i,j-1,1) + V_{n+1}(i,j-1,0) \tag{A.16}$$

From the definition of operator Γ_0, we have the following:

$$\Gamma_0 V_n(i,j,1) - \Gamma_0 V_n(i,j,0) - \Gamma_0 V_n(i,j-1,1) + \Gamma_0 V_n(i,j-1,0)$$

$$= \frac{\beta}{v} \left[\sum_{k \in S, k \neq i} \mu_i Q(i,k) \big(V_n(k,j,1) - V_n(k,j,0) - V_n(k,j-1,1) + V_n(k,j-1,0) \big) \right.$$
$$+ \lambda_{ij} \big(V_n(i,j+1,1) - V_n(i,j+1,0) - V_n(i,j,1) + V_n(i,j,0) \big)$$
$$\left. + (v - v_{i,j-1}) \big(V_n(i,j,1) - V_n(i,j,0) - V_n(i,j-1,1) + V_n(i,j-1,0) \big) \right] \tag{A.17}$$

From the induction hypothesis, (A.17) is less than or equal to zero for each $i \in I$ and $j \in \hat{J} \setminus \{0\}$. Lemma 6 implies that (A.16) is less than or equal to (A.17). Hence, (A.16) is less than or equal to zero for each $i \in I$ and $j \in \hat{J} \setminus \{0\}$, completing the induction and the proof of Lemma 7(a).

From Lemma 7(a), (A.17) is less than or equal to zero for each $i \in I$, $j \in \hat{J} \setminus \{0\}$, and $n \in \mathbb{N}_0$, implying that Lemma 7(b) holds. □

Proof of Theorem 2: Theorem 2 follows from Lemma 7(a). Since (A.12) holds for all $n \in \mathbb{N}_0$, it also holds for the infinite horizon function $V(i,j,u)$ where n tends to infinity. □

Appendix V.   Proof of Theorem 3

Lemma 8: For each $i \in I$, $j \in \hat{J} \setminus \{0\}$, and $n \in \mathbb{N}_0$, if action 1 is optimal in state $(i,j-1,0)$, then action 1 is also optimal in state $(i,j,0)$.

Proof of Lemma 8: Fix operating mode $i \in I$ and $n \in \mathbb{N}_0$. Suppose that $j \in \hat{J} \setminus \{0\}$.

From (A.10), the optimality of action 1 means that the cost associated with action 1 is less than or equal to the cost associated with action 0. Hence, if action 1 is optimal in state $(i,j-1,0)$, then:

$$c_i^{pd} + V_{n+1}(i,j-1,1) \leq \Gamma_0 V_n(i,j-1,0)$$

From Lemma 4(b), we have:

$$c_i^{pd} + V_{n+1}(i,j-1,1) \leq \Gamma_0 V_n(i,j,0)$$

If $V_{n+1}(i,j-1,1) = c_i^{pr} + V_{n+1}(i,0,0)$, then $V_{n+1}(i,j,1) = c_i^{pr} + V_{n+1}(i,0,0) = V_{n+1}(i,j-1,1)$ from Lemma 5. In this case, it holds that $c_i^{pd} + V_{n+1}(i,j,1) = c_i^{pd} + V_{n+1}(i,j-1,1) \leq \Gamma_0 V_n(i,j,0)$ and thus, action 1 is optimal in state $(i,j,0)$.

Otherwise, if $V_{n+1}(i,j-1,1) = \dfrac{\beta}{v}h + \Gamma_0 V_n(i,j-1,1)$, then:

$$c_i^{\mathrm{pd}} + \frac{\beta}{v}h + \Gamma_0 V_n(i,j-1,1) \le \Gamma_0 V_n(i,j-1,0) \tag{A.18}$$

From Lemma 7(b), we have:

$$\Gamma_0 V_n(i,j,1) - \Gamma_0 V_n(i,j,0) - \Gamma_0 V_n(i,j-1,1) + \Gamma_0 V_n(i,j-1,0) \le 0 \tag{A.19}$$

Inequalities (A.18) and (A.19) imply:

$$c_i^{\mathrm{pd}} + \frac{\beta}{v}h + \Gamma_0 V_n(i,j,1) \le \Gamma_0 V_n(i,j,0)$$

From (A.7), $V_{n+1}(i,j,1) \le \dfrac{\beta}{v}h + \Gamma_0 V_n(i,j,1)$. Thus,

$$c_i^{\mathrm{pd}} + V_{n+1}(i,j,1) \le \Gamma_0 V_n(i,j,0)$$

In this case, action 1 is optimal in state $(i,j,0)$, completing the proof. □

Proof of Theorem 3: The existence of an optimal replacement threshold $\psi(i) \le F$ in operating mode $i \in I$ follows from Lemma 5. The existence of an optimal spare part delivery threshold $\xi(i) \le F$ in operating mode $i \in I$ follows from Lemma 8. Since both Lemmas hold for all $n \in \mathbb{N}_0$, they also hold for the infinite horizon problem where n tends to infinity. □

References

Alagoz, O., Ayvaci, M.U.S., 2010. Uniformization in Markov Decision Processes, in: Cochran, J.J. (Ed.), Wiley Encyclopedia of Operations Research and Management Science. John Wiley & Sons, Inc.

Alam, M., Al-Saggaf, U.M., 1986. Quantitative reliability evaluation of repairable phased-mission systems using Markov approach. IEEE Trans. Rel. 35, 498–503. doi:10.1109/TR.1986.4335529

Armstrong, M.J., Atkins, D.R., 1998. A note on joint optimization of maintenance and inventory. IIE Trans. 30, 143–149. doi:10.1080/07408179808966446

Armstrong, M.J., Atkins, D.R., 1996. Joint optimization of maintenance and inventory policies for a simple system. IIE Trans. 28, 415–424. doi:10.1080/07408179608966288

Çekyay, B., Özekici, S., 2015. Optimal maintenance of semi-Markov missions. Probab. Eng. Inform. Sc. 29, 77–98. doi:10.1017/S0269964814000229

Çekyay, B., Özekici, S., 2012. Optimal maintenance of systems with Markovian mission and deterioration. Eur. J. Oper. Res. 219, 123–133. doi:10.1016/j.ejor.2011.12.037

Çekyay, B., Özekici, S., Cochran, J.J., Cox, L.A., Keskinocak, P., Kharoufeh, J.P., Smith, J.C., 2011. Condition-based maintenance under Markovian deterioration, in: Wiley Encyclopedia of Operations Research and Management Science. John Wiley & Sons, Inc.

Derman, C., 1970. Finite state Markovian decision processes. Academic Press, Inc., Orlando, FL, USA.

Driessen, M., Arts, J., van Houtum, G.J., Rustenburg, J.W., Huisman, B., 2014. Maintenance spare parts planning and control: a framework for control and agenda for future research. Prod. Plan. Control 1–20. doi:10.1080/09537287.2014.907586

Elwany, A.H., Gebraeel, N.Z., 2008. Sensor-driven prognostic models for equipment replacement and spare parts inventory. IIE Trans. 40, 629–639. doi:10.1080/07408170701730818

Elwany, A.H., Gebraeel, N.Z., Maillart, L.M., 2011. Structured replacement policies for components with complex degradation processes and dedicated sensors. Oper. Res. 59, 684–695. doi:10.1287/opre.1110.0912

Esary, J.D., Ziehms, H., 1975. Reliability analysis of phased missions, in: Proceedings of the Conference on Reliability and Fault Tree Analysis. Presented at the SIAM, pp. 213–236.

Flint, P., 1995. Too much of a good thing: better inventory management could save the industry millions while improving reliability. Air Transport World 32, 103–106.

Flory, J.A., Kharoufeh, J.P., Abdul-Malak, D.T., 2015. Optimal replacement of continuously degrading systems in partially observed environments. Nav. Res. Log. 62, 395–415. doi:10.1002/nav.21638

Harris, T.A., Barnsby, R.M., 2001. Life ratings for ball and roller bearings. Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology 215, 577–595. doi:10.1243/1350650011543817

Kabir, A.B.M.Z., Al-Olayan, A.S., 1996. A stocking policy for spare part provisioning under age based preventive replacement. Eur. J. Oper. Res. 90, 171–181. doi:10.1016/0377-2217(94)00246-0

Kawai, H., Koyanagi, J., Ohnishi, M., 2002. Optimal maintenance problems for Markovian deteriorating systems, in: Osaki, P.S. (Ed.), Stochastic Models in Reliability and Maintenance. Springer Berlin Heidelberg, pp. 193–218.

Kharoufeh, J.P., Solo, C.J., Ulukus, M.Y., 2010. Semi-Markov models for degradation-based reliability. IIE Trans. 42, 599–612. doi:10.1080/07408170903394371

Kim, K., Park, K.S., 1994. Phased-mission system reliability under Markov environment. IEEE Trans. Rel. 43, 301–309. doi:10.1109/24.295013

Kolesar, P., 1966. Minimum cost replacement under Markovian deterioration. Manage. Sci. 12, 694–706.

Kurt, M., Kharoufeh, J.P., 2010. Optimally maintaining a Markovian deteriorating system with limited imperfect repairs. Eur. J. Oper. Res. 205, 368–380. doi:10.1016/j.ejor.2010.01.009

Lam, M., 1995. An introduction to airline maintenance. Handbook of Airline Economics Aviation Week Group, 397–406.

Makis, V., Jiang, X., 2003. Optimal replacement under partial observations. Math. Oper. Res. 28, 382–394. doi:10.1287/moor.28.2.382.14484

Muckstadt, J.A., 2005. Analysis and algorithms for service parts supply chains. Springer, New York, NY :

Mura, I., Bondavalli, A., 1999. Hierarchical modeling and evaluation of phased-mission systems. IEEE Trans. Rel. 48, 360–368. doi:10.1109/24.814518

Özekici, S., 1995. Optimal maintenance policies in random environments. Eur. J. Oper. Res., OR Models for Maintenance Management and Quality Control 82, 283–294. doi:10.1016/0377-2217(94)00264-D

Puterman, M.L., 1994. Markov Decision Processes: Discrete Stochastic Dynamic Programming, 1st ed. John Wiley & Sons, Inc., New York, NY, USA.

Tinga, T., 2010. Application of physical failure models to enable usage and load based maintenance. Reliab. Eng. Syst. Safe. 95, 1061–1075. doi:10.1016/j.ress.2010.04.015

Tinga, T., Janssen, R., 2013. The interplay between deployment and optimal maintenance intervals for complex multi-component systems. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 227, 227–240. doi:10.1177/1748006X13480743

Turan, O., Ölçer, A.İ., Lazakis, I., Rigo, P., Caprace, J.D., 2009. Maintenance/repair and production-oriented life cycle cost/earning model for ship structural optimisation during conceptual design stage. Ships Offshore Struc. 4, 107–125. doi:10.1080/17445300802564220

Ulukus, M.Y., Kharoufeh, J.P., Maillart, L.M., 2012. Optimal replacement policies under environment-driven degradation. Probab. Eng. Inform. Sc. 26, 405–424. doi:10.1017/S0269964812000083

Unctad, 2013. Review of the maritime transport 2013, United Nations Publication. United Nations Conference on Trade and Development, Geneva: Switzerland.

Van Horenbeek, A., Buré, J., Cattrysse, D., Pintelon, L., Vansteenwegen, P., 2013. Joint maintenance and inventory optimization systems: A review. Int. J. Prod. Econ., Focusing on Inventories: Research and Applications 143, 499–508. doi:10.1016/j.ijpe.2012.04.001

Van Houtum, G.J., Kranenburg, B., 2015. Spare Parts Inventory Control under System Availability Constraints, International Series in Operations Research & Management Science. Springer US, Boston, MA.

Waldmann, K.-H., 1983. Optimal replacement under additive damage in randomly varying environments. Nav. Res. Log. 30, 377–386. doi:10.1002/nav.3800300302

Wang, L., Chu, J., Mao, W., 2009. A condition-based replacement and spare provisioning policy for deteriorating systems with uncertain deterioration to failure. Eur. J. Oper. Res. 194, 184–205. doi:10.1016/j.ejor.2007.12.012

Wang, L., Jian Chu, Weijie Mao, 2008. An optimum condition-based replacement and spare provisioning policy based on Markov chains. J of Qual in Maintenance Eng 14, 387–401. doi:10.1108/13552510810909984

Zhang, Z., Wu, S., Li, B., Lee, S., 2013. Optimal maintenance policy for multi-component systems under Markovian environment changes. Expert Syst. Appl. 40, 7391–7399. doi:10.1016/j.eswa.2013.07.003